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Stabilization of uncertain linear distributed delay systems with dissipativity constraints

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ABSTRACT

This paper examines the problem of stabilizing linear distributed delay systems with nonlinear distributed delay kernels and dissipativity constraints. Specifically, the nonlinear distributed kernel includes functions such as polynomials, trigonometric and exponential functions. By constructing a Liapunov–Krasovskii functional related to the distributed kernels, sufficient conditions for the existence of a state feedback controller which stabilizes the uncertain distributed delay systems with dissipativity constraints are given in terms of linear matrix inequalities (LMIs). In contrast to existing methods, the proposed scenario is less conservative or requiring less number of decision variables based on the application of a new derived integral inequality. Finally, numerical examples are presented to demonstrate the validity and effectiveness of the proposed methodology.

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1. Introduction

Among many models of Time Delay Systems (TDS) [1], distributed delay systems (DDS) cover a wide range of real time applications [2,3]. For a rigorous treatment and benchmark results on the frequency domain approaches for linear TDS or DDS, see the monograph [4] and the references therein. With regards to time domain approaches, the construction of Liapunov–Krasovskii functional [5] has been adopted as the most common method to undertake both stability analysis and controller synthesis. In particular, the Complete Liapunov–Krasovskii Functional (CKLF) [5,6], which provides both sufficient and necessary stability conditions for a delay system such as $\dot{\mathbf{x}}(t) = A_1\mathbf{x}(t) + A_2\mathbf{x}(t-r), r \geq 0$, incorporates most of the existing proposed functionals as special cases. For a thorough treatise on the fundamental theories of CKLF and its mathematical derivation, see [6] and references therein.

In contrast to constructing a quadratic function within the context of semi-definite programming, the decision variables of CLKF possess infinite dimension which gives rises to significant difficulties to produce numerical results. In addition, the similar problems have been encountered in dealing with the non-constant distributed delay terms. By assuming constant decision variables with constant distributed delay terms in particular, finite dimension constraints can be automatically obtained which leads to conventional stability conditions denoted by LMIs. There has been a significant series of literatures on this direction to perform either stability analysis or controller synthesis for linear DDS [7–9]. For a collection of the previous works on this topic, see the monographs [10,11]. On the other hand, the results in [12] have demonstrated that certain linear DDS can be transformed into a corresponding system with only discrete delays. However, the aforementioned method inherits obvious conservatisms due to the presence of additional dynamics required by adding new states.

An alternative synthesis approach, predicated on the discretization scheme proposed in [13], is presented in [14] considering linear DDS with a piecewise constant distributed delay term. Moreover, by using the application of full-block S-procedure [15], a novel synthesis result is presented in [16] to tackle systems with rational distributed delay kernels which are capable of dealing with general distributed terms via approximations. However, the derived stabilization conditions require (A, B) to be controllable in [16] (A is the delay free state space matrix and B is the input gain matrix), thereby ensuring the induced conservatisms cannot be ignored. Finally, a systematic way to construct controllers for linear delay systems, having forwarding or backstepping structures, has been investigated in [17].

In this paper we propose a method for stabilizing Linear DDS with nonlinear distributed kernels, which is achieved by constructing a standard complete Liapunov–Krasovskii functional. In addition, a quadratic supply function [18,11] and uncertainty with full block constraints [19] having nonconservative assumptions

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are incorporated to provide a broad characterization of both controller objectives and robustness. Furthermore, a new integral inequality is derived for the formulation of the synthesis conditions, which can be considered as a generalization of the recent proposed Bessel–Legendre inequality [20,21]. By applying the new derived integral inequality with Projection Lemma [22], convex synthesis conditions can be derived in terms of LMIs. Unlike existing methods, the proposed solution neither requires (A, B) to be controllable as in [16], nor demand forwarding or backstepping structures as in [17]. Furthermore, it can produce feasible results without considering approximations and with less decision variables in comparison with the stability analysis results in [23].

The paper is organized as follows. Some important preliminaries and the formulations of the synthesis problem are presented in Section 2. Section 3 contains the main results relating to controller synthesis. To demonstrate the validity and effectiveness of proposed methodologies, numerical examples are investigated in Section 4 before the final conclusion in Section 5.

Notation: The notations in this paper follow standard rules, though certain new symbols will be introduced for the sake of compactness: $\mathbb{T} := \{x \in \mathbb{R} : x \geq 0\}$; $\mathbb{S}^n := \{X \in \mathbb{R}^{n\times n} : X = X^\top\}$; $\mathbb{R}^{n\times n}_{[n]} := \{X \in \mathbb{R}^{n\times n} : \operatorname{rank}(X) = n\}$; notations $\|\mathbf{x}\|_q = \left(\sum_{i=1}^n |x_i|^q\right)^{\frac{1}{q}}$ and $\|f(\cdot)\|_p = \left(\int_{\mathbb{R}} |f(t)|^p \mathrm{d}t\right)^{\frac{1}{p}}$ and $\|\boldsymbol{f}(\cdot)\|_{p|q} = \left(\int_{\mathbb{R}} \|\boldsymbol{f}(t)\|_q^p \mathrm{d}t\right)^{\frac{1}{p}}$ are the norms associated with $\mathbb{R}^n$ and Lebesgue integrable functions spaces $\mathbb{L}_p(\mathbb{R};\mathbb{R})$ and $\mathbb{L}_p(\mathbb{R};\mathbb{R}^n_{\|q})$, respectively. $\mathbb{C}(\mathcal{X};\mathbb{R}^n)$ with $\sup_{\tau\in\mathcal{X}} \|\boldsymbol{f}(\tau)\|_2$ is the Banach space of continuous functions with an uniform norm. $\mathbf{Sy}(X) := X + X^\top$ is the sum of a matrix with its transpose. A column vector containing a sequence of objects is defined as $\mathbf{col}_{i=1}^n x_i := \left[\mathbf{row}_{i=1}^n x_i^\top\right]^\top := \left[x_1^\top \cdots x_i^\top \cdots x_n^\top\right]^\top$. We use $*$ to denote $[*]YX = X^\top YX$ or $X^\top Y[*] = X^\top YX$. The direct sum of two matrices and n matrices are defined as $X \oplus Y = \mathsf{Diag}(X, Y)$, $\bigoplus_{i=1}^n X_i = \mathsf{Diag}_{i=1}^n(X_i)$, respectively. Finally, $\otimes$ indicates the Kronecker product.

2. Preliminaries and problem formulations

Without losing generality, we only consider a system with one delay channel for the sake of simplicities. However, one can easily extend the corresponding Krasovskii functional to handle multiple delay channels simultaneously.

Consider a linear model of DDS

$$\begin{aligned}
\dot{\boldsymbol{x}}(t) &= A_1\boldsymbol{x}(t) + A_2\boldsymbol{x}(t-r) + B_2\boldsymbol{w}(t) \\
&\quad + \int_{-r}^{0} \widetilde{A}_3(\tau)\boldsymbol{x}(t+\tau)\mathrm{d}\tau + B_1\boldsymbol{u}(t) \\
\boldsymbol{z}(t) &= C_1\boldsymbol{x}(t) + C_2\boldsymbol{x}(t-r) + D_2\boldsymbol{w}(t) \\
&\quad + \int_{-r}^{0} \widetilde{C}_3(\tau)\boldsymbol{x}(t+\tau)\mathrm{d}\tau + D_1\boldsymbol{u}(t) \\
\boldsymbol{x}(\tau) &= \boldsymbol{\phi}(\tau), \quad \forall\tau \in \mathcal{O} := [-r, 0]_{\mathbb{R}}
\end{aligned} \tag{1}$$

to be stabilized, where $\boldsymbol{x}(t) \in \mathbb{R}^n$ is the solution of (1), $\boldsymbol{u}(t) \in \mathbb{R}^p$ denotes input signals, $\boldsymbol{w}(\cdot) \in \mathbb{L}_2(\mathbb{T};\mathbb{R}^q_{\|2})$ represents disturbance, $\boldsymbol{z}(t) \in \mathbb{R}^m$ is the regulated output, and $\boldsymbol{\phi}(\cdot) \in \mathbb{C}(\mathcal{O};\mathbb{R}^n)$ denotes initial condition. Matrices $A_1; A_2; \widetilde{A}_3(\tau) \in \mathbb{R}^{n\times n}$, $C_1; C_2; \widetilde{C}_3(\tau) \in \mathbb{R}^{m\times n}$, $B_1 \in \mathbb{R}^{n\times p}$, $B_2 \in \mathbb{R}^{n\times q}$, $D_1 \in \mathbb{R}^{m\times p}$, $D_2 \in \mathbb{R}^{m\times q}$ are given state space systems parameters with $n; m; p; q \in \mathbb{N}$. $r \in \mathbb{T}$ is a given constant specifying the length of delay channel. Finally, $\widetilde{A}_3(\tau)$ and $\widetilde{C}_3(\tau)$ satisfy the following assumption.

Assumption 1. There exists $\mathbf{m}(\cdot) \in \mathbb{C}^{[1]}(\mathcal{O};\mathbb{R}^\rho)$ with $\rho \in \mathbb{N}$ and $A_3 \in \mathbb{R}^{n\times n\rho}$, $C_3 \in \mathbb{R}^{m\times n\rho}$ such that $\forall\tau \in \mathcal{O}$, $\mathbb{R}^{n\times n} \ni \widetilde{A}_3(\tau) = A_3(\mathbf{m}(\tau)\otimes I_n)$ and $\mathbb{R}^{m\times n} \ni \widetilde{C}_3(\tau) = C_3(\mathbf{m}(\tau)\otimes I_n)$. In addition, $\mathbf{m}(\tau)$ satisfies the following properties:

$$\mathbf{m}(\tau) := \mathbf{col}_{i=1}^{\rho} m_i(\cdot), \qquad \frac{\mathrm{d}\mathbf{m}(\tau)}{\mathrm{d}\tau} = \mathsf{M}\mathbf{m}(\tau), \tag{2}$$

where $\mathsf{M} \in \mathbb{R}^{\rho\times\rho}$ and $m_i(\tau)$ are linear independent and that for all $i = 1\cdots\rho$ there exists an uncountable set $\mathcal{P} \subseteq \mathcal{O}$ such that $\forall\vartheta \in \mathcal{P}$, $m_i(\vartheta) \neq 0$. Similar assumptions can be found in [24–27].

Remark 1. Specifically, the elements inside of $\mathbf{m}(\tau)$ are the solutions of linear homogeneous differential equations with constant coefficients such as polynomials, trigonometric and exponential functions. In addition, there is no limitation on the size of the dimension of $\mathbf{m}(\tau)$ as long as it is able to cover all the elements in the distributed terms in (1). As for the generality of $\mathbf{m}(\tau)$, there are many applications can be modeled by (1) compatible with Assumption 1, for example, the compartmental dynamic systems with distributed delays mentioned in [28]. Furthermore, distributed delay systems concerning gamma distributions in [16,29] with a finite delay range can be stabilized by the proposed methods as well.

In this paper, we assume that all states are available for feedback and (1) is stabilized by a state feedback controller $\boldsymbol{u}(t) = K\boldsymbol{x}(t)$ with $K \in \mathbb{R}^{p\times n}$. Substituting $\boldsymbol{u}(t) = K\boldsymbol{x}(t)$ into (1) and considering Assumption 1 yields

$$\begin{aligned}
\dot{\boldsymbol{x}}(t) &= \Pi_1\boldsymbol{x}(t) + A_2\boldsymbol{x}(t-r) \\
&\quad + \int_{-r}^{0} A_3M(\tau)\boldsymbol{x}(t+\tau)\mathrm{d}\tau + B_2\boldsymbol{w}(t) \\
\boldsymbol{z}(t) &= \Omega_1\boldsymbol{x}(t) + C_2\boldsymbol{x}(t-r) \\
&\quad + \int_{-r}^{0} C_3M(\tau)\boldsymbol{x}(t+\tau)\mathrm{d}\tau + D_2\boldsymbol{w}(t)
\end{aligned} \tag{3}$$

as the corresponding closed loop system, where $\Pi_1 = A_1 + B_1K$ and $\Omega_1 = C_1 + D_1K$ and $M(\tau) := \mathbf{m}(\tau)\otimes I_n$.

To specify performance objectives for (3), we apply the quadratic form

$$s(\boldsymbol{z}(t), \boldsymbol{w}(t)) = -\begin{bmatrix}\boldsymbol{z}(t)\\ \boldsymbol{w}(t)\end{bmatrix}^\top \mathbf{J} \begin{bmatrix}\boldsymbol{z}(t)\\ \boldsymbol{w}(t)\end{bmatrix}, \tag{4}$$

with

$$\mathbf{J} = \begin{bmatrix} J_1^{-1} & J_2 \\ J_2^\top & J_3 \end{bmatrix} \in \mathbb{S}^{(m+q)}, \quad J_1^{-1} \succeq 0 \tag{5}$$

considered in [18] to be the supply rate function.

The supply function in (4) is able to characterize numerous optimization constraints such as $\mathbb{L}_2$ gain control:

$$\sup_{\boldsymbol{w}(\cdot)\in\mathbb{L}_2(\mathbb{T};\mathbb{R}^q_{\|2})} \left(\frac{\|\boldsymbol{z}(\cdot)\|_{2|2}}{\|\boldsymbol{w}(\cdot)\|_{2|2}}\right) < \gamma$$

$$J_1 = \gamma I_m, \qquad J_3 = -\gamma I_q, \qquad J_2 = \mathsf{O}_{m\times q}, \quad \gamma > 0 \tag{6}$$

and Sector Constraints when $m = q$ with

$$J_1^{-1} = I_m, \qquad J_2 = -\frac{1}{2}(\alpha+\gamma)I_m, \qquad J_3 = -\alpha\gamma I_m. \tag{7}$$

For the situation when $J_1^{-1} = J_3 = \mathsf{O}_m$ and $J_2 = -I_m$ with $m = q$, which corresponds to having the strict passivity constraint, the well posedness of J_1 does not need to be considered since there is no reason to apply Schur complement here given the fact that $\boldsymbol{z}^\top(t)J_1^{-1}\boldsymbol{z}(t) = 0$. As a result, one can use $J_1^{-1} = \mathsf{O}_m$ in (4) directly and no mathematical complications will be introduced in deriving the corresponding synthesis conditions.

The following lemmas and definition are required for the mathematical derivations in this paper.

Lemma 1. *$\forall P \in \mathbb{R}^{p\times q}$ and $\forall Q \in \mathbb{R}^{n\times m}$, we have*

$$(P\otimes I_n)(I_q\otimes Q) = (I_p\otimes Q)(P\otimes I_m). \tag{8}$$

Moreover, we have $\forall X \in \mathbb{R}^{n\times m}$, $\forall Y \in \mathbb{R}^{m\times p}$

$$(XY)\otimes I_n = (X\otimes I_n)(Y\otimes I_n). \tag{9}$$

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