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Linear quadratic regulation problem for discrete-time systems with multi-channel multiplicative noise[☆]

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ABSTRACT

This paper investigates the linear quadratic regulation (LQR) problem for discrete-time systems with multiplicative noise. Multiplicative noise is usually assumed to be a scalar in existing literature works. Motivated by recent applications of networked control systems and MIMO communication technology, we consider multi-channel multiplicative noise represented by a diagonal matrix. We first show that the finite horizon LQR problem can be solved using a generalized Riccati equation. We then prove the convergence of the generalized Riccati equation under the conditions of stabilization and exact observability, and obtain the solution to the infinite horizon LQR problem. Finally, we provide a numerical example to demonstrate the proposed approach.

1. Introduction

Recently, the control and estimation problems for systems with multiplicative noise have received much attention [1,2], due to that the signals contaminated by multiplicative noise are common in engineering and financial applications. Such examples can be found in image processing [3], communication systems, portfolio optimization, etc. Different from the additive noise, the second order statistics of the multiplicative noise is usually unknown as it depends on the control solution, which leads to additional difficulties. The LQR problem is one of the most important optimal control problems. The stochastic LQR problem was first studied in [4] and has been studied by many other researchers [5–17].

In the literature related to the topic of LQR of discrete-time linear stochastic systems two types of models are usually involved. One is the discrete-time linear systems with multiplicative noise [6–12], the other is the discrete-time linear systems subject to Markovian switching [13–17]. In [6], the author considered the finite horizon LQR problem for discrete-time systems subjected to state and control-dependent noise with additive noise using the dynamic programming technique. The indefinite LQR problem involving state and control-dependent noise has been introduced in [8]. On the basis of [8], [9] investigated the indefinite LQR problem for discrete-time multiplicative noise systems via semidefinite programming. In [10], the infinite-horizon LQ optimal control for a stochastic system with both state and control-dependent noise was considered. The authors derived the optimal control law under standard assumptions of mean-square stabilizability and exact observability. Assuming that the feedback signal is available, [11] investigated the optimal control and state estimation problems for multiplicative noise systems. The $\mathcal{H}$-representation and applications to generalized Lyapunov equations for linear stochastic systems was investigated in [12], where several topics are extensively discussed, such as observability and stabilization.

The authors in [13] considered the LQR problem of discrete-time Markov jump linear systems with multiplicative noise. The weighting matrices of the state and control in the performance criterion are allowed to be indefinite. A state feedback solution can be derived from a set of coupled generalized Riccati difference equations interconnected with a set of coupled linear recursive equations. Further, [14] investigated the existence of the maximal and mean square stabilizing solutions for a set of generalized coupled algebraic Riccati equations associated to the infinite-horizon LQR problem of discrete-time Markov jump linear

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systems with multiplicative noise. The authors in [15] studied the mean–variance optimal control problem for discrete-time systems subject to multiplicative noise and Markovian jumps. [16] extended the $\mathcal{H}_2$ control problem to discrete-time periodic systems with Markovian jumps and multiplicative noise. It is worth noting that the multiplicative noise considered in the aforementioned works [6–12] are all in scalar form. In other words, the multiplicative noise of each channel is assumed to be same, which is restricted and unrealistic.

In the past few years, networked control systems have attracted much interest from the control community. In [18], the authors studied the problem of remote mean-square stabilization of a MIMO system when independent fading channels are dedicated to each actuator and sensor. [19] addressed the mean-square stabilization problem for discrete-time networked control systems over fading channels. The model of the fading channel(s) was given in multiplicative form and possessed a diagonal structure. In [20], the authors investigated the LQ optimal control for discrete-time LTI systems with random input gains, and sufficient conditions are obtained to guarantee that the infinite-horizon LQ optimal control problem is solvable.

Motivated by the mentioned works, we consider the LQR problem for discrete-time systems with multi-channel multiplicative noise represented by a diagonal matrix, where the multiplicative noise of one channel is allowed to be different from another channel. To the best of our knowledge, there is no other work dealing with this problem in the literature. The main contributions of the paper are highlighted as follows: (i) We extend the scalar multiplicative case to the diagonal matrix multiplicative case. The multiplicative noise represented by a diagonal matrix makes the problem more complex. We present the criteria to verify the mean-square stabilization and exact observability for multi-channel multiplicative noise systems. By employing the matrix Kronecker product and Hadamard product flexibly, the criteria obtained in our paper has the similar structure with the scalar multiplicative noise case in [7,9,10]; (ii) Under the assumptions of mean square stabilization and exact observability, we present a different way in proving the convergence of the generalized Riccati equation by introducing a backward stochastic state-space model. The solution to the infinite horizon LQR problem is obtained in terms of a generalized algebraic Riccati equation; (iii) The obtained result in this paper will pay the way to investigate other related control problems for systems with multi-channel multiplicative noise, such as the indefinite LQR problem for Markovian jumps system, $\mathcal{H}_2/\mathcal{H}_\infty$ control, and so on.

The organization of this paper is as follows. Section 2 gives the problem formulation and introduces some preliminary. Section 3 presents the results of the finite horizon and infinite horizon LQR problem, and provides the proof of the convergence of the generalized difference Riccati equation under standard assumptions. Section 4 provides a numerical example to demonstrate the efficiency of the proposed approach. Section 5 gives some concluding remarks.

Notation: Throughout this paper, a real symmetric matrix $P > 0$ (≥ 0) denotes P being a positive definite (or positive semi-definite) matrix. I denotes an identity matrix of appropriate dimension. The superscripts "-1" and "T" represent the inverse and transpose of a matrix. $\mathcal{R}^n$ denotes the n-dimensional Euclidean space. $\mathcal{R}^{n\times m}$ is the set of all $n \times m$ real matrices. $\delta_{ij} = 0$ for $i \neq j$ and $\delta_{ii} = 1$. The Kronecker product is denoted by $\otimes$, and the Hadamard product by $\odot$. Assume $X = [X_1 \quad \cdots \quad X_n]$, vec$(X)$ is defined as $[X_1^T \quad \cdots \quad X_n^T]^T$. We associate the diagonal matrix with diagonal elements $\lambda_1, \ldots, \lambda_n$ defined by diag$\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$. Ker$(X)$ denotes the kernel space of a real matrix X. Furthermore, the mathematical expectation operator is denoted by E. tr is used to represent the trace of the matrix. Matrices, if the dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. Problem statement and preliminary

Consider the following system

$$x(k+1) = [A + \xi(k)A_0]x(k) + [B + \xi(k)B_0]u(k), \tag{1}$$

where $x(k) \in \mathcal{R}^n$, $u(k) \in \mathcal{R}^m$ are the state and the control input, respectively, the initial state $x(0)$ is assumed to be known, A, A_0, B, B_0 are matrices of appropriate dimensions, $\xi(k) = \text{diag}\{\xi_1(k), \ldots, \xi_n(k)\}$, whose elements are random processes with mean $E\{\xi_i(k)\} = 0$ and covariances $E\{\xi_i(k)\xi_j(s)\} = \sigma_{ij}\delta_{ks}$. Let $\mathcal{F}_k$ be the σ-algebra generated by the sequence $\{\xi(0), \xi(1), \ldots, \xi(k)\}$. For convenience, we denote $\Pi = [\sigma_{ij}]_{i,j=1,2,\ldots,n}$.

The cost function associated with system (1) is

$$J = E\left\{\sum_{k=0}^{\infty}\left[x^T(k)Qx(k) + u^T(k)Ru(k)\right]\right\}, \tag{2}$$

where E is the mathematical expectation over the noise $\{\xi(0), \xi(1), \ldots\}$, the weighting matrix R is positive definite and the matrix Q is non-negative definite.

Problem. Find the state feedback input sequences $\{u(k), k = 0, 1, \ldots, \infty\}$ in which $u(k)$ is $\mathcal{F}_{k-1}$-measurable such that the cost function J of (2) is minimized.

Remark 1. The state and control-dependent noises can be assumed to be different and correlated with each other in model (1). For simplicity, we assume that the state and control-dependent noises are the same, even though the later development and results can be adapted to the different and correlated case.

Remark 2. If we let $\xi_1(k) = \xi_2(k) = \cdots = \xi_n(k)$, then system (1) is given as

$$x(k+1) = [A + \xi_1(k)A_0]x(k) + [B + \xi_1(k)B_0]u(k). \tag{3}$$

The model (3) is similar with [6–10]. Considering that $\xi_1(k) = \xi_2(k) = \cdots = \xi_n(k)$ is restricted in many practical applications, the proposed model in our paper is more general. Meanwhile, the models about discrete-time linear systems with multiplicative noise in [13,17] are given as

$$x(k+1) = \left[A + \sum_{i=1}^{n}\xi_i(k)A_i\right]x(k) + \left[B + \sum_{i=1}^{n}\xi_i(k)B_i\right]u(k). \tag{4}$$

By decomposing the diagonal noisy matrix as a sum of scalar multiplicative noise, it follows from system (1) one has

$$x(k+1) = \left[A + \sum_{i=1}^{n}\xi_i(k)\epsilon_i A_{0i}\right]x(k) + \left[B + \sum_{i=1}^{n}\xi_i(k)\epsilon_i B_{0i}\right]u(k). \tag{5}$$

As such, the LQR problem in this paper can be settled by applying the existing result in [13,17]. However, due to the appearance of n correlated scalar multiplicative noise $\xi_i(k)$, the results are complex. Also, note that the matrices $\epsilon_i A_{0i}$ and $\epsilon_i B_{0i}$ are special (the elements are all zero except for the i row). Motivated by these points, we want to obtain some simple results by employing the technique of Hadamard product and Kronecker product flexibly, which have the similar structure as the results obtained from scalar multiplicative noise case (3) in [6–10].

For deterministic systems, it is well known that the conditions of stabilization and observation play crucial role in guaranteeing the convergence of the standard Riccati equation. Similar to the deterministic case, we define the stabilization and observation for multiplicative noise systems in the following.