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Iterative Learning Control for discrete nonlinear systems with randomly iteration varying lengths[☆]

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ABSTRACT

This note proposes ILC for discrete-time affine nonlinear systems with randomly iteration varying lengths. No prior information on the probability distribution of random iteration length is required prior for controller design. The conventional P-type update law is used with a modified tracking error because of randomly iteration varying lengths. A novel technical lemma is proposed for the strict convergence analysis in pointwise sense. An illustrative example verifies the theoretical results.

1. Introduction

Iterative Learning Control (ILC) is a kind of intelligent control approach that is suitable for controlled systems completing given task in a finite interval repeatedly. The inherent idea of ILC is learning from past experiences and performing to the current process. To be specific, in ILC, the control signal is updated iteratively using information generated from previous iterations, so that the system output could track the desired trajectory asymptotically along the iteration index. The concept of ILC is first proposed by Arimoto in [1] driven from human learning ability to robotic systems. As has been developed for three decades, ILC has become a hot field of intelligent control theory, which is fruitful both in theoretical analysis and practical applications [2–4]. However, in most of the reported results, the operation length and reference trajectory are usually unchanged in different iterations, so that the update law could improve the tracking performance gradually. This condition may limit the applicability of ILC, and it motivates us to consider ILC problem under iteration varying factors.

Some previous publications have discussed the problem of ILC with varying references. Saab et al. studied ILC for continuous-time nonlinear systems with slowly varying references in [5], where D-type, PD-type, and PID-type update law were used to generate the control signal for tracking problem, respectively. The reference of each iteration was assumed to have a small deviation with that of the previous iteration. Xu proposed a direct learning control approach in [6,7] to handle two cases of varying references. One case is that the references have an identical spacial pattern but different time scales, and while the other one is that the references have an identical time scale but different magnitudes scales. Recently, in [8] Chi et al. proposed an adaptive ILC approach to cope with a class of high-order discrete-time system with the references being iteration-varying. The authors provided iteration-recursive algorithms to estimate parameters and generate corresponding input signals. However, the iteration length is still unchanged in these studies except [7].

Thus one is interested in the tracking ability when the operation length varies randomly during different iterations. This situation also exists in some ILC applications [9,10]. The biomedical systems, functional electrical stimulation for upper limb movement and for gait assistance, were given in [9], and a humanoid robot study was provided in [10]. In these equipments, the learning process could not be with the same length every iteration because of complex factors and unknown dynamics. As one could see, the iteration-varying length would lead to varying outputs, which further results in different signals in different iterations. To cope with this issue, [11] introduced an iteration-average operator and then designed an ILC algorithm for discrete linear systems. It was

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shown that the tracking error and input error would converge to zero in mathematical expectation sense. However, few results on nonlinear system are reported.

This note proceeds to consider the ILC problem for nonlinear discrete-time systems with iteration varying lengths. There are three major differences between [11] and the current paper. First of all, an iteration-average operator was introduced in [11] for update law design, and thus all historical data should be stored for sustained updating. In this paper, the conventional P-type update law is used only with a modified tracking error. We will show that the simple P-type ensures a good tracking performance. Moreover, the conventional λ-norm technique was used in [11], while in this paper, we make a modification on the definition of λ-norm so that it becomes more appropriate for the randomly varying length problem. Last but not least, a stronger convergence result is obtained in this note. In [11], the expectation of the tracking error is shown to converge to zero, while this note provides the almost sure convergence of the tracking error. To be specific, the zero-error tracking performance is proved when the initial state is accurately reset, while for the case of initial shifts, it is shown that the tracking error is bounded in proportion to the bound of initial state bias.

In practical applications, the actual operation length may be greater or smaller than the expected length. If the actual length is greater than the expected length, then the redundant signals are discarded as none information could be gotten from this signals. Therefore, this case could be regarded as the full-length case as long as one directly cut the trajectory at the position of expected length. If the actual length is smaller than the expected length, then the signals at the missing time instances cannot be obtained, thus no information could be used to update the input. As a result, for expression to be concise, only the case that the actual length is no greater than the expected one is taken into account in this note.

The rest of the note is arranged as follows: Section 2 gives the problem formulation; Section 3 presents the design of ILC algorithm, while its convergence analysis is provided in Section 4; illustrative simulations are shown in Section 5 and Section 6 concludes this note.

Notations: $\|M\|$ denotes the Euclidean norm of a square matrix M. $\sigma(M)$ is the eigenvalue of M. $\mathbb{R}$ is the set of real numbers, while $\mathbb{R}^m$ is the m-dimensional space. $\mathbb{E}(\cdot)$ and $\mathbb{P}(\cdot)$ denote the mathematical expectation and probability, respectively. $\|\theta(t)\|_\lambda$ denotes the λ-norm of a vector $\theta(t)$, $\lambda > 0$, which is defined as $\|\theta(t)\|_\lambda = \sup_{t\in\mathcal{S}} \alpha^{-\lambda t}\mathbb{E}\|\theta(t)\|$ where $\alpha > 1$ is a suitable selected constant and $\mathcal{S}$ is a finite discrete set of t. $I_{n\times n}$ denotes the unit matrix with dimension $n \times n$, and the subscript $n \times n$ may be omitted when no misunderstanding is caused. Let $\mathbf{1}(event)$ be an indicator function meaning that it equals 1 if the event indicated in the bracket is fulfilled, and 0 if the event does not hold.

2. Problem formulation

Consider the following discrete affine nonlinear system

$$\begin{aligned} x_k(t+1) &= f(x_k(t)) + Bu_k(t) \\ y_k(t) &= Cx_k(t) \end{aligned} \tag{1}$$

where $k = 0, 1, \ldots$ denotes iteration index, t is time instance, $t \in \{0, 1, \ldots, N_d\}$, and N_d is the expected iteration length. $x_k(t) \in \mathbb{R}^n$, $u_k(t) \in \mathbb{R}^p$, and $y_k(t) \in \mathbb{R}^q$ denote state, input, and output, respectively. f is the nonlinear function. C and B are matrices with appropriate dimensions. Without loss of generality, it is assumed that CB is of full-column-rank.

Remark 1. Matrices B and C are assumed time-invariant in system (1) to make the expressions concise. They can be extended to the time-varying case, $B(t)$ and $C(t)$, and/or state dependent case, $B(x(t))$, without making any further effort (see the analysis details below). Moreover, it will be shown that the convergence condition is independent of $f(\cdot)$ in the following. This is the major advantage of ILC, that is, ILC focuses on the convergence property along iteration axis and requires little system information. In addition, it is evident that the nonlinear function could be time varying.

Let $y_d(t)$, $t \in \{0, 1, \ldots, N_d\}$ be the desired trajectory. $y_d(t)$ is assumed to be realizable, that is, there is a suitable initial state $x_d(0)$ and unique input $u_d(t)$ such that

$$\begin{aligned} x_d(t+1) &= f(x_d(t)) + Bu_d(t) \\ y_d(t) &= Cx_d(t). \end{aligned} \tag{2}$$

The following assumptions are required for the technical analysis.

A1. The nonlinear function $f(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$ satisfies global Lipschitz condition, that is, $\forall x_1, x_2 \in \mathbb{R}^n$,

$$\|f(x_1) - f(x_2)\| \le k_f\|x_1 - x_2\| \tag{3}$$

where $k_f > 0$ is the Lipschitz constant.

The global Lipschitz condition on nonlinear function is somewhat strong, although it is common in the ILC field for nonlinear systems. However, it should be pointed out that this assumption is imposed to facilitate the convergence derivations using λ-norm technique. With more efforts, the assumption could be extended to local Lipschitz case or continuous case [12,13].

A2. The identical initial condition is fulfilled, i.e., $x_k(0) = x_d(0), \forall k$.

The initial state may not be reset precisely every iteration in practical applications, but the bias is usually bounded. Thus one would relax the assumption A2 to the following one.

A3. The initial state could shift from $x_d(0)$ but should be bounded, i.e., $\|x_d(0) - x_k(0)\| \le \epsilon$ where ϵ is a positive constant.

Let N_d denote the expected length. The actual length, N_k, varies in different iterations randomly. Thus, two cases need to be taken into account, i.e., $N_k < N_d$ and $N_k \ge N_d$. For the latter case, it is observed that only the data at the first N_d time instances are used for input updating. In a consequence, without loss of any generality, one could regard the latter case as $N_k = N_d$. From another point of view, one could regard N_d as the maximum length of actual lengths. For the former case, the outputs at the time instance $N_k + 1, \ldots, N_d$ are missing, and therefore, they are not available for updating. In other words, the input signals for the former N_k time instances are only updated.

The control objective of this note is to design ILC algorithm to track the desired trajectory y_d, $t \in \{0, 1, \ldots, N_d\}$, based on the available output $y_k(t)$, $t \in \{0, 1, \ldots, N_k\}$, $N_k \le N_d$, such that the tracking error $e_k(t)$, $\forall t$ converges to zero with probability one as the iteration number k goes to infinity.

The following lemma is needed for the following analysis, and its proof is put in the Appendix.

Lemma 1. *Let η be a Bernoulli binary random variable with $\mathbb{P}(\eta = 1) = \overline{\eta}$ and $\mathbb{P}(\eta = 0) = 1 - \overline{\eta}$. M is a positive matrix. Then the equality $\mathbb{E}\|I - \eta M\| = \|I - \overline{\eta}M\|$ holds if and only if one of the following conditions is satisfied:* (1) $\overline{\eta} = 0$; (2) $\overline{\eta} = 1$; *and* (3) $0 < \overline{\eta} < 1$ *and* $0 < M \le I$.

3. ILC design

In this note, the minimum length is denoted by N_m. Then the operation length varies among the discrete integer set $\{N_m, \ldots, N_d\}$,