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Optimal control of linear systems with large and variable input delays

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ABSTRACT

This paper proposes an optimal control law for linear systems affected by input delays. Specifically we prove that when the delay functions are known it is possible to generate the optimal control for arbitrarily large delay values by using a DDE without distributed terms. The solution can be seen as a chain of predictors whose size depends on the maximum delay.

1. Introduction

The control and state estimation problems in presence of input or measurement delays have received growing attention due to its relevance in many emerging applications such as network control systems where delays must be taken into account in the transmission of input signals [1–5]. In the context of continuous-time systems it is known that the general solution to the control problem can be provided by means of operators on infinite dimensional spaces [6,7]. The optimal control problem has been studied and solved in this context [8–11]. In [12] it is shown that a suitable state feedback control which involves the integral of the past control law solves the infinite horizon optimal control problem for linear time-invariant systems with single input time-delay. In [13] the finite horizon optimal control problem of time-varying linear systems with multiple constant input delays has been solved.

However infinite dimensional approaches are difficult to implement, as they require to compute an integral term on-line. As explained in [7], obtaining this term as the solution to a differential equation must be discarded because it involves unstable pole-zero cancellation when the original system in unstable. The numerical quadrature rules to evaluate the integral term require special attention in the implementation [14] or approximation methods that yields suboptimal solutions [15,16].

Recently, finite dimensional or memoryless methods, meaning that the input is generated by an instantaneous state feedback as in the delay-free case, have been proposed for linear systems [17–23]. Some of these methods consider also time-varying delays. In [23] the LQ problem is solved with a memoryless feedback for known delay functions satisfying a delay bound. In this paper we extend the approach of [23] in two directions. The first extension is to overcome the problem of the delay bound by introducing a chain of predictors. In this way it is possible to generate a finite-dimensional stabilizing input for arbitrarily large delays, a result previously available only for systems not exponentially unstable [18]. The second extension is to extend the approach to the case of multiple delay functions, each acting on a specific input.

A basic assumption of our work is that the delay functions are known. This may be considered as a strong assumption in many practical situations, but we show that it is the price to pay for having the same performance as in the optimal delay free case. On the other hand, this assumption is not specific to our work but to any exact prediction/control approach in presence of delay. Consequently a contribution of this paper might also be considered to be the study of the conditions on the input delay under which the system can be optimally controlled as if the delay was not present. In this sense, we show that the *size* of the delay is not relevant as long as the delay is known and well behaved in a precise sense.

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We introduce the problem and the delay assumption in Section 2. The approach is illustrated in 3. With the aim of making easier to read the paper we first introduce the case of large delays with a single input in Section 3.1 before giving the solution for the more general case of multiple delayed inputs in Section 3.2. Section 4 considers output feedback control and Section 5 validates the method.

Notation. $\mathbb{R}_+$ is set of non-negative reals. $\sigma(A)$ denotes the set of eigenvalues of the square matrix A, and $\mu(A)$ the largest real part of its eigenvalues. $\Re(z)$ is the real part of $z \in \mathbb{C}$. $\mathcal{C}^n_\delta$ denotes the space of continuous functions that map $[-\delta, 0]$ in $\mathbb{R}^n$, with the uniform convergence norm, denoted $\|\cdot\|_\infty$.

2. Problem statement

In this paper we consider the following linear system with multiple input delays

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \sum_{i=1}^{p} B_i u_i(t-\delta_i(t)), \quad t > 0 \\ x(0) &= x_0 \\ u_i(t-\delta_i(t)) &= 0, \quad t < 0, \end{aligned} \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}$, $i = 1, \ldots, p$. Notice that system (1) has multiple inputs and each input has only one delay, differently from the case of single input with multiple delays considered, among others, by [7,24].

The delay functions $\delta_i : \mathbb{R}_+ \to [0, \bar{\delta}]$ are uniformly bounded by the known constant $\bar{\delta}$. We denote $\psi_i(t)_i = t - \delta_i(t)$ the time point at which the control signal applied at time t has been generated, that is, $u(t-\delta_i(t)) = u(\psi_i(t))$. Obviously, $\psi_i(t) \le t$. We require that the following two assumptions hold.

Assumption 1. Let $B = [B_1, \ldots, B_p]$. Then the pair (A, B) is controllable.

Assumption 2. The functions $\psi_i(t)$ are bijective, *i.e.* for $\forall t^* \ge \psi_i(0)\ \exists! t_i : t^* = \psi_i(t_i)$, $i = 1, \ldots, p$. Moreover, the inverse functions $t_i = \psi_i^{-1}(t^*)$ are known at time t^*.

Assumption 2 is necessary to ensure that when generating the input $u_i(\psi_i(t))$ at time $\psi_i(t)$ there is a known and unique time t_i at which the input will be received. Practical situation in which Assumption 2 holds are constant or continuous, slowly delays that satisfy $|\dot{\delta}(t)| < 1$. However, continuity or differentiability of $\delta_i(t)$ are not implied by Assumption 2, thus $\delta_i(t)$ could be fast-varying or even not continuous, as long as $\psi_i(t)$ are invertible and known functions (see for example $\delta(t)$ in Fig. 2). Assumption 2 is quite standard in this setting [20]. The only alternative to it is to use robust control with unknown input delay, but in this case the control is no longer optimal [1]. We look for the optimal controls $u_i(t)$ with respect to a quadratic functional in the infinite-horizon case, that can be written as

$$J = \int_0^\infty x^T(t)Qx(t) + \sum_{i=1}^{p} R_i u_i^2(\psi_i(t))dt, \tag{2}$$

where Q is an appropriate positive-definite symmetric matrix and R_i are positive scalars.

It is well known that, at least for constant delays $\delta_i(t) = \delta$, the optimal control of (1) can be achieved through the computation of distributed terms ([1], p. 202). Instead, we explore solutions based on optimal instantaneous state feedback of the kind

$$u_i(\psi_i(t)) = -K(\psi_i(t))x(\psi_i(t)), \tag{3}$$

and we show that the optimal control can be generated with such finite-dimensional feedback, even in presence of variable delays.

Remark 1. A different but related problem is when the delay affects the state measurement, but not the input, that is, at time t the input $u_i(t)$ can be immediately applied but must be generated using delayed information about the state, $\dot{x}(t) = Ax(t) + \sum_{i=1}^{p} B_i u_i x(t-\delta_i(t))$. The instantaneous state feedback (3) becomes $u_i(t) = -K(\psi_i(t))x(\psi_i(t))$. Thus, the method described in this paper can be applied also in this case and Assumption 2 can be relaxed to the knowledge of $\delta(t)$ at t.

3. Predictors for input delays

3.1. Systems with single input delay

In order to make the presentation easier we consider in the first place the case of a single delay and scalar input,

$$\dot{x}(t) = Ax(t) + Bu(\psi(t)) \tag{4}$$

with $u(t)$ scalar, $u(\psi(t)) = 0$ for $t < 0$, and $x(0) = x_0$.

Given a square matrix $A \in \mathbb{R}^{n\times n}$ we introduce the following scalar function of vector $K \in \mathbb{R}^n$ and scalar $\alpha \in \mathbb{R}_+$

$$\omega_A(\alpha, K) := \max\left\{\delta \in \mathbb{R}_+ : \int_0^\delta \left|Ke^{(A-BK)s}B\right| e^{\alpha s} ds \le 1\right\}, \tag{5}$$

with the convention that $\omega_A(\alpha, K) = \infty$ if the inequality is always satisfied. If $\omega_A(\alpha, K) < \infty$, due to the structure of the integrand in (5), larger values of α correspond to smaller values of $\omega_A(\alpha, K)$ and vice-versa. It is possible to show [21] that $\omega_A(\alpha, K)$ does not depend on B. but only on α, $\sigma(A)$ and $\sigma(A - BK)$, and it is therefore invariant to a change of coordinates.

The optimal control problem for system (4) was solved in [23] for delay functions uniformly bounded. We report the main result.

Theorem 1 ([23]). *Consider system (4) with the pair (A, B) controllable, $\delta(t) \le \bar{\delta}$ that satisfies Assumption 2 and the cost functional (2). Let $\bar{K}^o = R^{-1}B^TP$ be the optimal gain with no input delay, P steady-state solution of the Riccati equation*

$$A^TP + PA - PBR^{-1}B^TP + Q = 0, \tag{6}$$

and $\bar{A} = A - B\bar{K}^o$. If the delay bound satisfies $\bar{\delta} < \omega_A(-\mu(\bar{A}), \bar{K}^o)$, then the optimal control law is

$$u(\psi(t)) = \begin{cases} -\bar{K}^o e^{\bar{A}t}x_0, & t < \bar{\delta}, \\ -\bar{K}^o e^{\bar{A}(t-\psi(t))}x(\psi(t)), & t \ge \bar{\delta}. \end{cases} \tag{7}$$

Moreover the value of J for (4) with (7) is $x_0^TPx_0$.

In (7), by definition, $t - \psi(t) = \delta(t)$. In the time coordinate of the controller, control law (7) can be written, for $t \ge \bar{\delta}$

$$u(t) = -\bar{K}^o e^{\bar{A}(\psi^{-1}(t)-t)}x(t), \tag{8}$$

where $\psi^{-1}(t) - t = \delta(\psi^{-1}(t))$ is the delay with which the plant will receive the input, and $\psi^{-1}(t)$ is known in virtue of Assumption 2. From now on we use the time coordinate of the plant. It may be noticed that the idea behind (7) is to use $e^{\bar{A}\delta(t)}x(t-\delta(t))$ as a predictor of $x(t)$ This would yield, for $t \ge \bar{\delta}$, $u(\psi(t)) = -\bar{K}^o x(t) = u^o(t)$, where $u^o(t)$ is the optimal input for the delay-free case. If this finite-dimensional predictor works well, the optimal evolution is therefore the same as in the delay-free case. Theorem 1 provides a sufficient (sometimes necessary, see [23]) delay bound for the predictor. Our aim is to extend this solution to delays that are larger than $\omega_A(\bar{K}^o)$.

We resort to a *chain* of predictors, each in charge of extending the prediction provided by the exponential of $\bar{A}$ to a fraction