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Stabilization of hybrid neutral stochastic differential delay equations by delay feedback control

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ABSTRACT

This paper is concerned with the problem of exponential mean-square stabilization of hybrid neutral stochastic differential delay equations with Markovian switching by delay feedback control. A delay feedback controller is designed in the drift part so that the controlled system is mean-square exponentially stable. We discussed two types of structure controls; that is, state feedback and output injection. The stabilization criteria are derived in terms of linear matrix inequalities.

1. Introduction

A stochastic differential equation (SDE) with Markovian switching is known as a hybrid system which can be described by a set of SDEs with transitions between models determined by a Markovian chain in a finite mode set. As an important class of hybrid systems, SDEs with Markovian switching are usually used to model many practical systems, for example, electric power systems, the control system of a solar thermal central receiver, financial systems, etc. (see e.g. [1–5]). One of the important hot topics in the study of hybrid SDEs is the analysis of stability. A great number of significant results on this topic have been reported in the literature; see, for instance, [4–23] and the references therein.

There is now an intensive literature in the area of stabilization of SDEs. The problem of mean square exponential stabilization by state feedback controllers for a class of SDEs with Markovian switching was investigated in [24], while the problem of stabilization of hybrid stochastic differential delay equations (SDDEs) with Markovian switching by non-delay feedback controllers was addressed in [25]. The stabilization problem of hybrid SDDEs with Markovian switching by delay feedback controllers was addressed in [26]. Furthermore, the almost surely exponential stabilization problem of hybrid SDDEs by stochastic feedback controllers was investigated in [27]. Even though the stabilization of stochastic control systems has been widely studied [20,24–29], relatively little is known about the stabilization of hybrid neutral stochastic differential delay equations with Markovian switching. The purpose of this paper is to discuss the exponential mean-square stabilization of hybrid neutral stochastic differential delay equations with Markovian switching by delay feedback controllers.

Notation: Throughout this paper, for real symmetric matrices X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite). $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, and the notation $|\cdot|$ refers to the Euclidean vector norm. The notation M^T represents the transpose of the matrix M. If M is a matrix, its operator norm is denoted by $\|M\| = \sup\{|Mx| : |x| = 1\}$. If M is a symmetric matrix, $\lambda_{\max}(M)$ and $\lambda_{\min}(M)$ represent its largest and smallest eigenvalue, respectively. The symmetric terms in a symmetric matrix are denoted by $*$. $0_{m\times n}$ denotes zero matrix with $m \times n$ dimensions. $a \vee b$ denotes the maximum of a and b, while $a \wedge b$ denotes the minimum of a and b. $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ denotes a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$, where Ω is a sample space. $\mathcal{F}$ is the σ-algebra of subset of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. Let $\tau > 0$ and $C([-\tau, 0]; \mathbb{R}^n)$ denote the family of all continuous $\mathbb{R}^n$-valued function on $[-\tau, 0]$. Denote by $C^b_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$ the family of all $\mathcal{F}_0$-measurable bounded $C([-\tau, 0]; \mathbb{R}^n)$-valued random variables $\xi = \{\xi(\theta) : -\tau \leq \theta \leq 0\}$. $\mathbb{E}(\cdot)$ is the expectation operator with respect to some probability measure $\mathcal{P}$. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

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2. Problem formulation

In this paper, we consider the exponential mean-square stabilization problem of hybrid neutral stochastic differential delay equation (NSDDE) by delay feedback controllers. Given an unstable hybrid neutral stochastic differential delay equation

$$\begin{aligned} d[x(t)-D(x(t-\delta),r(t))] &= f(x(t),x(t-\tau),t,r(t))dt \\ &\quad + g(x(t),x(t-\tau),t,r(t))d\omega(t), \end{aligned} \tag{1}$$

where $\{r(t)\}$ is a continuous time Markovian process with right continuous trajectories taking values in a finite set $\mathcal{S} = \{1,2,\ldots,N\}$ with transition probabilities given by

$$\Pr\{r(t+\Delta t)=j \mid r(t)=i\} = \begin{cases} \pi_{ij}\Delta t + o(\Delta t), & i \neq j \\ 1+\pi_{ii}\Delta t + o(\Delta t), & i = j, \end{cases}$$

in which $\Delta t > 0$, $\lim_{\Delta t\to 0}(o(\Delta t)/\Delta t) = 0$, and $\pi_{ij} \geqslant 0$, for $j \neq i$, is the transition rate from mode i at time t to mode j at time $t+\Delta t$ and $\pi_{ii} = -\sum_{j=1,\, j\neq i}^{N}\pi_{ij}$. $\omega(t)$ is a scalar Brownian motion defined on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$, which is independent from the Markov chain $\{r(t),\ t \geq 0\}$ and satisfies $\mathbb{E}\{d\omega(t)\} = 0$, $\mathbb{E}\{d\omega^2(t)\} = dt$. The purpose of this paper is to design a delay feedback control $u(x(t),x(t-\tau),r(t))$ in the drift part, based on the current and past state and mode, so that the controlled system

$$\begin{aligned} &d[x(t)-D(x(t-\delta),r(t))] \\ &\quad = [f(x(t),x(t-\tau),t,r(t)) + u(x(t),x(t-\tau),r(t))]dt \\ &\qquad + g(x(t),x(t-\tau),t,r(t))d\omega(t), \end{aligned} \tag{2}$$

is exponentially stable in mean square. For the sake of simplicity, we only consider the underlying unstable hybrid system

$$\begin{aligned} d[x(t)-D(x(t-\tau),r(t))] &= f(x(t),x(t-\tau),t,r(t))dt \\ &\quad + g(x(t),x(t-\tau),t,r(t))d\omega(t). \end{aligned} \tag{3}$$

Assume that the initial data are given by $r(0) = r_0$ and

$$\{x(s) : -\tau \leq s \leq 0\} = \xi \in C^b_{\mathcal{F}_0}([-\tau,0];\ \mathbb{R}^n). \tag{4}$$

Denote by $x(t,\xi)$ the solution of Eq. (3) with initial conditions (4) and $r(0) = r_0$. Then by the theory of hybrid NSDDEs (see e.g. [8]), Eq. (3) on $t \geq 0$ is exponentially stable in mean square, if

$$\limsup_{t\to\infty} \frac{1}{t}\log(\mathbb{E}|x(t;\xi)|^2) < 0$$

for any $\xi \in C^b_{\mathcal{F}_0}([-\tau,0];\mathbb{R}^n)$. Set $\eta(t) = x(t) - D(x(t-\tau),r(t))$, and design a linear delay feedback controller

$$\begin{aligned} &u(x(t-\tau),r(t)) = F(r(t))G(r(t))\eta(t-\tau) \\ &\quad = F(r(t))G(r(t))\big(x(t-\tau) - D(x(t-2\tau),r(t))\big), \end{aligned} \tag{5}$$

where $F(r(t)) : \mathcal{S} \to \mathbb{R}^{n\times l}$, $G(r(t)) : \mathcal{S} \to \mathbb{R}^{l\times n}$, and one of them is given while the other needs to be designed. These two cases are usually known as (see e.g. [26,27]):

- State feedback: design $F(\cdot)$ when $G(\cdot)$ is given.
- Output injection: design $G(\cdot)$ when $F(\cdot)$ is given.

According to Eq. (3), the controlled system becomes

$$\begin{aligned} &d[x(t)-D(x(t-\tau),r(t))] \\ &\quad = [f(x(t),x(t-\tau),t,r(t)) + u(x(t-\tau),r(t))]dt \\ &\qquad + g(x(t),x(t-\tau),t,r(t))d\omega(t) \\ &\quad = [f(x(t),x(t-\tau),t,r(t)) + F(r(t))G(r(t)) \\ &\qquad \times \big(x(t-\tau) - D(x(t-2\tau),r(t))\big)]dt \\ &\qquad + g(x(t),x(t-\tau),t,r(t))d\omega(t). \end{aligned} \tag{6}$$

The controlled system (6) is a neutral stochastic differential delay equation with Markovian switching, where some initial data are required to be known. In general, we always impose the initial condition $\{x(s) : -2\tau \leq s \leq 0\} = \xi \in C^b_{\mathcal{F}_0}([-2\tau,0];\mathbb{R}^n)$ and fix the initial state r_0 arbitrarily for Markov chain $r(t)$ but let the initial data ξ vary in $C^b_{\mathcal{F}_0}([-2\tau,0];\mathbb{R}^n)$ for the NSDDE (6) (see e.g. [3,15,26]). In this paper, we shall regard the controlled system (6) as an NSDDE on $t \geq 2\tau$ with initial data $\{x(s) : 0 \leq s \leq 2\tau\}$, then by the theory of hybrid NSDDEs (see e.g. [15,26]), we have $\mathbb{E}(|x(t)|^2) < \infty$, for both $0 \leq t \leq 2\tau$ and $t \geq 2\tau$. These can be interpreted as follows: let Eq. (3) evolve from time 0 to 2τ and observe the whole segment $\{x(t) : 0 \leq t \leq 2\tau\}$ with fixed initial state r_0 arbitrarily for Markov chain and $\xi \in C^b_{\mathcal{F}_0}([-\tau,0];\mathbb{R}^n)$. As time evolves, we design the delay feedback control $u(x(t-\tau),r(t)) = F(r(t))G(r(t))\big(x(t-\tau)-D(x(t-2\tau),r(t))\big)$ from the moment of 2τ based on the past observation $\{x(t) : 0 \leq t \leq 2\tau\}$. Accordingly we shall regard the controlled system (6) as an NSDDE on $t \geq 2\tau$ with initial data $\{x(t) : 0 \leq t \leq 2\tau\}$ (see e.g. [26]). The solution of Eq. (6) with initial data $\xi \in C^b_{\mathcal{F}_0}([0,2\tau];\mathbb{R}^n)$ is denoted by $x(t,\xi)$, then we have the following definition:

Definition 1. For any initial data $\xi \in C^b_{\mathcal{F}_0}([0,2\tau];\mathbb{R}^n)$ and fixed initial state r_0 arbitrarily for Markov chain $r(t)$, Eq. (6) is said to be exponentially stable in mean square if

$$\limsup_{t\to\infty} \frac{1}{t}\log(\mathbb{E}|x(t;\xi)|^2) < 0.$$

Remark 1. Usually, we design the feedback controller $u(x(t),r(t))$ which only depends on the current state $x(t)$. However, there exists time delay τ between the time when the system state is observed and the time when the feedback control reaches the system in practice. Naturally, a delay feedback control $U(x(t-\tau),r(t))$ which depends on the past states $x(t-\tau)$ should be considered. In [26], the authors studied the stabilization of non-neutral stochastic differential delay equations with Markovian switching by delay feedback control $u(x(t-\tau)) = F(r(t)G(r(t)))x(t-\tau)$. However, $\eta(t) = x(t) - D(x(t-\tau),r(t))$ is regarded as a whole state in this paper; it is thus more natural to design the delay feedback control $u(x(t-\tau),r(t)) = F(r(t))G(r(t))\eta(t-\tau)$.

Remark 2. In this paper, the linear delay feedback control is with the structure of the form $u(x(t-\tau),r(t)) = F(r(t))G(r(t))\eta(t-\tau)$. When $G(r(t))$ is given, the feedback control is a state feedback. In the case when $F(r(t))$ is given, it is a special form of output feedback.

3. Stabilization of linear hybrid NSDDE

In this section, we are given an n-dimensional unstable linear hybrid NSDDE

$$\begin{aligned} d[x(t)-D(r(t))x(t-\tau)] &= [A(r(t))x(t) + B(r(t))x(t-\tau)]dt \\ &\quad + [C(r(t))x(t) + H(r(t))x(t-\tau)]d\omega(t) \end{aligned} \tag{7}$$

on $t \geq \tau$. For notational simplicity, in the sequel, a matrix $D(r(t))$ will be denoted by D_i for each possible $r(t) = i$, $i \in \mathcal{S}$; for example, $A(r(t)) = A_i$, $B(r(t)) = B_i$, $C(r(t)) = C_i$, $H(r(t)) = H_i$, and so on. Here, we assume $\|D_i\| < 1$ for all $i \in \mathcal{S}$. We design a delay feedback control $u(x(t-\tau),r(t))$ in the drift part so that the controlled system

$$\begin{aligned} &d[x(t)-D(r(t))x(t-\tau)] \\ &\quad = [A(r(t))x(t) + B(r(t))x(t-\tau) + u(x(t-\tau),r(t))]dt \\ &\qquad + [C(r(t))x(t) + H(r(t))x(t-\tau)]d\omega(t), \end{aligned} \tag{8}$$