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Multi-consensus of multi-agent networks via a rectangular impulsive approach[☆]

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ABSTRACT

A multi-consensus problem is studied in multi-agent networks. The interaction mechanism of competition/abstention/cooperation among agents is introduced. Three rectangular impulsive protocols are proposed to solve multi-consensus of second order multi-agent networks with a directed topology. These algorithms have the performance of Dirac impulsive control and discrete-time control. Necessary and sufficient conditions are obtained for the stationary multi-consensus and the dynamic multi-consensus. Numerical examples are provided to illustrate the effectiveness of the obtained criteria.

1. Introduction

In the past two decades, distributed cooperative control of multi-agent networks has received attractive attentions from various research communities. Coordination control means that local communication and cooperation among individual agents may lead to certain desired global behaviors. To uncover the underlying mechanisms of collective behaviors, various mathematical or physical models have been proposed [1]. The collective activities of nature have inspired the designs of some practical engineering applications such as the formation control of multi-robot [2,3], the distributed computation [4], and the coordination control of distributed sensor networks [5,6]. As one of the most typical collective behaviors of multi-agent networks, consensus means that a group of agents converge to a common value of interest under some distributed protocols.

In real applications, the information transmission among agents is not continuous due to the unreliability of communication channels, the limited sensing ability of agents, and the constraints of total cost. The Dirac impulsive control and discrete-time control have been studied profoundly in multi-agent networks. On the one hand, the existing impulsive algorithms [7,8] have a faster convergence speed and cause abrupt changes of states at sampling instants. However, some practical multi-agent systems cannot bear the sudden changes of states. On the other hand, the discrete-time algorithms [9,10] converge slowly due to the sampled informations.

In many multi-agent networks, there may be multiple consistent states due to different environments, situations or even the time when the agents are carrying out a cooperative task. For example, in a military coordinated operation, the navy, the army, and the air need to finish different combat tasks, and then a common objective is achieved. Suitable protocols are designed to realize that the states of multiple agents in each subnetwork asymptotically converge to an individual consistent value when there exist information exchanges not only among the agents in the same subnetwork but also among the ones in different subnetworks, referred to as *multi-consensus* hereafter [11].

Multi-consensus of complex dynamic network can be divided into the following two types: first, the consistent value of each subnetwork is related to the initial value for a specific grouping;

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second, in the process of network evolution, the states of vertices degenerate into different groups, and the states of different groups will ultimately tend to an individual consistent value. Compared with group consensus [12] and cluster consensus [13] which only aim at a specific grouping, multi-consensus includes a wider concept and is more practical significance.

In coordination applications, the information flow may be directed, either due to heterogeneity, nonuniform communication powers, or sensing with a limited field of view. The case of directed topology is much more challenging than that of undirected one due to the fact that the adjacency matrix of a digraph is nonsymmetric. In this paper, multi-consensus of second order multi-agent networks with a directed topology is studied via a rectangular impulsive approach, in which a large multi-agent network is decomposed into some non-overlapping subnetworks. Necessary and sufficient conditions are derived based on matrix theory, under which the stationary multi-consensus and the dynamic multi-consensus can be reached in the presence of information exchanges among subnetworks.

2. Preliminaries and problem formulation

Notation: $\mathbf{0}_n = [0, \ldots, 0]^T \in \mathbb{R}^n$ and $\mathbf{1}_n = [1, \ldots, 1]^T \in \mathbb{R}^n$. Let $z \in \mathbb{C}$, $\mathrm{Re}(z)$ and $\mathrm{Im}(z)$ represent, respectively, the real and imaginary part, $\bar{z}$ denote the conjugate of z. $I_n \in \mathbb{R}^{n\times n}$ denotes the identity matrix and $O_{m\times n} \in \mathbb{R}^{m\times n}$ denotes the all-zero matrix. Let $diag(a_1, \ldots, a_n)$ be the diagonal matrix with diagonal entries $a_1, \ldots, a_n$.

2.1. Preliminaries

A weighted digraph $G = (V, E, W)$ of order n consists of a vertex set $V = \{1, \ldots, n\}$, a link set $E \subseteq V \times V$, and a nonnegative weighted adjacency matrix $W = [w_{ij}] \in \mathbb{R}^{n\times n}$. $e_{ij} = (j, i)$ indicates a directed link from vertex j to vertex i. $w_{ij} > 0$ if and only if $e_{ij} \in E$, and $w_{ij} = 0$ otherwise. Moreover, assume that there are no self-loops, i.e., $w_{ii} = 0$ for all $i \in V$. The in-neighbor set of vertex i is denoted by $N_i = \{j \in V : (j, i) \in E\}$. We call $d_i \triangleq \sum_{j=1}^{n} w_{ij}$ the in-weight of the vertex i and $D \triangleq diag(d_1, \ldots, d_n)$ the in-weight matrix of the digraph G. The Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{n\times n}$ of the digraph G is defined as $L \triangleq D - W$.

Before moving on, we need the following lemmas.

Lemma 1 ([14]). *Let $A_{11}, A_{12}, A_{21}, A_{22} \in \mathbb{R}^{n\times n}$ and $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$. Then, $\det(A) = \det(A_{11}A_{22} - A_{12}A_{21})$ if A_{11}, A_{12}, A_{21} and A_{22} commute pairwise.*

Lemma 2. *All the roots of $s^2 + as + b = 0$, where $a, b \in \mathbb{C}$, are within the circle with a radius r if and only if all the roots of $(r^2 + ar + b)z^2 + 2(r^2 - b)z + r^2 - ar + b = 0$ are in the open left half plane (LHP).*

Proof. By applying bilinear transformation $\frac{s}{r} = \frac{z+1}{z-1}$, the equation $s^2 + as + b = 0$ can be rewritten as

$$(r^2 + ar + b)z^2 + 2(r^2 - b)z + r^2 - ar + b = 0.$$

Notice that this bilinear transformation maps the interior of the circle with a radius r onto the open LHP.

Lemma 3 ([15]). *Consider a polynomial $Q(z) = z^2 + (m + \mathbf{i}n)z + p + \mathbf{i}q$, where m, n, p, and q are real constants. Then, $Q(z)$ is Hurwitz stable if and only if $m > 0$ and $mnq + m^2p - q^2 > 0$.*

2.2. Problem formulation

Definition 1. A network $G_k = (V_k, E_k, W_k)$ is said to be a subnetwork of the network $G = (V, E, W)$ if $V_k \subseteq V$, $E_k \subseteq E$, and W_k inherits W.

A network $G = (V, E, W)$ contains n vertices and consists of m $(m \geqslant 2)$ subnetworks $G_k = (V_k, E_k, W_k)$ $(k = 1, \ldots, m)$ with n_k vertices, where $\sum_{k=1}^{m} n_k = n$. Assuming that $V_i \neq \emptyset$, $\cup_{i=1}^{m} V_i = V$, and $V_i \cap V_j = \emptyset$ for any $i \neq j$, then the network is combined by m subnetworks plus a link set among them. Denote the vertex indexes of the kth subnetwork by $V_k = \{1+\sum_{i=1}^{k} n_{i-1}, \ldots, \sum_{i=1}^{k} n_i\}$, where $n_0 = 0$.

Consider n agents with second order discrete-time dynamics described as

$$\begin{cases} x_i(t_{l+1}) = x_i(t_l) + hv_i(t_l) + \dfrac{1}{2}h^2u_i(t_l), & \forall i \in V, \\ v_i(t_{l+1}) = v_i(t_l) + hu_i(t_l), \end{cases} \tag{1}$$

where $x_i(t_l)$, $v_i(t_l)$, and $u_i(t_l)$ are the position, velocity, and control input of agent i at the time $t = t_l$, respectively. Notice that (1) is the exact discrete-time dynamics based on zero order hold in a sampled-data setting. The sampling period $h = t_{l+1} - t_l$ is a positive constant.

Definition 2. Denoting the index of the subnetwork in which agent i lies by $\hat{i}$, two types of multi-consensus are defined for second order multi-agent networks as follows:

(1) The network is said to reach a stationary multi-consensus if for arbitrary initial values, the states satisfy

$$\begin{cases} \lim\limits_{l\to\infty} |x_i(t_l) - x_j(t_l)| = 0, & \forall \hat{i} = \hat{j}, \\ \lim\limits_{l\to\infty} v_i(t_l) = 0, \end{cases}$$

and

$$\lim_{l\to\infty} |x_i(t_l) - x_j(t_l)| > 0, \quad \forall \hat{i} \neq \hat{j}.$$

(2) The network is said to reach a dynamic multi-consensus if for arbitrary initial values, the states satisfy

$$\begin{cases} \lim\limits_{l\to\infty} |x_i(t_l) - x_j(t_l)| = 0, & \forall \hat{i} = \hat{j}, \\ \lim\limits_{l\to\infty} |v_i(t_l) - v_j(t_l)| = 0, \end{cases}$$

and

$$\lim_{l\to\infty} \sup |v_i(t_l) - v_j(t_l)| > 0, \quad \forall \hat{i} \neq \hat{j},$$

where $0 < \lim_{l\to\infty} \sup |v_i(t_l)| \leqslant \gamma_k$ and γ_k is a positive constant.

The dynamic multi-consensus includes two cases: first, if the positions of agents asymptotically converge to $a_k t + b_k$, where $a_k, b_k \in \mathbb{R}$, then we say that the network reaches the first dynamic multi-consensus; second, if the positions of agents asymptotically converge to $A_k \sin(\omega t + \theta_k)$, where $A_k, \omega, \theta_k \in \mathbb{R}$, then we say that the network reaches the second dynamic multi-consensus.

3. Rectangular impulsive protocols

In the following, we introduce a concept of intelligent impact factor (IIF), and denote the influence of agent j to agent i by ς_{ij}, where $\varsigma_{ij} \in \{-1, 0, 1\}$. When agent i competes with agent j, agent i expects to stay away from agent j, let $\varsigma_{ij} = -1$. When agent i abstains from agent j, agent j has no influence on agent i, let $\varsigma_{ij} = 0$. When agent i cooperates with agent j, agent i actively achieves consensus with agent j, let $\varsigma_{ij} = 1$.

Assumption 1. The agents in the same subnetwork can cooperate, and the agents in the different subnetworks can compete, abstain or cooperate. Moreover, the competition/abstention/cooperation of agent i to agent j does not indicate the competition/abstention/cooperation of agent j to agent i, respectively.