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Second-order consensus in time-delayed networks based on periodic edge-event driven control☆

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ABSTRACT

The second-order consensus problems in multi-agent systems with undirected topologies and time-invariant delays are considered in this paper. The time-delay robustness of a consensus protocol based on periodic edge-event driven control is investigated, and the studied protocol can largely reduce the communication and controller-updating costs. The relationship between time delays and event-detecting period is characterized. By Lyapunov methods, it is shown that the protocol can solve state consensus problems when the interaction topology is connected, and the system does not exhibit Zeno behavior. Finally, simulations are given to demonstrate the effectiveness of our theoretical results.

1. Introduction

In the past decade, consensus problems in multi-agent systems have attracted a large number of researchers' attention, and have been intensively studied. And so far, numerous results have been obtained and applied in many areas, such as coordination control [1–4], formation control [5], flocking [6], rendezvous [7], and network connectivity preservation [8].

Consensus control is an important and challenging research topic in multi-agent systems, and lots of achievements have been made for networks of first-order integrators [3,2,5,1,9]. In [10], Vicsek et al. proposed a simple but compelling discrete-time model, called the Vicsek model, which ensures the headings of all agents converge to a common value. Jadbabaie et al. investigated the Vicsek model and provided a theoretical explanation in [1]. In [2], Olfati-Saber and Murray investigated consensus problems with directed interaction topologies and time delays by Lyapunov methods. In [4], Ren and Beard extended the results in [1,2] and showed that a state consensus can be achieved asymptotically under dynamically changing directed interaction topologies if the union of interaction topologies contain a spanning tree frequently enough. And then lots of results of consensus theory following their work were obtained [3,8]. It is worth mentioning that there has been a growing interest in consensus problems of double integrators recently [11,12]. In [11], Ren and Atkins investigated consensus problems of double integrators under directed interaction topologies, and proposed a second-order consensus protocol. They also showed that a state consensus can be achieved only if the interaction topology has a spanning tree. In [12], Xie and Wang studied second-order consensus problems in multi-agent systems with fixed and switching topologies, and introduced a different consensus protocol.

Most existing results on consensus problems require that agents continuously monitor and update the states of their neighbors, which may require high costs of communication and controller updates. To reduce these costs, sampled-data consensus has been intensively studied in the past several years, and many protocols were designed based on periodic data-sampling [13–16]. Periodic sampled-data control considers the behavior of systems at sampling instants, and every agent should communicate with all of its neighbors at each of sampling instants. However, in most cases, it is not necessary for every agent to do that. Thus, an alternative approach to scheduling data-sampling, called event-driven control, was proposed [17–23]. In event-driven control, controller updates are determined by certain conditions which are usually formulated based on agent states, thus event-driven

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control could have a better performance in cooperative control. In [20], Dimarogonas et al. proposed an event-based protocol, which is based on the ratio of a certain measurement error with respect to the norm of a function of states, to reduce controller-updating costs. However, this approach requires that each agent continuously monitor the states of its neighbors. In order to avoid keeping track of state errors, a self-triggered approach was also presented in [20]. In [19,22,24], Xiao et al. proposed edge-event based consensus protocols, which can further reduce the communication costs and controller-updating costs. In [23], Cao et al. extended the results to consensus control of double integrators, and proposed two edge-event triggering rules to ensure that all agents reach a consensus on states. In [25], Heemels et al. proposed a periodic event-triggered control strategy to study linear systems where communication resources are limited.

In this paper, we focus on a multi-agent system with double integrator dynamics as well as transmission time delays, and model the multi-agent system by an undirected graph, in which each edge connecting two adjacent agents represents the communication link between them. We assume that all the information links share a common time-invariant transmission delay. We develop a periodic edge-event driven control method for distributed cooperative consensus. Our main contribution is the presentation and time-delay robustness analysis of a consensus protocol, which combines periodic event-detection and edge-event driven control. By the Lyapunov-based approach, the relationship between time delays and event-detecting period is characterized. It is worth mentioning that the edge-events are only triggered at periodic time instants, which implies that the time intervals between two consecutive edge-events of any edge have a lower bound; in other words, the system does not exhibit Zeno behavior. Our proposed protocol has the advantages of both time-driven control and event-driven control, and has a satisfactory performance with reduced controller-updating costs and communication costs.

The rest of the paper is organized as follows. Some preliminaries in graph theory are presented in Section 2. In Section 3, the problem is formulated and an edge-event triggering rule is designed. In Section 4, the main result and the consensus analysis are presented. Numerical simulations are performed to verify the theoretical analysis in Section 5. Conclusions are finally given in Section 6.

2. Preliminaries

In this section, we list some basic concepts in graph theory. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a graph with vertex set $\mathcal{V} = \{v^1, v^2, \dots, v^n\}$ and edge set $\mathcal{E} \subseteq \{(v^i, v^j) | v^i, v^j \in \mathcal{V}\}$. Graph $\mathcal{G}$ is called undirected if and only if $(v^i, v^j) \in \mathcal{E}$ implies $(v^j, v^i) \in \mathcal{E}$. Graph $\mathcal{G}$ has a self-loop if there exists an edge (v^i, v^i). A sequence of distinct vertices $v^{i_1} v^{i_2} \cdots v^{i_m}$ such that $(v^{i_k}, v^{i_{k+1}}) \in \mathcal{E}$ for all $k \in \{1, 2, \dots, m-1\}$ is called a path from v^{i_1} to v^{i_m}, and graph $\mathcal{G}$ is called connected if for any two distinct vertices v^i and v^j, there exists a path from v^i to v^j. Suppose that there are m edges in graph $\mathcal{G}$. Label these edges with 1 through m and assign each of these edges an arbitrary orientation. Then, the $n \times m$ incidence matrix $D = [d_{ij}]$ is defined as follows:

$$d_{ij} = \begin{cases} -1, & \text{if } v^i \text{if is the tail of the } j\text{if th oriented edge} \\ 1, & \text{if } v^i \text{if is the head of the } j\text{if th oriented edge} \\ 0, & \text{otherwise.} \end{cases}$$

Associate the undirected graph $\mathcal{G}$ with a symmetric nonnegative matrix $A = [a_{ij}]$ such that $a_{ij} > 0$ if and only if $(v^j, v^i) \in \mathcal{E}$. Then, $A = [a_{ij}]$ is called the weight matrix of graph $\mathcal{G}$. The degree matrix $\Delta = [\Delta_{ij}]$ of graph $\mathcal{G}$ with A is a diagonal matrix with $\Delta_{ij} = 0$ for all $i \neq j$ and $\Delta_{ii} = \sum_{k=1, k \neq i}^{n} a_{ik}$. The graph Laplacian of $\mathcal{G}$ is defined as follows [2]:

$$L = \Delta - A.$$

Moreover, Let Λ be a $m \times m$ diagonal matrix, with the weight of the ith edge in the ith diagonal position, then the graph Laplacian and the incidence matrix satisfy the following equality:

$$L = D \Lambda D^T.$$

Notations: Let $\mathbf{0} = [0, 0, \dots, 0]^T \in \mathbb{R}^n$, $\mathbf{1} = [1, 1, \dots, 1]^T \in \mathbb{R}^n$, $\mathbf{0}_{n \times m} = [\mathbf{0}, \mathbf{0}, \dots, \mathbf{0}] \in \mathbb{R}^{n \times m}$, and let $\|x\| = \sqrt{x^T x}$, where $x \in \mathbb{R}^n$.

3. Problem formulation

In this paper, we study a multi-agent system consisting of n double integrators. Label these agents with 1 through n, and let $x_i(t), v_i(t) \in \mathbb{R}$ denote the position and velocity of agent i, respectively, $i = 1, 2, \dots, n$. The dynamics of each agent is given as follows:

$$\begin{cases} \dot{x}_i(t) &= v_i(t) \\ \dot{v}_i(t) &= u_i(t) \end{cases}, \quad i = 1, 2, \dots, n, \tag{1}$$

where $u_i(t)$ is a protocol to be designed based on the state information received by agent i from its neighbors.

The interaction topology of multi-agent system (1) is modeled by an undirected graph $\mathcal{G}$ without self-loops. Vertex v^i represents agent i, and edge (v^i, v^j) represents an information link between agents i and j. Agent j is called a neighbor of agent i if edge (v^j, v^i) exists. Let N_i be the index set of all neighbors of agent i, $i = 1, 2, \dots, n$, i.e. $N_i = \{j | (v^j, v^i) \in \mathcal{E}\}$.

Assume there is a sequence of time $t_0, t_1, t_2, \dots$ with the property that $t_{s+1} = t_s + h$, $s = 0, 1, 2, \dots$, where $h > 0$ is the event-detecting period. Let $y(t) = D^T x(t)$ and $w(t) = D^T v(t)$ be the state information of edges. Based on the definition of matrix D, for the qth edge, $1 \leq q \leq m$, there must exist two vertices v^i and v^j, such that $y_q(t) = x_i(t) - x_j(t)$ and $w_q(t) = v_i(t) - v_j(t)$. Let $s^{ij}(t)$ be an integer and denote the time index in $\{t_s | s \in \mathbb{N}\}$, such that $t_{s^{ij}(t)}$ is the most recent time up to t, at which the edge-event of (v^i, v^j) is triggered. Let $e_q(t)$ and $f_q(t)$ be the virtual state information of the qth edge, and their values are defined as follows:

$$\begin{cases} e_q(t) &= y_q(t_{s^{ij}(t)}) \\ f_q(t) &= w_q(t_{s^{ij}(t)}) \end{cases}, \quad q = 1, 2, \dots, m.. \tag{2}$$

Based on the definition above, $e_q(t)$ and $f_q(t)$ denote the most recent state information of the qth edge when the edge-event of (v^i, v^j) is triggered.

At each time in the sequence $t_0, t_1, t_2, \dots$, each agent decides whether or not to exchange state information with its neighbors based on the pre-assigned event-triggering conditions. If the states of two adjacent agents satisfy the pre-assigned conditions, these two agents exchange state information over the information link and their controllers update consequently. In our model, all the data exchanges are treated independently of others, and we call them edge-events.

Assume that there exist a common transmission time-invariant delay τ, $\tau < h$, on all information links. We suppose that the instant state information of each edge can be obtained, but each agent can only get the delayed state information of the incident edges, which means that there are no time delays in the state information of agents, but there are time delays in the virtual state information of edges. This assumption is coincident with that there is a common transmission time delay between any two agents. Assume there are no time delays for event detectors, and the edge-event triggering rule is given as follows: