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Realization of full column rank precompensators using stabilizing static state feedback[☆]

Eduardo Castañeda-Toledo[a], Vladimír Kučera[b,c,*], Javier Ruiz-León[a]

[a] Centro de Investigación y de Estudios Avanzados del Instituto Politécnico Nacional, Guadalajara, Mexico
[b] Faculty of Electrical Engineering, Czech Technical University in Prague, Czech Republic
[c] Czech Institute of Informatics, Robotics, and Cybernetics, Czech Technical University in Prague, Czech Republic

HIGHLIGHTS

- Suppose a compensator is specified so as to alter the transfer function of the system
- Suppose state feedback is applied to the system to realize the compensation
- Non-square compensators result in non-regular state feedback
- Such a feedback law is not unique and affects the controllability of the system
- Conditions are determined for a stable match to exist

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ABSTRACT

In some control problems, it is convenient to use a precompensator to alter the transfer function of the system such that the transfer function of the compensated system has a specified property. Then, if possible, the action of the compensator on the system is realized by a static state feedback law applied to the system. When the compensator is square and non-singular, the state feedback is regular and the problem has already been solved. Non-square compensators, however, result in non-regular state feedback. In this paper, necessary and sufficient conditions are presented for the compensated system to be stabilizable by such a non-regular static state feedback law.

1. Introduction

A control law that is widely used in theory and applications is static state feedback. In some problems, however, the specifications are given in terms of the closed-loop transfer function. Examples include decoupling, model matching, and disturbance rejection. Although these problems can be solved using the state space formalism [1], an alternative way to tackle them is to use a precompensator to alter the transfer function of the system so as the transfer function of the compensated system has a specified property. Then, it is investigated whether there exists a state feedback having the same input–output effect that the given precompensator.

Thus, the key problem consists in finding conditions under which the desired compensator is static state feedback realizable. In fact, necessary and sufficient conditions exist in the literature. The seminal work was published by Hautus and Heymann [2] for nonsingular compensators. The realizability conditions are stated in terms of a polynomial matrix description of the system. Isidori and Morse [3] gave necessary and sufficient conditions in terms of the McMillan degree of a rational matrix. Castro and Ruiz-León [4] presented equivalent realizability conditions using structural invariants of the system. The general case of non-square full column rank compensators was considered by Kučera and Herrera [5], Herrera [6,7], and Castañeda and Ruiz-León [8], where different realizability conditions are presented. In fact, the conditions given in [5] are based on a polynomial matrix description of the system, the conditions in [6] are expressed in terms of a constant basis for the left kernel of a rational matrix, and the conditions presented in [8] make use of structural invariants of the system.

When a compensator is static state feedback realizable, then stability of the compensated closed-loop system is an issue. The first necessary and sufficient condition for the compensated system to be asymptotically stable was given by Kučera [9] in the

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* Correspondence to: Faculty of Electrical Engineering, Czech Technical University in Prague, Technická 2, 16627 Prague 6, Czech Republic. Tel.: +420 224 355 700.
E-mail addresses: castaned@gdl.cinvestav.mx (E. Castañeda-Toledo), kucera@fel.cvut.cz (V. Kučera), jruiz@gdl.cinvestav.mx (J. Ruiz-León).

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case of nonsingular compensators. The condition is expressed in terms of a polynomial matrix description of the system. For non-square full column rank compensators, a similar stability condition was provided by Kučera and Herrera [5] which, however, turned out to be only sufficient. Thus, stabilizability by non-regular static state feedback, when realizing the action of a non-square full column rank compensator on the given system, remains an open problem.

In this paper, we present necessary and sufficient conditions under which the compensated system can be stabilized. The difficulty of the problem arises from two facts. Firstly, non-regular state feedback that realizes a given non-square full rank compensator is not unique. Secondly, non-regular state feedback affects controllability of the compensated system. To solve the problem, we show that the controllable part of the compensated system is the same for any feedback used to realize the compensator, and that the freedom inherent in this feedback can be used to alter the uncontrollable part of the compensated system. Thus, conditions are obtained under which the compensated system can be rendered asymptotically stable.

2. Preliminaries

Throughout the paper, R denotes the field of real numbers. Accordingly, R^n stands for the n-vector space over R and $\mathrm{R}^{m\times r}$ stands for the set of $m \times r$ matrices with entries in R. Finally, $\mathrm{R}_p(s)$ denotes the ring of proper rational functions over R and $\mathrm{R}_p^{m\times r}(s)$ denotes the set of $m \times r$ matrices with entries in $\mathrm{R}_p(s)$.

Let (A, B, C, D) be a state space representation of a linear time-invariant differential system described by the equations

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) + Du(t), \end{aligned} \tag{1}$$

where $x \in \mathrm{R}^n$, $u \in \mathrm{R}^m$ and $y \in \mathrm{R}^p$ are the state, input and output vectors of the system, respectively. The transfer function matrix of the system is given by

$$T(s) = C(sI - A)^{-1}B + D. \tag{2}$$

Consider a static state feedback law (F, G) of the form

$$u(t) = Fx(t) + Gv(t), \tag{3}$$

where $v \in \mathrm{R}^r$ is a new input vector, $F \in \mathrm{R}^{m\times n}$ and $G \in \mathrm{R}^{m\times r}$ with $\operatorname{rank} G = r$. The closed-loop system defined by (1) and (3) then gives rise to the transfer function matrix

$$T_{F,G}(s) = (C + DF)(sI - A - BF)^{-1}BG + DG. \tag{4}$$

If $r = m$ (matrix G is square and nonsingular), then (3) is said to be regular state feedback, and if $r < m$, then (3) is said to be non-regular state feedback.

After some algebraic manipulations, (4) can be expressed as

$$T_{F,G}(s) = T(s)W(s), \tag{5}$$

where

$$W(s) = \left[I - F(sI - A)^{-1}B\right]^{-1} G \tag{6}$$

is a proper rational matrix. In the case of regular state feedback, (6) is a biproper matrix, i.e. a square proper rational matrix whose inverse exists and is also proper rational. In the case of non-regular state feedback, (6) is column biproper, i.e.

$$\operatorname{rank} \lim_{s\to\infty} W(s) = r$$

holds so that $W(s)$ can be completed to a biproper matrix.

Thus, the action of state feedback (F, G) on the system (A, B, C, D) can be represented in transfer function form as the postmultiplication of the system transfer function matrix $T(s)$ by a proper rational matrix $W(s)$.

The converse problem, i.e. under which conditions a proper rational matrix that postmultiplies $T(s)$ can be realized using a static state feedback law applied to the system, is known as feedback realizability of compensators. Then, a given proper rational compensator $W(s)$ is said to be feedback realizable if there exists a static state feedback control law (F, G) such that (6) holds.

Feedback realization of compensators is useful in problems where, even when state feedback is to be applied, the specifications are given in terms of the transfer function matrix of the closed-loop system. For example, in decoupling where the closed-loop transfer function matrix is to be diagonal, or in model matching where it is to match a desired transfer function matrix.

Conditions for static state feedback realization of compensators are well known. The following result applies to square nonsingular compensators.

Lemma 1 ([2]). *Given a system (A, B, C, D), let $Q(s), P(s)$ be right coprime polynomial matrices such that*

$$(sI - A)^{-1}B = Q(s)P^{-1}(s)$$

and let $W(s) \in \mathrm{R}_p^{m\times m}(s)$ be a nonsingular compensator. Then $W(s)$ is feedback realizable if and only if

(a) *$W(s)$ is biproper, and*
(b) *$W^{-1}(s)P(s)$ is a polynomial matrix.* □

The required regular static state feedback is linked to a constant solution (U, V) of the polynomial equation [9]

$$W^{-1}(s)P(s) = UP(s) + VQ(s),$$

where U is a nonsingular matrix. Then the regular static state feedback realizing the compensator is given by

$$F = -U^{-1}V, \qquad G = U^{-1}.$$

The following result applies to non-square full column rank compensators.

Lemma 2 ([6]). *Given a system (A, B, C, D) and let $W(s) \in \mathrm{R}_p^{m\times r}(s)$ be a full column rank compensator. Then $W(s)$ is feedback realizable if and only if the left Kronecker indexes[1] $\{\mu_i\}$ of the matrix*

$$H(s) := \begin{bmatrix} (sI - A)^{-1}BW(s) \\ \bar{W}(s) \end{bmatrix}$$

where $\bar{W}(s)$ is the strictly proper part of $W(s)$, satisfy the following two conditions:

(a) *For some integer q,*

$$\begin{aligned} &\mu_1 = \cdots = \mu_{m+q} = 0 \\ &0 < \mu_{m+q+1} \le \mu_{n+m-p} \\ &p := \operatorname{rank} H(s). \end{aligned}$$

(b) *Among the rows corresponding to $\mu_1, \ldots, \mu_{m+q}$ in a minimal basis for the left kernel of $H(s)$, there exist m rows that*

[1] A minimal basis for the left kernel of a proper rational matrix $H(s)$ is given by a set of polynomial vectors such that the sum of their degrees is minimal. The minimal degrees $\{\mu_i\}$ of the polynomial vectors in a minimal basis for the left kernel of $H(s)$ correspond to the left Kronecker indexes of $H(s)$ [10].