Systems & Control Letters 89 (2016) 16–23

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Adaptive iterative learning control for high-order nonlinear multi-agent systems consensus tracking

Xu Jin

The Daniel Guggenheim School of Aerospace Engineering, Georgia Institute of Technology, 270 Ferst Drive, Atlanta, GA, 30332-0150, United States

ARTICLE INFO

Article history:
Received 26 May 2015
Received in revised form
25 November 2015
Accepted 9 December 2015
Available online 4 January 2016

Keywords:
Consensus
Adaptive iterative learning control
Fault tolerant
Multi-agent system
Nonlinear system

ABSTRACT

In this work, we present a new distributed adaptive iterative learning control (AILC) scheme for a class of high-order nonlinear multi-agent systems (MAS) under alignment condition with both parametric and nonparametric system uncertainties, where the actuators may be faulty and the control input gain functions are not fully known. Backstepping design with the composite energy function (CEF) structure is used in the discussion. Through rigorous analysis, we show that under this new AILC scheme, uniform convergence of agents output tracking error over the iteration domain is guaranteed. In the end, an illustrative example is presented to demonstrate the efficacy of the proposed AILC scheme.

1. Introduction

Multi-agent systems (MAS), or networked cooperative systems, have attracted considerable attention from the research community in the past decade or so, due to their widespread applications and cross-disciplinary nature [1–3]. In particular, there are many applications in which tracking a desired trajectory by all the agents is desirable. In such problems, the desired trajectory can be seen as generated by a virtual leader, and is not influenced by all the other agents in the network. Furthermore, the full information of the desired trajectory may only be available to a subset of the agents in the network. In the literature, this is usually called leader-following consensus [4], model reference consensus [5], leader-following control [6] and so on.

It is well known now that iterative learning control (ILC) is effective in handling repeated control processes. Because of its structural simplicity and effective learning ability in the process of controller design, ILC has been widely used in industries for control of repetitive motions, such as robotic manipulators, hard disk drives, chemical plants, and so forth [7–10]. By taking advantage of the repetitive nature in the learning process, ILC algorithms can improve the tracking performance progressively, so that to achieve perfect tracking asymptotically or exponentially as the iteration number increases. The use of ILC for MAS is a relatively new field and has been reported in a few previous works in the literature. In [11], a high-order internal model (HOIM) based ILC scheme for MAS formation is studied. In [12], a distributed ILC scheme is developed for MAS with switching topology. In [13], the finite-time output consensus problem of MAS is considered with ILC schemes. However, these works are based on the contraction mapping approach. This approach relies on the restrictive assumption of perfect resetting, or known as identical initial condition, which means that the tracking error at the start of each iteration should be reset to zero perfectly, but this condition may not be practically implementable. A slight mismatch in the initial condition may result in divergence of tracking error [14]. Among numerous efforts that aim at mitigating this constraint, the alignment condition appears to be practical for a number of applications, in which the final state of the previous iteration becomes the initial state of the current iteration. The reference trajectory is spatially closed, meaning that the starting point of the reference trajectory is also the end point in each iteration. ILC under alignment condition for MAS consensus tracking have been studied in several works, for example, [15] for first order MAS, and [16] for second order systems, all for systems with undirected graph topologies. Besides, in these works the system uncertainty terms are assumed to satisfy parametric forms, which exclude the uncertainties or disturbances that may only be norm-bounded. High-order nonlinear MAS with direct graph topologies have not been rigorously studied in the literature, and remain as an open question for further studies. The work [17] is one of the few that address the high-order MAS problems in ILC domain. However, this work also considers undirected graph topology only. Furthermore, the control input

E-mail address: xu.jin@gatech.edu.

http://dx.doi.org/10.1016/j.sysconle.2015.12.009
0167-6911/

gain function is totally known, and the agents actuator system is assumed to be free from potential faults, which limits the practical application into real engineering systems. To the best of our knowledge, there is no previous work that addresses high-order nonlinear multi-agent systems under directed graph topology with uncertain control input gain functions and actuator faults in the ILC frame, without assuming the identical initial condition.

In this work, to address all the important and challenging issues mentioned above, we present a novel distributed adaptive iterative learning control (AILC) scheme for MAS output consensus tracking problems for a class of high-order nonlinear MAS with a directed graph topology under alignment condition. Both parametric and nonparametric system uncertainties are considered. Full information about the desired output trajectory is only available to a subset of all the agents. The control input gain functions for the agents are not totally known. Furthermore, the agents actuators may be subject to both multiplicative and additive actuator faults, where the multiplicative faults may represent the loss of efficiency over time, and additive actuator faults may represent the faults presented in the control input channel. Nonparametric uncertainties such as norm-bounded nonlinear uncertainties can be effectively handled, where the unknown bounds of the uncertainties can be handled by a corresponding ILC update law. Through rigorous analysis, we show that under this new AILC scheme, uniform convergence of the agents output tracking error over the iteration domain is guaranteed. The main contributions of this work can be summarized as follows: (1) a novel distributed AILC scheme for high-order nonlinear MAS with a directed graph topology has been proposed to guarantee uniform tracking error convergence over the iteration domain; (2) the proposed algorithm can effectively deal with both parametric and nonparametric uncertainties, where the parametric uncertainties contain time-varying functions, and the nonparametric uncertainties satisfy norm-bounded conditions; (3) both multiplicative and additive actuator faults are considered; (4) state-dependent control input gain functions which are not fully known can be effectively dealt with.

2. Basic graph theory and notations

A weighted graph is represented by $G = (V, E)$, where $V = \{v_1, \ldots, v_N\}$ is a nonempty set of nodes/agents, $E \subseteq V \times V$ is the set of edges/arcs. $(v_i, v_j) \in E$ indicates that agent j can get information from agent i, but not necessarily vice versa. In this case, agent i is called a neighbor of agent j. The topology of a weighted graph G is often represented by the adjacency matrix $A = [a_{ij}] \in \mathscr{R}^{N\times N}$, where $a_{ij} = 1$ if $(v_j, v_i) \in E$, otherwise $a_{ij} = 0$. Throughout this work, it is assumed that $a_{jj} = 0, j = 1, \ldots, N$, and the topology is fixed, i.e., A is time-invariant. G is a directed graph, or digraph in short. Define $d_i = \sum_{j=1}^{N} a_{ij}$ as the weighted in-degree of node i and $D = diag(d_1, \ldots, d_N) \in \mathscr{R}^{N\times N}$ as the in-degree matrix. The graph Laplacian matrix is $L = [l_{ij}] = D - A \in \mathscr{R}^{N\times N}$.

In this paper, $|\cdot|$ is the absolute value of a real number; $\|\cdot\|$ is the Euclidean norm of a vector; matrix $P > 0 (P \geq 0)$ means P is positive definite (positive semidefinite); θ^T denotes the transpose of the vector θ.

3. Problem formulation

Consider $N (N > 2)$ agents with distinct high-order nonlinear dynamics. Dynamics of the jth $(j = 1, \ldots, N)$ agent at the kth iteration is described as

$$\begin{aligned}\dot{x}_{k,j,i}(t) &= x_{k,j,i+1}(t), \quad i = 1, \ldots, n-1,\\ \dot{x}_{k,j,n}(t) &= b_j(\bar{x}_{k,j})(\rho_j(t)u_{k,j}(t) + \phi_j(t))\\ &\quad + d_j(\bar{x}_{k,j}, t) + \Theta_j^T(t)F_j(\bar{x}_{k,j}),\\ y_{k,j}(t) &= x_{k,j,1}(t), \end{aligned} \tag{1}$$

where $k = 1, 2, \ldots$ is the iteration index, $t \in [0, T], T > 0$ represents the operation time in each iteration. $x_{k,j,i}(t) \in \mathscr{R}$ is the ith state variable of the agent j at the kth iteration, $\bar{x}_{k,j} = [x_{k,j,1}, \ldots, x_{k,j,n}]^T \in \mathscr{R}^n$ is the state vector of the agent j. $d_j(\bar{x}_{k,j}, t) \in \mathscr{R}$ are bounded nonparametric uncertainties. $\Theta_j(t) \in \mathscr{R}^m$ are the unknown time-varying functions that are iteration independent, $F_j(\bar{x}_{k,j}) \in \mathscr{R}^m$ are the known nonlinear state dependent functions. $y_{k,j}(t)$ is the jth agent output of at the kth iteration. $\rho_j(t)u_{k,j}(t) + \phi_j(t) \in \mathscr{R}$ is the actuator signal of the node j, where $u_{k,j}(t) \in \mathscr{R}$ is the control action to be designed. Here we consider both multiplicative actuator faults $\rho_j(t)$ and additive actuator faults $\phi_j(t)$. If $\rho_j(t) = 1$ and $\phi_j(t) = 0$, we say that the jth agent is free from actuator faults [18].

Remark 1. The agents' model (1) is a widely discussed formulation in the nonlinear system literature, which is also sometimes referred to as the Brunovsky canonical form. For example, certain dynamic models for the flight of helicopters [19], chaotic systems [20,21], electro-hydraulic systems [22], can be converted into the form of (1).

Remark 2. The formulation of nonparametric term $d_j(\bar{x}_{k,j}, t)$ and parametric term $\Theta_j^T(t)F_j(\bar{x}_{k,j})$ represents a wide range of system uncertainties. In the adaptive fuzzy control literature, it is often assumed that a continuous function $f(x)$ defined on a compact set can be represented to an arbitrary degree of precision as $f(x) = \Theta^T F(x) + \varepsilon(x)$, where in this case $F(x)$ is known and called the base functions, Θ is the ideal neural network weight vector that is unknown and constant, and $\varepsilon(x)$ is the neural network approximation error which is unknown and can be arbitrarily small [23,24]. Compared with such a formulation, the formulation presented in (1) is more general, as the parametric unknown term can be time-varying, and the nonparametric unknown term can be both time and state dependent.

The desired output trajectory for the MAS (1) is defined as

$$y_r(t) = \sum_{l=1}^{n_r} \omega_{r,l}(t)\psi_{r,l} + \varphi_{r,2} = \varpi_{r,1}^T(t)\varphi_{r,1} + \varphi_{r,2}, \tag{2}$$

where $\varpi_{r,1}(t) = [\omega_{r,1}(t), \ldots, \omega_{r,n_r}(t)]^T \in \mathscr{R}^{n_r}$ is a vector of basis functions that is accessible by all agents, whereas $\varphi_{r,1} = [\psi_{r,1}, \ldots, \psi_{r,n_r}]^T \in \mathscr{R}^{n_r}$ and $\varphi_{r,2} \in \mathscr{R}$ are constant parameters that are only accessible by certain agents. Collectively, we can express (2) as $y_r(t) = \varpi_r^T(t)\varphi_r$, where $\varpi_r(t) = [\varpi_{r,1}^T(t), 1]^T \in \mathscr{R}^{n_r+1}$, and $\varphi_r = [\varphi_{r,1}^T, \varphi_{r,2}]^T \in \mathscr{R}^{n_r+1}$.

Under the alignment condition, the desired output trajectory satisfies the spatial closeness condition, meaning $y_r(0) = y_r(T)$. Furthermore, we also have $y_r^{(l)}(0) = y_r^{(l)}(T)$ for $l = 1, \ldots, n-1$, where $y_r^{(l)}(t)$ denotes the lth order derivative of $y_r(t)$ with respective to time. This means, in each iteration of operation, the desired system trajectory will end where it will start from in the next iteration. Alignment condition also implies that the resetting effort is not necessary at the beginning of each iteration, which is unlike the traditional identical initial condition, where the initial tracking error has to be reset to zero at the start of each iteration. Hence, under alignment condition, $x_{k,j,i}(0) = x_{k-1,j,i}(T)$ for $j = 1, \ldots, N$ and $i = 1, \ldots, n$, which means the system will start from where it stopped in the previous iteration.

Remark 3. The formulation of the desired output trajectory (2) can be seen in many works like [25,26]. Notice that $y_r(t)$ can be regarded as periodic with period T, since $y_r(0) = y_r(T)$, and $y_r(t)$ is iteration independent. We know from trigonometric form of the Fourier series, that $y_r(t)$ can be written as $y_r(t) = a_0 + \sum_{l=1}^{\infty}(a_l \cos(\frac{2\pi l}{T}t) + b_l \sin(\frac{2\pi l}{T}t))$, where a_0, a_l and b_l are called