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Consensusability of multi-agent systems with time-varying communication delay ☆

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ABSTRACT

This paper studies the consensusability problem of continuous-time multi-agent systems with time-varying communication delay in undirected network. We design a consensus protocol based on the low gain solution of a parametric algebraic *Riccati* equation (ARE) and not use the precise information of the amount of time-varying delay. By studying the joint effect of agent dynamic and network topology, sufficient conditions are given for consensus when all poles of the system matrix are on the closed left-half plane. In case of non-zero poles on the imaginary axis, maximal admissible upper bound of the time-varying delay is given in terms of both the agent dynamic and network topology; otherwise, consensus can be achieved for the time-varying delay with arbitrarily large upper bound. Finally, simulation results are presented to demonstrate the effectiveness of the theoretical results.

1. Introduction

Consensus problem as one of the fundamental problems in the area of distributed coordination control, has attracted much interest in recent years. This problem has wide applications, such as formation control [1,2], flocking [3], distributed filtering [4], and synchronization of coupled chaotic oscillators [5]. Consensus problem means to design a networked interaction protocol, such that all the agents reach an agreement asymptotically or in a finite time. The common property is that each individual agent lacks global knowledge of the whole systems and can only gain relative information from its neighbors in the network [6]. Based on the circumstances, both relative states [7] and relative outputs [8] can be used.

The technique for studying consensus protocol is always based on the relation between the structure of agent dynamic and network communication. In the last decades, some theoretical results have been established, to name a few. For Vicsek's model, where all the agents move in a plane at the same speed but with different headings, Jadbabaie et al. [9] provided a consensus protocol to guarantee all the agents eventually move in the same direction. Ren et al. [10] utilized matrix analysis and algebraic graph, extending the results in [9] from the bidirectional case to unidirectional case and giving more relaxable conditions. Moreau [11] used a set-valued Lyapunov approach to study consensus problem with unidirectional time-dependent communication links. Ma et al. [7] and You et al. [12] respectively provided a necessary and sufficient condition for reaching consensus of continuous and discrete cases with generic linear agent dynamic under a common consensus protocol.

In practice, networked multi-agent systems delay is unavoidable in the process of communicating information, which should be taken into consideration. So far, there are many works to address delay issues. Olfati-Saber et al. [13] investigated the consensus problem in frequency domain and provided a necessary and sufficient condition on the upper bound of the communication delay. Sun et al. [14] dealt with average consensus problem in undirected networks with time-varying communication delays. Liu et al. [15] studied the consensus problem with delays and noises in transmission channels. For the second order case, Qin et al. [16] presented a sufficient condition of all the agents reaching consensus under a constant communication delay. Under the assumption that all the poles of the system matrix are on the closed left-half plane, Wang et al. [17] studied the consensus of high-order multi-agent systems that were coupled through networks with communication delay. For the general linear multi-agent systems, Zeng et al. [18] considered the consensus problem with communication and input delays. Zhang et al. [19] investigated the case of communication delay and focused on searching an allowable delay bound.

This paper considers the consensusability of dynamic multi-agent systems with communication delay and only the distributed information is utilized to design protocol. Our main contribution is to derive the consensus when all the poles of the system matrix

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lie on the closed left-half plane and the delay is time-varying. Thus the system is more general than the one studied in [13–15]. Due to the delay, the technique adopted in [7] that without communication delay is invalid. The method of Lyapunov–Krasovskii functional analysis used in [16,19] cannot work since the time-varying property of delay. The way by analyzing the characteristic function of the delayed system in [17,18] is also unavailable because of the time-varying delay. To overcome these difficulties, we design a consensus protocol based on low gain feedback [20,21] which depends on the solution of a parametric ARE. It should be emphasized that the design of the control gain is much more challenging than that in [21] due to the simultaneous stabilization of a number of systems in this paper.

The rest of this paper is organized as follows. Some preliminary results of graph theory are reviewed in Section 2. Section 3 states the problem to be investigated. In Section 4, sufficient conditions on consensusability of multi-agent are given. Simulation studies are given in Section 5. Some concluding remarks are drawn in the last section.

The following notations will be used throughout this paper. The sets of real numbers, positive numbers are denoted by $\mathcal{R}$ and $\mathcal{R}^+$. For any positive integer N, let $\mathcal{N} = \{1, \ldots, N\}$. The notation A^T represents the transpose of matrix A. Symbol $diag\{A_1, \ldots, A_N\}$ is a diagonal matrix with main diagonal block matrices A_i, $i \in \mathcal{N}$, and the off-diagonal blocks are zero matrices. For a positive scalar τ, let $\mathscr{C}_{n,\tau}$ denote the Banach space of continuous vector function mapping the interval $[-\tau, 0]$ into $\mathcal{R}^n$. Denote $\otimes$ the *Kronecker* product [22].

2. Preliminaries

In this section, we provide a brief introduction about algebraic graph theory.

Let $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ be an undirected graph, where $\mathcal{V} = \{1, \ldots, N\}$ is the index of N vertices; $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represents the edges, and weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathcal{R}^{N\times N}$ is a nonnegative 0-1 matrix. An edge of $\mathcal{G}$ is denoted by $e_{ij} = (i, j)$. Self-edge (i, i) is not allowed, i.e., $(i, j) \in \mathcal{E}$ if and only if $a_{ij} = 1$. The set of neighbors of vertex i is denoted by $N_i = \{j \in \mathcal{V}, (i, j) \in \mathcal{E}\}$. An undirected graph is connected if any two distinct vertices of the graph can be connected via a path that follows the edges of the graph. The degree of vertex i is defined as $deg(i) = \sum_{j=1}^{N} a_{ij}$, and the *Laplacian* matrix of $\mathcal{G}$ is defined as $L_{\mathcal{G}} = D - \mathcal{A}$, where $D = diag\{deg(1), \ldots, deg(N)\}$. So, every row sum of $L_{\mathcal{G}}$ is zero, i.e., $L_{\mathcal{G}}\mathbf{1}_N = 0$, where $\mathbf{1}_N = [1, 1, \ldots, 1]^T \in \mathcal{R}^N$ is a right eigenvector corresponding to eigenvalue 0. If $\mathcal{G}$ is connected, all the eigenvalues of $L_{\mathcal{G}}$ are positive except one 0 eigenvalue, and can be written as $0 = \lambda_1 < \lambda_2 \le \cdots \le \lambda_N$.

3. Problem statement

In this paper, we consider a system consisting of N agents indexed by $1, 2, \ldots, N$, respectively. The dynamic of the ith agent is described as:

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad i \in \mathcal{N}, \tag{1}$$

where $x_i(t) \in \mathcal{R}^n$ and $u_i(t) \in \mathcal{R}^m$ are the state and input of agent i. $A \in \mathcal{R}^{n\times n}$ and $B \in \mathcal{R}^{n\times m}$ are constant matrices. It is assumed that all the poles of A are on the closed left-half plane. In addition, initial value $x_i(0)$ is known to agent i and its neighbors.

We regard the above N agents as vertices of a network topology, and the relationships among them can be conveniently described by an undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ with $\mathcal{V} = \{1, 2, \ldots, N\}$ and $\mathcal{A} = [a_{ij}] \in \mathcal{R}^{N\times N}$.

Note that the communication delay is common in network control. Let $\tau(t) : [0, \infty) \to \mathcal{R}^+$ denote the time-varying delay during the information communication. Thus, it is available to design the control protocol $u_i(t)$ using relative states at time $t - \tau(t)$. The proposed control protocol is as follows:

$$u_i(t) = K\sum_{j=1}^{N} a_{ij}[x_j(t - \tau(t)) - x_i(t - \tau(t))], \tag{2}$$

where $K \in \mathcal{R}^{m\times n}$ is a fixed control gain and independent of the agent index i. To make the system (1) operational under protocol (2), we additionally set the initial values $x_i(t) = 0$ for any $t < 0$ and $i \in \mathcal{N}$. Now, we introduce the definition of consensusability.

Definition 1. The multi-agent system (1) is said to be consensusable under protocol (2) if for any finite $x_i(0)$, $\lim_{t\to\infty} \|x_j(t) - x_i(t)\| = 0$, $\forall i, j \in \mathcal{N}$. Furthermore, if $\lim_{t\to\infty} x_i(t) = \frac{1}{N}\sum_{j=1}^{N} x_j(0)$, we say protocol $u_i(t)$ asymptotically solves the average consensus problem.

Problem Statement: Our aim is to find the feedback gain K in the distributed protocol (2) such that the multi-agent system (1) with time-varying communication delay are consensusable.

From the assumption that all the poles of A are on the closed left-half plane, when A is stable, i.e., $Re(\lambda(A)) < 0$, it is trivial to choose the feedback gain $u_i(t) = \mathbf{0}$ to make the multi-agent systems achieve consensus. Without loss of generality, assume (A, B) have the following form after an appropriate coordinate transformation, $A = \begin{bmatrix} A_0 & 0 \\ 0 & A_s \end{bmatrix}$, $B = \begin{bmatrix} B_0 \\ B_s \end{bmatrix}$, where $A_0 \in R^{n_0\times n_0}$ contains all eigenvalues of A that are on the imaginary axis and $A_s \in R^{n_1\times n_1}$ contains all the eigenvalues of A that have negative real part, with $n_1 + n_0 = n$. Denote $x_i(t) = [(x_i^0(t))^T, (x_i^s(t))^T]^T$, $K = [K_0, K_s]$. Let $K_s = \mathbf{0}$, where $\mathbf{0}$ is a compatible dimension matrix with zero elements. Then, it follows that

$$x_i^0(t) = A_0x_i^0(t) + B_0K_0\sum_{j=1}^{N} a_{ij}[x_j^0(t - \tau(t)) - x_i^0(t - \tau(t))],$$

$$x_i^s(t) = A_sx_i^s(t) + B_sK_0\sum_{j=1}^{N} a_{ij}[x_j^0(t - \tau(t)) - x_i^0(t - \tau(t))].$$

Since $Re(\lambda(A_s)) < 0$, consensusability of the multi-agent systems can be directly obtained only from $\lim_{t\to\infty} \|x_i^0(t) - x_j^0(t)\| = 0$, $\forall i, j \in \mathcal{N}$, which is only related with A_0. Therefore, it is natural to confine the following assumption for the system matrix:

Assumption 1. All the eigenvalues of A lie on the imaginary axis.

Remark 1. For the delay-free case, i.e., $\tau(t) = 0$, Ma et al. [7] designed the consensus protocol based on the solution of an ARE. Due to the presence of delay, this method will not work. Because of the non-zero system matrix, all agents may not achieve average consensus and the method used in [14,23] cannot work. In the rest of the paper, we shall develop a new approach to derive the consensusability of the multi-agent systems.

4. Main results

Assume λ_i, $i = 2, \ldots, N$ be the non-zero eigenvalues of $L_{\mathcal{G}}$ and satisfy $\lambda_2 \le \cdots \le \lambda_N$. Denote $\delta \triangleq \min_{2\le i\le N}\{\lambda_i\} = \lambda_2$ and $\sigma_i \triangleq \lambda_i\delta^{-1}$, then it is easy to know $\sigma_i \ge 1$, $i = 2, \ldots, N$. Based on low gain matrix $P(\gamma)$ [21], we construct consensus gain $K = \delta^{-1}B^T P(\gamma)$, where γ is a scalar and matrix $P(\gamma)$ is the positive definite solution of the following ARE

$$A^TP(\gamma) + P(\gamma)A - P(\gamma)BB^TP(\gamma) = -\gamma P(\gamma). \tag{3}$$

Thus, protocol (2) becomes

$$u_i(t) = \delta^{-1}B^TP(\gamma)\sum_{j=1}^{N} a_{ij}[x_j(t - \tau(t)) - x_i(t - \tau(t))]. \tag{4}$$