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Robust exponential input-to-state stability of impulsive systems with an application in micro-grids[☆]

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ABSTRACT

This paper studies the robust exponential input-to-state stability (robust e-ISS) for impulsive systems. New notions of *input-to-state exponent* (IS-e) and e-*property* are proposed. Based on the established relation between IS-e and e-property, and the method of variation of constants formula, the equivalent conditions for robust e-ISS have been derived. Then the notion of *robust event*-e-*ISS* is defined. The sufficient conditions and the robust regions for robust e-ISS and robust event-e-ISS are also derived by using the IS-e of every subsystem. It shows the whole system may have robust event-e-ISS while every subsystem may have no ISS. It also shows the external disturbances may lead to relatively small robust regions. The results are then specialized to derive the equivalent conditions of interval e-ISS for interval impulsive systems. As an application, the result is used to test the ISS for a controlled micro-grid.

1. Introduction

Input-to-state stability (ISS) analysis aims to investigate how external disturbance inputs affect the system stability. Since the notion of ISS was proposed in the late 1980s [1], ISS analysis of dynamical systems with disturbances or external inputs has quickly become one of the active research topics in system analysis and design. Moreover, ISS has been successfully employed in the stability analysis and control synthesis of nonlinear systems with disturbance inputs and complex structure, see [2–10]. Quite a few recent works focus on the ISS of impulsive systems, see e.g. [11–19]. In particular, [11] established the relation between robust stability and the existence of ISS-Lyapunov functions. However, it should be pointed out that an ISS-Lyapunov function is not easy to find, especially for those impulsive systems with unstable subsystems. And it often requires the impulsive system to have finite number of time-invariant subsystems. Moreover, there are fewer results of robust ISS reported.

The objective of this paper is to study the robust exponential ISS (robust e-ISS) for impulsive systems with uncertainties. The system studied here is time-varying and may have an infinite number of subsystems. Instead of finding an ISS-Lyapunov function, we aim to investigate the robust e-ISS from the dynamic characteristic of subsystems and the hybrid structure of impulsive systems. For this goal, we propose notions of *input-to-state exponent* (IS-e) for general dynamical systems, and e-*property* for the transition matrix function (TMF) of the nominal system. The relation between IS-e and e-property is then established. By this relation and the method of variation of constants formula, the equivalent conditions for the robust e-ISS are derived. In the case of zero external inputs, the impulsive system has robust e-ISS if and only if its nominal system is exponentially stable or if and only if the TMF has negative e-property. In the case of non-zero external inputs, the minimal dwell time (MiDT) condition is necessary to derive such equivalent conditions for the robust e-ISS. Then the notion of robust event-e-ISS is given. And the sufficient conditions for the two types of robust e-ISS including robust regions are derived via using IS-e of every subsystem. It shows the whole system may have robust event-e-ISS while every subsystem has no ISS. The maximal dwell time (MaDT) condition is used when there exist subsystems with no ISS. It also shows the external inputs may lead to relatively small robust regions compared with no external inputs. The results are then specialized to derive the equivalent conditions of interval e-ISS for interval impulsive systems.

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As an application, the result is applied to power systems. Recently, the expected high depth of penetration of distributed generation (DG) in the utility distribution grid has brought about concepts of "micro-grid" and "smart grid", see [20,21]. The full benefits of high depth of penetration of DG units are gained if a micro-grid can be operated in both grid-connected and islanded modes [22]. Moreover, the non-50/60 Hz power output of many DG units means that power-electronic converter interfaces are required [23]. However, the switching between grid-connected and islanded modes and switching operation of converter interfaces, result in hybrid behavior between continuous dynamics and switching events in a micro-grid or smart grid. Thus, a micro-grid is a typical impulsive hybrid system, see [24]. In a micro-grid, in order to enable an electronically-interfaced DG unit and its local load to retain uninterrupted power supply (UPS) in both grid-connected and islanded modes, the main problem is to how to control the coupling voltage-sourced converter (VSC). In this paper, we present a hybrid control for a VSC which is used in the UPS application. Then we use the obtained results on impulsive systems to test the ISS for the controlled micro-grid.

The rest of this paper is organized as: In Section 2, we give the preliminaries. In Section 3, we give the equivalent conditions of robust e-ISS. In Section 4, we specialize the results to the interval impulsive systems. In Section 5, we study the application on micro-grids. Conclusions are in Section 6.

2. Preliminaries

Let $\mathbb{R}^n$ denote the n-dimensional real vector space, $\mathbb{R}^+ = [0, +\infty)$, and $\mathbb{N} = \{0, 1, 2, \ldots\}$. Let $PC[\mathbb{R}^+, \mathbb{R}]$ denote the class of piecewise continuous functions from $\mathbb{R}^+$ to $\mathbb{R}$, with discontinuities of the first kind. For a matrix $A = (a_{ij})_{m\times n}$, let $\|A\| = [\lambda_{\max}(A^TA)]^{\frac{1}{2}}$, $\|A\|_1 = \max_{1\le j\le n}\{\sum_{i=1}^m a_{ij}\}$ and $\|A\|_\infty = \max_{1\le i\le m}\{\sum_{j=1}^n a_{ij}\}$. Let $\lambda_i(X)$, $i = 1, \ldots, n$, be all eigenvalues of the $n\times n$ matrix X and $\lambda_{\max}(X)$ ($\lambda_{\min}(X)$) the maximum (minimum) of $\lambda_i(X)$.

For a subset $\mathcal{I} \subseteq \mathbb{R}_+$, let $X|_{\mathcal{I}}$ denote the limitation of X on $\mathcal{I}$, i.e., $X|_{\mathcal{I}} : \mathcal{I} \to \mathbb{R}^{n\times n}$. For a constant $a > 0$, we denote the set of matrix functions:

$$\Omega_a \triangleq \{X|X : \mathbb{R}_+ \to \mathbb{R}^{n\times n}, \|X(s)\| \le a, \ \forall s \in \mathbb{R}_+\}.$$

For constant $\alpha \in \mathbb{R}$, we define: $\mathcal{B}(\alpha) = \begin{cases} 1, & \alpha \ge 0, \\ 0, & \alpha < 0. \end{cases}$ A function $\psi : \mathbb{R}_+ \to \mathbb{R}_+$ is of class-$\mathcal{K}$ ($\psi \in \mathcal{K}$) if it is continuous, zero at zero and strictly increasing. It is of class-$\mathcal{K}_\infty$ if it is of class-$\mathcal{K}$ and is unbounded. A function $\varphi : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ is of class-$\mathcal{KK}$ if $\varphi(\cdot, t)$ is of class-$\mathcal{K}$ for $t \ge 0$ and $\varphi(s, \cdot)$ is of class-$\mathcal{K}$ for $s \ge 0$. And for a function $w : \mathbb{R}_+ \to \mathbb{R}^n$, and $t_0, t, \hat{t} \in \mathbb{R}_+$ with $t_0 \le t \le \hat{t}$, define $\|w\|_{[t]} = \sup_{s\in[t_0,t]}\{\|w(s)\|\}$, $\|w\|_{[t,\hat{t}]} = \sup_{s\in[t,\hat{t}]}\{\|w(s)\|\}$, and $\|w\|_\infty = \sup_{t\ge t_0, t\in\mathbb{R}_+}\{\|w\|_{[t]}\}$.

Consider the impulsive system with form:

$$S_1 : \begin{cases} \dot{y}(t) = A_k(t)y(t) + \tilde{A}_k(t)y(t) + \omega_c(t), \\ \quad t \in \mathcal{I}_k = (t_k, t_{k+1}], \\ \Delta y(t) = C_k(t)y(t) + \tilde{C}_k(t)y(t) + \omega_d(t), \\ \quad t = t_k, \ k \in \mathbb{N}, \end{cases} \tag{1}$$

where $\Delta y(t) = y(t^+) - y(t)$; $A_k(t), C_k(t) \in \mathbb{R}^{n\times n}$ are known matrices; $\tilde{A}_k(t), \tilde{C}_k(t)$ are uncertain matrices; ω_c, ω_d are the external inputs with $\|\omega_c\|_\infty < +\infty$, $\|\omega_d\|_\infty < +\infty$; $\mathcal{I}_k = (t_k, t_{k+1}]$ where $\{t_k : k \in \mathbb{N}\}$ is the impulsive instant sequence satisfying $0 \le t_0 < t_1 < \cdots < t_k < t_{k+1} < \cdots$.

We assume the solution to S_1 exists globally and uniquely on $[t_0, \infty)$ and is continuous except at $t_k, k \in N$, at which it is left-hand continuous, i.e., $y(t_k) = y(t_k^-)$. Let $y(t) = y(t, t_0, y_0, \omega_c, \omega_d)$ be the solution of S_1 with initial condition $y(t_0^+) = y_0$. For S_1, there are several special cases: if $\tilde{A}_k = \tilde{C}_k = 0$, i.e., no uncertain matrices, then

$$S_2 : \begin{cases} \dot{y}(t) = A_k(t)y(t) + \omega_c(t), & t \in \mathcal{I}_k, \\ \Delta y(t) = C_k(t)y(t) + \omega_d(t), & t = t_k, \ k \in \mathbb{N}; \end{cases} \tag{2}$$

and if $\tilde{A}_k = \tilde{C}_k = 0$ and $\omega_c = \omega_d = 0$, then S_1 is changed to its *nominal system*:

$$S_3 : \begin{cases} \dot{x}(t) = A_k(t)x(t), & t \in \mathcal{I}_k, \\ \Delta x(t) = C_k(t)x(t), & t = t_k, \ k \in \mathbb{N}. \end{cases} \tag{3}$$

For every $k \in \mathbb{N}$, we also consider the related linear system on $\mathcal{I}_k$ as:

$$S_4 : \dot{x}(t) = A_k(t)x(t), \quad t \in \mathcal{I}_k. \tag{4}$$

Definition 2.1. System S_1 is said to have *robust exponential ISS* (robust e-ISS) with decay rate $\alpha > 0$ if there exist constants $\gamma_k > 0$, $\beta_k > 0$, $M > 0$, $K_c > 0$, $K_d > 0$ such that for any $\tilde{A}_k|_{\mathcal{I}_k} \in \Omega_{\gamma_k}$, $\tilde{C}_k|_{\hat{\mathcal{I}}_k} \in \Omega_{\beta_k}$,

$$\|y(t)\| \le Me^{-\alpha(t-t_0)}\|y_0\| + K_c\|\omega_c\|_\infty + K_d\|\omega_d\|_\infty, \\ t \ge t_0, \ t \in \mathcal{I}_k, \ k \in \mathbb{N}. \tag{5}$$

Remark 2.1. (i) If S_1 has robust e-ISS, then under $\omega_c = \omega_d = 0$, S_1 is robustly exponentially stable. In the literature, some replaced $\|\omega\|_\infty$ by $\|\omega\|_{[t]}$.

(ii) In the literature for robustness of linear system with uncertain matrices $\tilde{A}_k$ and $\tilde{C}_k$, it is often assumed there exist common matrices E and F such that

$$[\tilde{A}_k, \tilde{C}_k] = E[\Sigma_{\tilde{A}_k}, \Sigma_{\tilde{C}_k}]F$$

with $\Sigma_X^T\Sigma_X \le I$, $X = \tilde{A}_k, \tilde{C}_k$. If this assumption holds, then robust regions of $\tilde{A}_k$ and $\tilde{C}_k$ in Definition 2.1 are: $\Omega_{\gamma_k} = \Omega_{\beta_k} = \Omega_a$, where $a = \|E\|\,\|F\|$.

Now, we introduce the notion of input-to-state exponent (IS-e) for the general system:

$$\dot{x}(t) = f(t, x(t), u(t)), \quad t \ge t_0, \tag{6}$$

where $x(t)$ is the state, $u(t)$ is the external input, and function f satisfies the required conditions such that the solution to (6) exists uniquely.

Definition 2.2. (i) System (6) is said to have *input-to-state exponent* (IS-e) if there exist $K > 0$, $\alpha \in \mathbb{R}$, and $\varphi \in \mathcal{KK}$ such that the solution $x(t)$ satisfies

$$\|x(t)\| \le Ke^{\alpha(t-t_0)}\|x(t_0)\| + \varphi(\|u\|_{[t]}, (t-t_0)^{\mathcal{B}(\alpha)}), \\ t \ge t_0. \tag{7}$$

(ii) In (7), system (6) is said to have *input-to-state positive exponent* (IS-e$^+$) if $\alpha > 0$, *input-to-state zero exponent* (IS-e^0) if $\alpha = 0$, and *input-to-state negative exponent* (IS-e$^-$) if $\alpha < 0$.

Definition 2.3. System S_1 is said to have IS-e with exponent α if for some $M > 0$, and $\varphi_c, \varphi_d \in \mathcal{KK}$,

$$\|y(t)\| \le Me^{\alpha(t-t_0)}\|y_0\| + \varphi_c(\|\omega_c\|_{[t]}, (t-t_0)^{\mathcal{B}(\alpha)}) \\ + \varphi_d(\|\omega_d\|_{[t]}, (t-t_0)^{\mathcal{B}(\alpha)}).$$

And S_1 is said to have IS-e$^+$ if $\alpha > 0$, IS-e^0 if $\alpha = 0$, and IS-e$^-$ if $\alpha < 0$.

Remark 2.2. (i) The notion of IS-e$^+$ characterizes possible exponential growth of the state and exponential propagation of the disturbance. It is related to the notion of *forward completeness* of nonlinear system, see [25] and [26].