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Observability of singular time-delay systems with unknown inputs

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ABSTRACT

In this manuscript we give sufficient conditions guaranteeing the observability of singular linear systems with commensurable delays affected by unknown inputs appearing in both the state equation and the output equation. These conditions allow for the reconstruction of the entire state vector using past and actual values of the system output. The obtained conditions coincide with known necessary and sufficient conditions of singular linear systems without delays.

1. Introduction

The description of a variety of practical systems by means of singular systems, also called descriptive, implicit, or differential algebraic systems, has been shown to be useful since several decades ago as it is well explained in [1]. Such systems, as many others, may contain time delay terms in the state, input, and/or system output, a compendium of new researching results for singular systems with time delays has been recently published, Gu et al. [2]. Despite the increasing research on problems such as solvability, stability and controllability, up to the authors knowledge, there is quite a few works dedicated to the study of the observability of singular systems with time delays, with or without inputs. For singular systems with a time-delay in the state (without inputs), a condition is found in [3] that ensures the observability of the system (interpreted as the reconstruction of the initial condition). However, such a condition seems to be difficult to check since it involves an integral on time that depends on a parameter within an infinite set. More papers, not too many, can be found addressing the observer design problem for singular linear time-delay systems with unknown inputs (SLSUI). In [4], a Luenberger-like observer design is proposed by using a virtual discrete time system with the matrices of the originally considered continuous time system. In [5] a Luenberger-like observer is proposed for singular delayed linear systems with unknown inputs not affected by time-delays.

Considering singular linear systems with unknown inputs (without delays), in [6], conditions under which the trajectory of the state vector can be reconstructed were given, as well as a formula to reconstruct the state in terms of the system output and a finite number of its derivatives, provided any state trajectory is smooth. Regarding linear systems with commensurate delays and affected by unknown inputs, sufficient conditions allowing for the reconstruction of the state vector were obtained in [7]. Hence, this paper takes advantage of those both results, to tackle the observability problem of a general class of singular linear time-delay systems with unknown inputs. The main contribution of this paper is the obtaining of sufficient (checkable) conditions allowing for the reconstruction of the trajectory of the state vector.

The remainder of the paper is organized as follows. In Section 2, the class of singular systems considered along the manuscript is presented, as well as the problem we aim to address. The main result is given in Section 3, which in turn is divided into 4 subsections. In Section 3.1, the studied system is transformed, and the state vector is split into two parts so that previous results may be used. Observability conditions for the first part of the transformed state vector are obtained straightforwardly in Section 3.2. Observability conditions are deduced for the second part of the transformed state vector in Section 3.3. Finally, with the results of Sections 3.2 and 3.3, observability conditions for the original system are given in Section 3.4. In Section 4, a formula for the reconstruction of the state vector is obtained. Finally, an academic example that illustrates the theoretical obtained results is given in Section 5.

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Notation. $\mathbb{R}$ is the field of real numbers. $\mathbb{R}[\delta]$ is the polynomial ring over the real field $\mathbb{R}$. I_n is the identity matrix of dimension n by n. A square matrix $A(\delta)$ with terms in $\mathbb{R}[\delta]$ is called unimodular if its determinant is a nonzero constant. A matrix $A(\delta)$ of n by m dimension is called left invertible if there exists a matrix, denoted by $A^{+}(\delta)$, such that $A^{+}(\delta)A(\delta) = I_m$. For a matrix $F(\delta)$ (over $\mathbb{R}[\delta]$), rank $F(\delta)$ denotes the rank of $F(\delta)$ over $\mathbb{R}[\delta]$. The degree of a polynomial $p(\delta) \in \mathbb{R}[\delta]$ is denoted by $\deg p(\delta)$. For a matrix $M(\delta)$, $\deg M(\delta)$ (the degree of $M(\delta)$) is defined as the maximum degree of all the entries $m_{ij}(\delta)$ of $M(\delta)$. The limit from below of a time valued function is denoted as $f(t_{-})$.

2. System description and problem formulation

The system considered along the paper belongs to the class of delay systems whose dynamics is governed by the following equations

$$E\dot{x}(t) = \sum_{i=0}^{k_a} A_i x(t-ih) + \sum_{i=0}^{k_b} B_i w(t-ih) \tag{1a}$$

$$y(t) = \sum_{i=0}^{k_c} C_i x(t-ih) + \sum_{i=0}^{k_d} D_i w(t-ih) \tag{1b}$$

where h is a positive real number. At a time t, $x(t) \in \mathbb{R}^n$, $y(t) \in \mathbb{R}^p$, and $w(t) \in \mathbb{R}^m$. The initial condition $\varphi(t)$ is a piecewise continuous function $\varphi(t) : [-kh, 0] \to \mathbb{R}^n$ ($k = \max\{k_a, k_b, k_c, k_d\}$), hence $x(t) = \varphi(t)$ on $[-kh, 0]$. We also consider that $w(\cdot) \in \mathscr{D}^m$, which is the set of admissible vector functions mapping from $\mathbb{R}$ to $\mathbb{R}^m$ for which (1a) has a solution (such a solution might be not unique).

All matrices A_i, B_i, C_i, and D_i are constant of suitable dimension. It is assumed that the rank of $E \in \mathbb{R}^{n\times n}$ is strictly less than n, i.e., E is not invertible. The main goal of this work is to find testable conditions under which the estimation of $x(t)$ may be carried out based on the knowledge of the system output $y(t)$. For the studied system (1) the following assumptions are imposed.

A1. Every solution of (1a) is piecewise differentiable.

Now, let us introduce the following definition dealing with the observability of the system.

Definition 1. System (1) is said to be backward unknown input observable (**BUIO**) on $[t_1, t_2]$ if for each $\tau \in [t_1, t_2]$ there exist $t_1' < t_2' \le \tau$ such that, for every input $w \in \mathscr{D}^m$ and every initial condition φ,

$$y(t;\varphi, w) = 0 \quad \text{for all } t \in \left[t_1', t_2'\right] \text{ implies } x(\tau_{-};\varphi, w) = 0.$$

In the previous definition we use the word *backward* since for the reconstruction of $x(\tau)$ we require the use of values of $y(t)$ obtained at time instants previous than or equal to τ, which obeys the requirement of causality imposed in practice. Thus, based on the previous definition, the aim of this paper is to find out conditions under which the system (1) is BUIO.

Following a standard way, we define the backward shift operator $\delta : x(t) \mapsto x(t-\tau)$. Hence, the system (1) may be rewritten as follows,

$$E\dot{x}(t) = A(\delta)x(t) + B(\delta)w(t) \tag{2a}$$

$$y(t) = C(\delta)x(t) + D(\delta)w(t) \tag{2b}$$

where $A(\delta) = \sum_{i=0}^{k_a} A_i\delta^i$, and $B(\delta) = \sum_{i=0}^{k_b} B_i\delta^i$, $C(\delta) = \sum_{i=0}^{k_c} C_i\delta^i$, and $D(\delta) = \sum_{i=0}^{k_d} D_i\delta^i$ are defined likewise. Using this notation, the matrices of system (2) may be considered as matrices over the polynomial ring $\mathbb{R}[\delta]$, which allow us to use the tools of the algebra of polynomials.

3. Sufficient observability conditions

3.1. Transformed system

By using a simple matrix decomposition, the matrix E can be transformed into the following form

$$\begin{bmatrix} I_q & 0 \\ 0 & 0 \end{bmatrix} \tag{3}$$

that is, there exist two invertible matrices (over $\mathbb{R}$), denoted as R and S, such that RES is equal to the matrix in (3). Then, (2) can be transformed to the following form:

$$\dot{z}_1(t) = A_{11}(\delta)z_1(t) + A_{12}(\delta)z_2(t) + B_1(\delta)w(t) \tag{4a}$$

$$0 = A_{21}(\delta)z_1(t) + A_{22}(\delta)z_2(t) + B_2(\delta)w(t) \tag{4b}$$

$$y(t) = C_1(\delta)z_1(t) + C_2(\delta)z_2(t) + D(\delta)w(t) \tag{4c}$$

where $\begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} = S^{-1}x(t)$, $z_1 \in \mathbb{R}^q$, $z_2 \in \mathbb{R}^{n-q}$. In order to study the observability of the whole system, we will start for studying the observability of z_1 and z_2. For this, we consider z_2 as an input vector. Let $\mathscr{D}^{n-q+m}$ be the set of functions mapping from $\mathbb{R}$ to $\mathbb{R}^{n-q+m}$ for which (4a)–(4b) have a solution.

Definition 2. The state $z_1(t)$ of (4) is said to be **BUIO** on $[t_1, t_2]$ if for each $\tau \in [t_1, t_2]$ there exist $t_1' < t_2' \le \tau$ such that, for every input $\left[z_2^T \quad w^T\right]^T \in \mathscr{D}^{n-q+m}$ and every initial condition φ,

$$y(t;\varphi, z_2, w) = 0 \quad \text{for all } t \in \left[t_1', t_2'\right]$$
$$\text{implies } z_1(\tau_{-};\varphi, z_2, w) = 0. \tag{5}$$

Definition 3. The state $z_2(t)$ of (4) is said to be **BUIO** on $[t_1, t_2]$ if for each $\tau \in [t_1, t_2]$ there exist $t_1' < t_2' \le \tau$ such that, for any input $w(t) \in \mathscr{D}^m$ and every initial condition φ,

$$y(t;\varphi, w) = 0 \quad \text{for all } t \in \left[t_1', t_2'\right] \text{ implies } z_2(\tau_{-};\varphi, w) = 0.$$

3.2. Algebraic condition for the reconstruction of z_1

If we define $\bar{y}(t) = \begin{bmatrix} 0 \\ y(t) \end{bmatrix}$, $\bar{y}(t) \in \mathbb{R}^{p+n-q}$, and the input and output distribution matrices are expressed in the following form,

$$\bar{B}(\delta) = \begin{bmatrix} A_{12}(\delta) & B_1(\delta) \end{bmatrix}, \qquad \bar{C} = \begin{bmatrix} A_{21}(\delta) \\ C_1(\delta) \end{bmatrix}$$

$$\bar{D}(\delta) := \begin{bmatrix} A_{22}(\delta) & B_2(\delta) \\ C_2(\delta) & D(\delta) \end{bmatrix}$$

then, system (4) takes the following compact form

$$\dot{z}_1 = A_{11}(\delta)z_1 + \bar{B}(\delta)\begin{bmatrix} z_2 \\ w \end{bmatrix} \tag{6a}$$

$$\bar{y} = \bar{C}(\delta)z_1 + \bar{D}(\delta)\begin{bmatrix} z_2 \\ w \end{bmatrix}. \tag{6b}$$

Since $y(t) = 0$ if, and only if, $\bar{y}(t) = 0$, then, the implication (5) in Definition 2 may be restated as: $\bar{y}(t;\varphi, z_2, w) = 0$ for all $t \in \left[t_1', t_2'\right]$ implies $z_1(\tau_{-};\varphi, z_2, w) = 0$.

Matrix recursive algorithm

Following the approach used in [7], we define the following recursive algorithm, which generates matrices $\{\Delta_k(\delta)\}$: