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Disturbance decoupling problem for multi-agent systems: A graph topological approach

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ABSTRACT

This paper studies the disturbance decoupling problem for multi-agent systems with single integrator dynamics and a directed communication graph. We are interested in topological conditions that imply the disturbance decoupling of the network, and more generally guarantee the existence of a state feedback rendering the system disturbance decoupled. In particular, we will develop a class of graph partitions, which can be described as a "topological translation" of controlled invariant subspaces in the context of dynamical networks. Then, we will derive sufficient conditions in terms of graph partitions such that the network is disturbance decoupled, as well as conditions guaranteeing solvability of the disturbance decoupling problem. The proposed results are illustrated by a numerical example.

1. Introduction

Analysis and design of multi-agent systems and networks of dynamical agents have turned to an extremely popular research target in the last decade. Studying consensus/synchronization and designing feedback protocols to achieve consensus/synchronization have perhaps been the most popular framework in this direction; see e.g. [1–4].

Motivated by the fact that agents are subjected to external disturbances in practice, consensus analysis in the presence of disturbance has been carried out in the literature, and consensus protocols which are robust against external disturbances have been proposed; see e.g. [5,4]. Another example of dynamical networks in which one seeks for disturbance rejection is balancing demand–supply in distribution networks; see e.g. [6,7]. In this framework, storage variables correspond to vertices, flow inputs corresponding to edges, disturbances amount for inflows and outflows at certain vertices, and the objective is to achieve load balancing, that is convergence of storage variables to the same value by regulating the flow inputs.

It is well-known that relative information of the agents play a crucial role in the context of distributed control. In fact, reaching consensus and achieving a desired formation heavily relies on the efficient and accurate transmission of relative information. In this paper, we investigate the conditions under which communication in certain channels is not affected by the external disturbances acting on some of the agents. To achieve this, we consider the possibility of applying state feedback controllers to some agents, called *leaders*.

An important issue in studying networks of dynamical agents is to deduce certain network properties from the "network topology" which is typically given in terms of the so-called "communication graph" of the network. For instance, it is well-known that connectivity of the communication graph plays a crucial role in the consensus problem (see e.g. [1]). Recently, studying network properties from a topological perspective has attracted the attention of many researchers see e.g. [8–11]. A notable instance is controllability analysis; see e.g. [12,11,13,10,14,15]. In this framework, agents are labeled as *leaders* and *followers*. Leaders are agents through which external input signals are injected to the network, and the rest of the agents are called followers. Then, controllability analysis amounts to investigate the possibility of deriving the states of the agents to arbitrary values by appropriate input signals applied to the leaders. Graph partitions, and in particular "almost equitable partitions", has been proven to be a useful tool in controllability analysis [12] and also model order reduction [16]. These partitions can be considered as a topological translation of L-invariant subspaces, with L denoting the Laplacian matrix of the network communication graph; see e.g. [17,12].

In this paper, we study the "disturbance decoupling problem" of diffusively coupled leader–follower networks, where each vertex

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has single integrator dynamics and some agents are affected by external disturbances. As mentioned earlier, roughly speaking, the disturbance decoupling problem (DDP) for a classical linear system with inputs and outputs, amounts to find a feedback (typically, state feedback) such that the output of the closed-loop system is not affected by disturbance signals acting on the states of the system, see e.g. [18]. If such a feedback exists, then we say the DDP for the system is solvable.

Despite being potentially attractive, only very few problems have been formulated in terms of disturbance decoupling problem. An exception is [19], where DDP has been related to formation control of nonholonomic mobile robots. The notion of disturbance decoupling problem has also been studied in the context of a dynamic game contest; see [20].

Our contribution comes from but goes beyond the disturbance decoupling solution in the *geometric approach* for linear systems. The geometric approach to linear system synthesis was inaugurated by the recognition of controlled invariant subspaces, due independently to Basile and Marro [21] and to Wohnam and Morse [22]. Disturbance decoupling problem is in fact an immediate application of the controlled invariance property, see e.g. [23]. To the authors' best knowledge, the current manuscript is the first attempt to study the disturbance decoupling problem for networks of dynamical agents from a topological perspective. Studying network properties such as controllability and DDP from a topological perspective provides valuable insights into the structure/behavior of the network, and it will facilitate the design.

In this paper, we introduce a new class of partitions, namely almost equitable partitions with respect to a cell, in order to provide an appropriate topological translation for controlled invariant subspaces in the context of dynamical networks. Then, by using this extended notion of almost equitability, we derive sufficient (topological) conditions for the network to be disturbance decoupled. More precisely, we consider both open loop and closed loop disturbance decoupling problem. In the first case, we investigate if the network is already disturbance decoupled without applying input signals to the leaders. In the latter case, we consider the solvability of the DDP for the network that amounts to find (if possible) a state feedback controller rendering the network disturbance decoupled. In particular, we establish sufficient topological conditions guaranteeing the network to be disturbance decoupled (open loop) as well as conditions guaranteeing the solvability of DDP (closed loop). A crucial point in the context of distributed control is to exploit the relative (local) information of the states of the agents rather than absolute (global) information of the states. As desired, it will be observed that in case the DDP for the network is solvable then the controller rendering the network disturbance decoupled is indeed using relative information of the states of the agents.

The structure of this paper is as follows. In Section 2, some preliminary materials are provided, and the open loop and closed loop disturbance decoupling problems for multi-agent systems are formulated. In Section 3, the notion of almost equitability with respect to a cell is proposed and characterized in terms of controlled invariant subspaces. In Section 4, we establish sufficient conditions guaranteeing the network to be disturbance decoupled as well as conditions guaranteeing the solvability of DDP. A brief discussion on algorithms verifying the proposed topological conditions is provided in Section 5. To illustrate the proposed results, a numerical example is provided in Section 6. Finally, the paper ends with a summary in Section 7.

2. Diffusively coupled multi-agent systems and disturbance decoupling

2.1. Leader–follower diffusively coupled multi-agent systems with disturbance

In this paper, we consider a multi-agent system consisting of $n > 1$ agents labeled by the set $V = \{1, 2, \ldots, n\}$. We assign three subsets of V as follows: $V_{\mathrm{L}} = \{\ell_1, \ell_2, \ldots, \ell_m\}$ where $m \leqslant n$, $V_{\mathrm{F}} = V \setminus V_{\mathrm{L}}$ and $V_{\mathrm{D}} = \{w_1, w_2, \ldots, w_r\}$ where $r \leqslant n$.

We associate the dynamics

$$\dot{x}_i(t) = \begin{cases} z_i(t) + u_k(t) + d_l(t) & \text{if } i = w_l \in V_{\mathrm{D}} \\ z_i(t) + u_k(t) & \text{otherwise} \end{cases} \tag{1}$$

to each agent $i = \ell_k \in V_{\mathrm{L}}$, and

$$\dot{x}_i(t) = \begin{cases} z_i(t) + d_l(t) & \text{if } i = w_l \in V_{\mathrm{D}} \\ z_i(t) & \text{otherwise} \end{cases} \tag{2}$$

to each agent $i \in V_{\mathrm{F}}$, where $x_i \in \mathbb{R}$ represents the state of agent $i \in V$, z_i indicates the coupling variable of agent $i \in V$, $u_k \in \mathbb{R}$ is an external control input signal received by agent $i = \ell_k \in V_{\mathrm{L}}$, and $d_l \in \mathbb{R}$ is taken as an external disturbance signal influencing agent $i = w_l \in V_{\mathrm{D}}$.

Considering the roles of the defined subsets of V, we refer to V_{L} as the *leader set*, V_{F} as the *follower set*, and V_D as the *disturbance set*. Correspondingly, we say i is a *leader* if $i \in V_L$, and i is a *follower* if $i \in V_F$.

We consider a simple directed graph $G = (V, E)$, where V is the vertex set and $E \subseteq V \times V$ is the arc set of G. For the sake of simplicity and clarity of the presentation, we restrict ourselves to unweighted graphs. For two distinct vertices $i, j \in V$, we have $(i, j) \in E$ if there is an arc from i to j with i being the *tail* and j being the *head* of the arc. Then i is said to be a *neighbor* of j. The coupling variable z_i admits the following *diffusive coupling* rule:

$$z_i(t) = -\sum_{(j,i) \in E} (x_i(t) - x_j(t)). \tag{3}$$

By defining $x(t) = \mathrm{col}(x_1(t), x_2(t), \ldots, x_n(t))$, $u(t) = \mathrm{col}(u_1(t), u_2(t), \ldots, u_m(t))$ and $d(t) = \mathrm{col}(d_1(t), d_2(t), \ldots, d_r(t))$, we write the above leader–follower diffusively coupled multi-agent system (1)–(3) into a compact form as follows:

$$\dot{x}(t) = -Lx(t) + Mu(t) + Sd(t) \tag{4}$$

where L is the in-degree Laplacian of the simple directed graph G (see e.g. [8, p. 26]), the matrix $M \in \mathbb{R}^{n \times m}$ is defined by

$$M_{ik} = \begin{cases} 1 & \text{if } i = \ell_k \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

and the matrix $S \in \mathbb{R}^{n \times l}$ is defined by

$$S_{il} = \begin{cases} 1 & \text{if } i = w_l \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

To introduce the output variables, we consider another simple directed graph $G_y = (V, E_y)$ and define the output $y(t)$ of the system (4) as follows:

$$y(t) = R^\top x(t) \tag{7}$$

where R is the incidence matrix of G_y (see e.g. [8, p. 23]). Observe that the output variables (7) capture the differences between the state components of certain pairs of agents determined by the arc set E_y of G_y. In particular, an arc from i to j in G_y corresponds to the output variable $x_i - x_j$ in (7).

In this paper, we study the so-called *disturbance decoupling problem* for multi-agent system (4) by establishing graph topological conditions. Roughly speaking, our aim is to investigate the effect of the disturbance signal d on the output y, given by (7).

For a formal description of the problem and discussing the proposed results, we first review the disturbance decoupling problem and its solution for ordinary linear systems.