Systems & Control Letters 65 (2014) 1–12

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Two triggered information transmission algorithms for distributed moving horizon state estimation☆

Jing Zhang, Jinfeng Liu *

Department of Chemical & Materials Engineering, University of Alberta, Edmonton, AB T6G 2V4, Canada

ARTICLE INFO

Article history:
Received 2 August 2013
Received in revised form
17 October 2013
Accepted 2 December 2013
Available online 14 January 2014

Keywords:
Nonlinear systems
Distributed state estimation
Moving horizon estimation
Information update algorithms
Chemical processes

ABSTRACT

In this work, we consider the reduction of information transmission frequency of distributed moving horizon estimation (DMHE) for a class of nonlinear systems in which interacting subsystems exchange information with each other through a shared communication network. Specifically, algorithms based on two event-triggered methods are proposed to reduce the number of information transmissions between the subsystems in a DMHE scheme. In the first algorithm, a subsystem sends out its current information when a triggering condition based on the difference between the current state estimate and a previously transmitted one is satisfied; in the second algorithm, the transmission of information from a subsystem to other subsystems is triggered by the difference between the current measurement of the output and its derivatives and a previously transmitted measurement. In order to ensure the convergence and ultimate boundedness of the estimation error, we also propose to redesign the local moving horizon estimator of a subsystem to account for the possible lack of state updates from other subsystems explicitly. A chemical process is utilized to demonstrate the applicability and performance of the proposed approaches.

1. Introduction

Due to the increasing global competition, large-scale complex chemical processes are common appearances in the modern process industry due to their economic efficiency. Distributed model predictive control (DMPC) has emerged as an attractive control approach to handle the scale and interactions of large-scale complex chemical processes (e.g., [1,2]). It has been demonstrated that DMPC can achieve improved closed-loop performance while preserving the flexibility of the decentralized framework [3,4]. However, most of the existing DMPC designs were developed under the assumption that the state measurements of subsystems are available. It is in general difficult to measure all the state variables in a process system. This makes state estimation very important in feedback control. For nonlinear systems, observer design problems are challenging and have attracted significant attention (e.g., [5–8]). However, the majority of these results were obtained in a centralized framework.

It is desirable to adopt distributed (or decentralized) state estimation methods along with a distributed optimal control system like DMPC to maintain the structural flexibility. In the literature, there are existing results on decentralized observer designs for different classes of systems (e.g., [9–11]) and distributed Kalman filtering based on consensus algorithms with applications to sensor networks (e.g., [12–14]). These results are primarily developed for linear systems. Recently, a distributed moving horizon estimation (DMHE) approach was first developed for linear systems [15] and then extended to nonlinear systems [16]. Along this line of work, in [17,18], DMHE schemes based on subsystem models were developed for both linear and nonlinear systems. Because the above DMHE schemes were developed based on the classical centralized moving horizon estimation (MHE) [19,20], they maintain the advantages of MHE including the ability to handle nonlinearities, constraints and optimality considerations explicitly. However, as in the centralized MHE, the convergence of the estimates of the above DMHE schemes to the actual system state requires a reliable approximation of the arrival cost which is in general a difficult task for constrained nonlinear systems. Moreover, the convergence rates of the estimates given by the above DMHE schemes to the actual system states are not tunable which is not favorable from an output feedback control point of view.

To address the above issues, in our previous work [21], we developed a robust DMHE design for a class of nonlinear systems with bounded uncertainties. In the design of a local MHE, an auxiliary nonlinear deterministic observer was taken advantage of to calculate a confidence region for the state estimate every sampling time. The estimate of the local MHE is restricted within the confidence region. This approach was demonstrated to reduce the effect of poorly approximated arrival cost and was proved to lead

☆ Financial support from Natural Sciences and Engineering Research Council of Canada is gratefully acknowledged.

* Corresponding author. Tel.: +1 780 492 1317; fax: +1 780 492 2881.
E-mail address: jinfeng@ualberta.ca (J. Liu).

0167-6911/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.sysconle.2013.12.003

to bounded estimation error with potential tunable convergence rates. It, however, requires the subsystems to exchange information every sampling time. The frequent information transmission requirement may impede the application of the DMHE to processes that have a shared communication network with limited capacity. Moreover, extensive information exchanging may reduce the robustness of the system due to data dropouts in the communication network.

Motivated by the above observations, in this work, we propose two algorithms to reduce the number of information transmissions between subsystems based on the DMHE framework developed in [21] via event-triggered approaches. Event-triggered approaches have been widely used in the design of control systems that have shared communication and computation resources (e.g., [22–24]). Event-triggered approaches have also been adopted in state estimation in wireless sensor networks (see, e.g., [25,26]) to reduce information transmission in the network while maintaining stability and performance. In these results, a centralized model in general is used to design the centralized/distributed estimators. Reviews of results on event-triggered feedback in control and estimation can be found in [27,28]. Within process control, in [29], a coordinated quasi-decentralized framework was developed to minimize the information exchange for networked control systems. The quasi-decentralized framework was recently utilized in the design of an output feedback model predictive control (MPC) with an adaptive forecast-triggered communication strategy to minimize the number of information transmissions from a supervisory observer to local controllers [30]. In [31], an event-based approach was adopted to reduce the evaluation times of a centralized MHE–MPC system. In [31], the triggering condition was designed in a centralized manner based on output and its derivative measurements.

The contributions of the present work are as follows: (1) the design of triggering conditions for state estimation system in the DMHE framework for nonlinear systems; (2) the development of two different triggering conditions for a DMHE scheme; and (3) the rigorous stability analysis of the DMHE scheme with the two triggering conditions. Specifically, in this work, in the first proposed algorithm, a subsystem sends out its current information when a triggering condition based on the difference between the current state estimate and a previously transmitted state estimate is satisfied; in the second proposed algorithm, the transmission of information from a subsystem to other subsystems is triggered by the difference between the current measurement of the output and its derivatives and a previously transmitted measurement. In order to ensure the convergence and ultimately boundedness of the estimation error, the local MHE of a subsystem also needs to be redesigned to account for the possible lack of state updates from other subsystems. Sufficient conditions for the proposed DMHE implemented following the two algorithms to ensure the convergence and ultimately boundedness of the estimation error are derived. The application to a chemical process illustrates the effectiveness of the proposed approaches in reducing the number of information transmissions between the subsystems while maintaining the estimation performance.

Notation. The operator $|\cdot|$ denotes Euclidean norm while $|\cdot|_Q$ indicates the weighted Euclidean norm, defined as $|x|_Q = \sqrt{x^TQx}$ where Q is a positive definite square matrix. A function $f(x)$ is said to be locally Lipschitz with respect to x if there exists a positive constant L_f^x such that $|f(x') - f(x'')| \leq L_f^x|x' - x''|$ for all x' and x'' in a given region of x and L_f^x is the Lipschitz constant. A function $\beta(r, s)$ is said to be a class $\mathcal{KL}$ function if for each fixed s, $\beta(r, s)$ is strictly increasing and satisfies $\beta(0, s) = 0$ with respect to r, and for each fixed r, it is decreasing with respect to s, and $\beta(r, s) \to 0$ as $s \to \infty$. The symbol diag(v) denotes a diagonal matrix whose diagonal elements are the elements of vector v. The superscript $^{(s)}$ denotes the s-th order time derivative of a function. The symbol $L_f h$ denotes the Lie derivative of function h with respect to f, defined as $L_f h(x) = \frac{\partial h}{\partial x} f(x)$ while $L_f^r h$ denotes r-th order Lie derivative, defined as $L_f^r h(x) = L_f L_f^{r-1} h(x)$. A matrix (or vector) A^+ denotes the pseudoinverse of a matrix (or vector) A.

2. Preliminaries

2.1. Problem formulation

In this work, we consider a class of nonlinear systems composed of m interconnected subsystems. Each subsystem can be described by the following state-space model:

$$\begin{aligned} \dot{x}_i(t) &= f_i(x_i(t), w_i(t)) + \tilde{f}_i(X_i(t)) \\ y_i(t) &= h_i(x_i) + v_i(t) \end{aligned} \tag{1}$$

where $i = 1, \ldots, m$, $x_i(t) \in \mathbb{R}^{n_{x_i}}$ denotes the vector of state variables of subsystem i, $w_i(t) \in \mathbb{R}^{n_{w_i}}$ denotes disturbances associated with subsystem i, and the vector function f_i characterizes the dependence of the dynamics of x_i on itself and the associated disturbances. The vector function $\tilde{f}_i$ characterizes the interactions between subsystem i and other subsystems. The vector $y_i \in \mathbb{R}^{n_{y_i}}$ is the measured output of subsystem i and $v_i \in \mathbb{R}^{n_{v_i}}$ is a measurement noise vector. The state vector $X_i(t)$ is composed of subsystem states involved in the expression of the interaction of subsystem i with other subsystems. Specifically, we use $\mathbb{I}_i$, $i = 1, \ldots, m$, to denote the set of subsystem indices whose corresponding subsystem states are involved in X_i. It should be noted that x_i may be involved in X_i. Moreover, we use the set $\mathbb{C}_i$, $i = 1, 2 \ldots, m$, to denote the set of subsystems whose dynamics has dependence on x_i. The subsystem states x_i, $i = 1, \ldots, m$, are assumed to be contained in convex compact sets such that $x_i \in \mathbb{X}_i$, $i = 1, \ldots, m$. It is also assumed that the system disturbances and measurement noise are bounded such that $w_i \in \mathbb{W}_i$ and $v_i \in \mathbb{V}_i$, $i = 1, \ldots, m$, where $\mathbb{W}_i := \{w_i \in \mathbb{R}^{n_{w_i}} : |w_i| \leq \theta_{w_i}\}$, $\mathbb{V}_i := \{v_i \in \mathbb{R}^{n_{v_i}} : |v_i| \leq \theta_{v_i}\}$ with θ_{w_i} and θ_{v_i}, $i = 1, \ldots, m$, known positive real numbers.

The entire nonlinear system state vector and measured output vector are denoted as x and y which are composed of the states and outputs of the m subsystems, respectively. That is $x = [x_1^T \cdots x_i^T \cdots x_m^T]^T \in \mathbb{R}^{n_x}$ and $y = [y_1^T \cdots y_i^T \cdots y_m^T]^T \in \mathbb{R}^{n_y}$. The entire system can be described as follows:

$$\begin{aligned} \dot{x}(t) &= f(x(t), w(t)) + \tilde{f}(x(t)) \\ y(t) &= h(x(t)) + v(t) \end{aligned} \tag{2}$$

where $f, \tilde{f}, w, h$, and v are appropriate compositions of $f_i, \tilde{f}_i, w_i, h_i$, and v_i, $i = 1, \ldots, m$, respectively.

2.2. Modeling of measurements

In this work, we consider that the outputs of the m subsystems, y_i, $i = 1, \ldots, m$, are sampled synchronously and periodically at time instants $\{t_{k\geq 0}\}$ such that $t_k = t_0 + k\Delta$ with $t_0 = 0$ the initial time, Δ a fixed sampling time interval and k positive integers. It is also assumed that the measurements of the time derivatives of the outputs, $\dot{y}_i, \ldots, y_i^{(n-1)}$, $i = 1, \ldots, m$, are available at each sampling time. The two assumptions imply that a measurement of the following vector is available for each subsystem at each sampling time:

$$Y_i(t) = \begin{bmatrix} y_i(t) \\ \dot{y}_i(t) \\ \vdots \\ y_i^{(n-1)}(t) \end{bmatrix}$$