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Adaptive asymptotic tracking control of uncertain nonlinear system with input quantization☆

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ABSTRACT

Asymptotic tracking control of uncertain nonlinear system with input quantization is an important, yet challenging issue in the field of adaptive control. So far, there is still no result available in addressing this issue even for the case of time-invariant reference signal. In this paper, we solve this problem by proposing a new tuning function control scheme which is designed on the basis of a novel decomposition of hysteresis quantizer. It is proved that the proposed scheme ensures the global boundedness of all closed-loop signals and the asymptotic convergence of tracking error to zero. Moreover, an explicit bound for the L_2-norm of the tracking error is derived, which shows that the transient performance can also be improved with the proposed scheme.

1. Introduction

In recent years, quantized control has received considerable attentions because of its theoretical importance and practical value in networked systems, hybrid systems, multi-agent systems, etc., as seen from [1–11]. In these systems, a common feature is that their main components are connected by shared networks with limited bandwidth, and then the quantizer is usually placed after the controller or after the sensor to respectively reduce the communication rate of control signal or feedback signal to be sent over the networks. It was shown by [12] that the coarsest quantizer which can quadratically stabilize a linear system should follow a logarithmic law. By treating the logarithmic quantizer as a sector-bound uncertainty and combining with a quadratic Lyapunov function approach, a comprehensive study on quantized feedback control of linear systems has been conducted in [13]. In [14], a quantization-dependent Lyapunov function approach was further proposed and the obtained stability condition is less restrictive comparing with [13]. In [15], the quantized feedback control was generalized to the nonlinear system through a cyclic-small-gain theorem. It is worthy to point out that most of the works mentioned above are on the basis of robust approaches. More results on robust quantized control could be found in [16–22].

Apart from the robust control, adaptive control is also an effective approach in handling the system uncertainties [23]. In [24], an adaptive quantized control scheme based on logarithmic quantizer was proposed to stabilize linear uncertain systems. The adaptive quantized control for nonlinear uncertain system was further studied in [25]. Also in this work, a hysteresis quantizer was newly proposed to avoid the possible chattering induced by logarithmic quantizer. Despite these contributions, the stability established in [25] depends on a restrictive condition about control input, which is difficult to be checked beforehand since the control signal is available only after the controller is put in operation. Such a restrictive condition was removed in [26] by using a novel backstepping design approach. However, this approach is limited to the nonlinear system that unknown parameters only exist in the last subsystem and all system functions should satisfy global Lipschitz continuity condition. Another restriction in [26] also commonly in [24,25] is that the obtained stabilization schemes are difficult to be generalized to the trajectory tracking control problem, let alone to achieve the asymptotic tracking performance. To the best knowledge of authors, so far, no result has been reported to remove these two restrictions. In addition, we find that there is also no result currently available in developing an adaptive control scheme to guarantee the transient tracking performance of nonlinear system when input quantization is considered.

In this paper, we fill in these gaps by proposing a new tuning function control scheme for a class of uncertain nonlinear systems with input quantization. It is found that the restrictions in [24–26] mentioned above mainly arise from an unsatisfactory

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decomposition of hysteresis quantizer, in which the disturbance-like term cannot be ensured bounded by a control-independent constant. Such a finding stimulates us to decompose the hysteresis quantizer through a new way shown as in Lemma 1 latter, such that the restrictions in [24–26] could be removed. Irrespective of the advantages, this new decomposition also brings about a dynamical control coefficient which obstructs the adaptive control design. To address this issue, we firstly design a new adaptive control framework which has a useful property given as in Lemma 3 such that the dynamical control coefficient is transformed into its lower bound in backstepping procedure, and then employ the adaptive approach to handle the transformed bounds to accommodate the input quantization effect. With our proposed adaptive controller, the parameterized uncertainties are allowed to exist in each subsystem and the system nonlinear functions are not required to satisfy the global Lipschitz continuity condition. In this sense, the investigation of this paper enlarges the classes of the nonlinear system that can be handled by using adaptive quantized control. Moreover, the proposed controller can be applied to the trajectory tracking control problem with a guarantee that all closed-loop signals are globally bounded and the tracking error converges to zero asymptotically. In addition, an explicit bound for the L_2-norm of the tracking error is derived, which shows that the transient performance can also be improved with the proposed scheme.

2. Problem statement

2.1. Nonlinear system model

Consider a class of uncertain nonlinear systems with input quantization as follows.

$$\begin{aligned}
\dot{x}_1 &= x_2 + \theta^T\varphi_1(x_1)\\
\dot{x}_2 &= x_3 + \theta^T\varphi_2(x_1, x_2)\\
&\vdots\\
\dot{x}_{n-1} &= x_n + \theta^T\varphi_{n-1}(x_1, \dots, x_{n-1})\\
\dot{x}_n &= \psi_0(x) + q(u) + \theta^T\varphi_n(x)\\
y &= x_1
\end{aligned} \tag{1}$$

where $x = [x_1, \dots, x_n]^T \in \mathbb{R}^n$ are system states, $u \in \mathbb{R}$ is the control input, and $y \in \mathbb{R}$ is the system output. $\varphi_i \in \mathbb{R}^p$ and $\psi_0 \in \mathbb{R}$ are known smooth functions. $\theta \in \mathbb{R}^p$ are unknown structural parameters. $q(u) \in \mathbb{R}$ represents the output of a hysteresis quantizer given latter.

Remark 1. Comparing with [26] which also focuses on the adaptive quantized control design, the system model studied in this paper is much more general because the parameterized uncertainty exists in each subsystem, as seen from model (1).

The control objective of this paper is to design an adaptive control law u for nonlinear system (1) such that

- All the closed-loop signals are globally bounded;
- The output $y(t)$ can track the given reference signal $y_r(t)$ with a tracking error converging to zero.

Assumption 1. The reference signal $y_r(t)$ and its first nth-order time derivatives $y_r^{(i)} (i = 1, \dots, n)$ are known, smooth, and bounded.

Remark 2. It is worthy to point out that Assumption 1 is very common in the existing literatures on backstepping control design, see for instance [27,28,23,29–32].

2.2. Asymmetric hysteresis quantizer

In quantized control systems, the quantizer is usually placed after the controller to reduce the communication rate of control information to be sent over the networks, as seen from [17,19,24–26,33]. In this paper, we newly propose an asymmetric hysteresis quantizer, given as below.

$$q(u) = \begin{cases}
u_i^+, & \dfrac{u_i^+}{1+\delta_+} < u \le u_i^+, \dot{u} < 0 \text{ or},\\
& u_i^+ < u \le \dfrac{u_i^+}{1-\delta_+}, \dot{u} > 0;\\
u_i^- & u_i^- \le u < \dfrac{u_i^-}{1+\delta_-}, \dot{u} > 0 \quad \text{or},\\
& \dfrac{u_i^-}{1-\delta_-} \le u < u_i^-, \dot{u} < 0;\\
u_i^+(1+\delta_+), & u_i^+ < u \le \dfrac{u_i^+}{1-\delta_+}, \dot{u} < 0 \quad \text{or},\\
& \dfrac{u_i^+}{1-\delta_+} < u \le \dfrac{1+\delta_+}{1-\delta_+}u_i^+, \dot{u} > 0;\\
u_i^-(1+\delta_-), & \dfrac{u_i^-}{1-\delta_-} \le u < u_i^-, \dot{u} > 0 \quad \text{or},\\
& \dfrac{1+\delta_-}{1-\delta_-}u_i^- \le u < \dfrac{u_i^-}{1-\delta_-}, \dot{u} < 0;\\
0, & 0 \le u < \dfrac{u_{\min}^+}{1+\delta_+} \quad \text{or},\\
& \dfrac{u_{\min}^+}{1+\delta_+} \le u \le u_{\min}^+, \dot{u} > 0,\\
& \dfrac{u_{\min}^-}{1+\delta_-} < u \le 0 \quad \text{or},\\
& u_{\min}^- \le u \le \dfrac{u_{\min}^-}{1+\delta_-}, \dot{u} < 0;\\
q(u(t^-)), & \dot{u} = 0
\end{cases} \tag{2}$$

where $q(u(t^-))$ is the status prior to $q(u(t))$. u_i^+ and u_i^- are quantized values given by

$$u_i^+ = \left(\frac{1-\delta_+}{1+\delta_+}\right)^{1-i} u_{\min}^+, \qquad u_i^- = \left(\frac{1-\delta_-}{1+\delta_-}\right)^{1-i} u_{\min}^- \tag{3}$$

$i = 1, 2, \dots$. $u_{\min}^+$ and $u_{\min}^-$ are dead-zone parameters as seen in Fig. 1. The constants $\delta_+, \delta_- \in (0, 1)$ determine the coarseness of the quantizer. The larger the selected δ_+ and δ_- are, the coarser the quantizer is.

Remark 3. To reduce communication burden in a network, the control signal with larger change rate needs to be quantized by the coarser quantizer. However, the practical control signal may have different change rates in positive control domain ($u > 0$) and negative control domain ($u < 0$), which thus requires different coarseness of quantizer in these two domains. Obviously, this point cannot be effectively and flexibly achieved by the symmetric quantizers used as in [13–15,24–26,33,34]. Given this reason, we propose an asymmetric hysteresis quantizer in this paper, as seen in (2).

3. Adaptive quantized control design

3.1. Decomposition of hysteresis quantizer

According to the sector-bound property used as in [25,26], the hysteresis quantizer (2) can be decomposed into the following form.

$$q(u) = u + \Delta(u) \tag{4}$$