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Global stabilization of high-order nonlinear time-delay systems by state feedback☆

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ABSTRACT

We consider the problem of global stabilization by state feedback for a class of high-order nonlinear systems with time-delay. By developing a novel dynamic gain-based backstepping approach, a state feedback controller independent of the time-delay is explicitly constructed with the help of appropriate Lyapunov–Krasovskii functionals. The precise knowledge (even the upper bound) of the time-delay is not required. It is proved that the states of the nonlinear time-delay systems can be regulated to the origin while all the closed loop signals are globally bounded. Finally, both physical and academic examples are given to illustrate the applications of the proposed scheme.

1. Introduction

In this paper, we consider the state feedback control for a class of high-order nonlinear time-delay systems via the Lyapunov–Krasovskii method:

$$\begin{aligned}
&\dot{x}_i(t) = x_{i+1}^{p_i}(t) + g_i(\bar{x}_i(t)) + f_i(\bar{x}_i(t-d)), \\
&\quad i = 1, \dots, n-1, \\
&\dot{x}_n(t) = u(t) + g_n(x(t)) + f_n(x(t-d)), \\
&x(\tau) = \zeta(\tau), \quad \tau \in [-d, 0], \qquad (1)
\end{aligned}$$

where $x = [x_1, \dots, x_n]^T \in \mathbb{R}^n$ is the state vector, $\bar{x}_i = [x_1, \dots, x_i]^T$, $u \in \mathbb{R}$ is the system input, p_i are odd integers, g_i and f_i are locally Lipschitz functions and not necessary to be completely known, the nonnegative constant d denotes the unknown time-delay, and $\zeta(\tau) \in \mathbb{R}^n$ is a continuous function vector defined on $[-d, 0]$.

High-order nonlinear system (1) in the absence of time-delay has been widely studied in [1–3]. It is pointed out in [1] that global stabilization of the system (1) without time-delay can be achieved by state feedback under a high-order version of feedback linearizable condition.

When $p_i = 1, 1 \leq i \leq n-1$, the system (1) reduces to the well-known strict feedback system with time-delay. In [4], the author extended the backstepping method [5,6] to such a nonlinear time-delay system and designed a memoryless state feedback controller using the Lyapunov–Krasovskii method which, however, was proved incorrect later by [7,8]. The problem existing in [4] is that the Lyapunov functions in the traditional backstepping design are no longer effective for the nonlinear time-delay systems and the virtual control signals are too hard to design. By employing the idea of changing supply functions [9], this problem was solved for a class of lower-triangular systems in [10] with the knowledge of the upper bound of the time-delay. A more general class of nonlinear time-delay systems was investigated in [11], but the delay amplitude knowledge was still required. When some restrictive growth conditions [12,13] are imposed on the system nonlinearities, global output feedback stabilization can be achieved. Note that the problem in [4] can be avoided for these output feedback schemes because the construction of the Lyapunov–Krasovskii functional is necessary in one step.

When $p_i > 1$, the system (1) is in the high-order strict-feedback form [1] with time-delay. While the Jacobian linearization is uncontrollable, the mentioned system is not feedback linearizable. If we apply adding a power integrator technique [1] to the controller design by using the Lyapunov–Krasovskii method, the same problem in [4] will occur and become more complicated for such high-order systems. Under restrictive growth conditions, output feedback control was considered in [14,15] for high-order time-delay systems which, by the same reason as those for $p_i = 1$ [12,13], can also avoid the problem in [4]. Indeed, compared with output feedback control, *state feedback control* for the nonlinear

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time-delay systems is a more challenging issue. It remains unclear and open how a memoryless state feedback controller can be constructed for the system (1) to achieve global asymptotic stabilization in the sense that, for any initial conditions $x(t_0)$, $-d \le t_0 \le 0$, the state x converges to the origin and all the closed loop signals are bounded. Hence, a new tool is required for the global stabilization problem.

The main objective of this paper is to give a solution to global state feedback stabilization of the high-order nonlinear time-delay system (1) and in particular, to circumvent the long-standing problem in [4]. For this purpose, we develop a novel dynamic gain-based backstepping approach which, in addition, introduces a dynamic gain in each step of the recursive design. A key feature of the proposed dynamic gain-based backstepping technique is that the Lyapunov function is chosen in a new recursive manner with appropriately designed gain-based Krasovskii functionals. Owing to the introduction of the dynamic gains, additional negative terms appear in the derivative of the Lyapunov function, which can be used to counteract stronger system nonlinearities. As a result, a memoryless state feedback controller can be constructed. Moreover, the upper bound of the time-delay is not required to be known a priori. It is shown that the system states can be regulated to the origin while all the closed loop signals are globally bounded. Finally, numerical examples are given to show the effectiveness of the proposed scheme.

2. Preliminary lemmas

In this section, we shall introduce two technical lemmas which are useful for the controller design and stability analysis.

Lemma 1. *For any positive integers m, n and any positive function $f(x, y) : \mathbb{R}^2 \to \mathbb{R}^+$, the following inequality holds*

$$|x|^m |y|^n \le \frac{m}{m+n} f(x, y) |x|^{m+n} + \frac{n}{m+n} f^{-m/n}(x, y) |y|^{m+n}. \quad (2)$$

Proof. The reader may refer to [1] for the proof of this lemma.

Lemma 2. *For any nonnegative smooth function $f(x_1, \ldots, x_n)$ and positive integer k, there exist nonnegative smooth functions $f_i(x_1, \ldots, x_i)$, $1 \le i \le n$, such that*

$$f(x_1, \ldots, x_n) \sum_{i=1}^{n} |x_i|^k \le \sum_{i=1}^{n} f_i(x_1, \ldots, x_i) |x_i|^k. \quad (3)$$

Proof. Define a nonnegative continuous function[1]

$$\bar{f}(r) = \max_{x \in B_r} |f(x)|, \quad B_r = \left\{x : \|x\|^2 \le r,\ r \ge 0\right\}, \quad (4)$$

where $x = [x_1, \ldots, x_n]^T$. It is known that any continuous function $\bar{f}(r) : [0, +\infty) \to \mathbb{R}$ can be dominated by a smooth nondecreasing function $f^*(r)$, i.e., $\bar{f}(r) \le f^*(r)$. Note that $f(x) \le \bar{f}\left(\|x\|^2\right)$. Then, we can obtain that

$$\begin{aligned} f(x) &\le \bar{f}\left(\|x\|^2\right) \\ &\le f^*\left(\|x\|^2\right) \\ &= f^*\left(x_1^2 + \cdots + x_n^2\right) \\ &\le f^*\left(nx_1^2\right) + \cdots + f^*\left(nx_n^2\right). \end{aligned} \quad (5)$$

For $1 \le i, j \le n$, we have

$$\begin{aligned} f^*\left(nx_i^2\right)\left|x_j\right|^k &= \left[f^*\left(nx_i^2\right) - f^*(0)\right]\left|x_j\right|^k + f^*(0)\left|x_j\right|^k \\ &\le f_{ij}(x_i)|x_i|\left|x_j\right|^k + f^*(0)\left|x_j\right|^k \\ &\le \bar{f}_{ij}(x_i)|x_i|^{k+1} + \left|x_j\right|^{k+1} + f^*(0)\left|x_j\right|^k, \end{aligned} \quad (6)$$

where to obtain the last inequality, Lemma 1 has been used by considering $f_{ij}(x_i)|x_i|$ as one part, and $f_{ij}(x_i)$ and $\bar{f}_{ij}(x_i)$ are some nonnegative smooth functions. With (5) and (6), it is not difficult to prove that Lemma 2 holds.

3. Main results

To achieve global stabilization of the nonlinear time-delay systems, the following assumptions have been made for (1).

A.1: $p_1 \ge \cdots \ge p_{n-1} \ge p_n = 1$.
A.2: The nonlinearities $g_i(\bar{x}_i)$ and $f_i(\bar{x}_{id})$, $1 \le i \le n$, satisfy[2]

$$\begin{aligned} |g_i(\bar{x}_i)| &\le \gamma_i(\bar{x}_i)\left(|x_1|^{p_i} + \cdots + |x_i|^{p_i}\right), \\ |f_i(\bar{x}_{id})| &\le \sigma_i(\bar{x}_{id})\left(|x_{1d}|^{p_i} + \cdots + |x_{id}|^{p_i}\right), \end{aligned} \quad (7)$$

for known nonnegative smooth functions γ_i and σ_i.

Remark 1. Without considering the time-delay, the above hypotheses can be viewed as a high-order version of feedback linearizable condition, whose global stabilization problem has been solved in [1] by means of adding a power integrator. However, the presence of time-delay makes it impossible to implement the conventional recursive design. Specifically, the virtual controllers cannot be well defined to counteract the time-delay nonlinearities, which may result from the utilization of inappropriate Lyapunov functions. Therefore, it is necessary to find appropriate Lyapunov functions for global stabilization of the nonlinear time-delay systems. In this paper, a new recursive design method is developed by introducing one dynamic gain and the corresponding gain-based Lyapunov function at each step. Together with Krasovskii functionals, the designed controller guarantees global stability of the closed loop system.

Remark 2. It should be pointed out that when the functions $g_i(\bar{x}_i) + f_i(\bar{x}_{id})$ are replaced by $f_i(x, x_d)$ in the right-hand side of (1), the condition

$$|f_i(x, x_d)| \le \sigma_i(\bar{x}_i, \bar{x}_{id}) \sum_{j=1}^{i}\left(\left|x_j\right|^{p_i} + \left|x_{jd}\right|^{p_i}\right) \quad (8)$$

is equivalent to (7). In fact, according to the inequalities (5) and (6) in the proof of Lemma 2, we have $|f_i(x, x_d)| \le \sigma_{i1}(\bar{x}_i)\sum_{j=1}^{i}\left|x_j\right|^{p_i} + \sigma_{i2}(\bar{x}_{id})\sum_{j=1}^{i}\left|x_{jd}\right|^{p_i}$ for some nonnegative smooth functions $\sigma_{i1}(\bar{x}_i)$ and $\sigma_{i2}(\bar{x}_{id})$. Here, the expressions $g_i(\bar{x}_i) + f_i(\bar{x}_{id})$ are used only for the sake of a better understanding of our proposed method.

Now, we are ready to present the main results of the paper.

Theorem 1. *Consider the nonlinear time-delay system (1) satisfying the hypotheses A.1 and A.2. Then, a $(n-1)$th-order memoryless state feedback controller can be constructed to globally asymptotically stabilize the mentioned system in the sense that, for any continuous initial conditions $x(t_0)$, $-d \le t_0 \le 0$, the state x converges to the origin while all the signals of the closed loop system are bounded.*

[1] Throughout the paper, we use $\|\cdot\|$ to denote the Euclidean norm of a vector or the corresponding induced norm of a matrix.

[2] In this paper, for simplicity, we let ς_d denote the corresponding delay term $\varsigma(t-d)$. For instance, $x_{1d} = x_1(t-d)$ and $f_i(\bar{x}_{id}) = f_i(\bar{x}_i(t-d))$.