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Distributed MPC of constrained linear systems with time-varying terminal sets

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ABSTRACT

This paper proposes a decoupling strategy for the Distributed Model Predictive Control (DMPC) for a network of dynamically-coupled linear systems. Like most DMPC approaches, the proposed approach has a terminal set and uses a Lyapunov matrix for the terminal cost in the online optimization problem for each system. Unlike them, the terminal set changes at every time step and the Lyapunov matrix is not block-diagonal. These features result in a less conservative DMPC formulation. The proposed method is easy to implement when the network is strongly connected (or when a central collector is used). Otherwise, the computations of the terminal set require the online solutions of a series of linear programming problems but can be speeded up significantly by preprocessing. Numerical examples showing these results are provided.

1. Introduction

This paper considers the Distributed Model Predictive Control (DMPC) of a network of M systems, each of which is of the form

$$x^i(t+1) = \sum_{j\in\mathbb{Z}^M} A^{ij}x^j(t) + B^i u^i(t), \tag{1}$$

$$x^i(t) \in X^i,\ u^i(t) \in U^i,\ i = 1, \dots, M,\ t \in \mathbb{Z}^+ \tag{2}$$

where $x^i \in \mathbb{R}^{n_i}$, $u^i \in \mathbb{R}^{m_i}$ are the state and input of the ith system, $A^{ij} \in \mathbb{R}^{n_i\times n_j}$ is the system dynamics relating x^i and its coupled states of x^j, X^i and U^i are the corresponding state and control constraints respectively.

The study of DMPC of network system has received considerable attention recently and several approaches have been proposed for its solution, see [1–7]. One typical approach is to treat the $A^{ij}x^j$ where $i \neq j$ as a disturbance to the i system, see [8–11]. Others (Chapter 7 and 11 of [4,12]) propose the use of the dual decomposition approach to handle the coupled dynamics. In these approaches, appropriate terminal constraints and terminal costs are needed; the choices of which are also active research areas. Clearly, and the most conservative choice of the terminal constraint is the origin [13–15]. Less conservative approaches include the use of a static ellipsoidal terminal sets [16–18] and a time-varying ellipsoidal set induced from a block diagonal Lyapunov matrix [5,19]. In the latter case, each diagonal block of the Lyapunov matrix also determines the terminal cost function of the corresponding system. In cases where M is small or moderate, restricting Lyapunov matrix to be block-diagonal can be restrictive. This work focuses on such systems and proposes an approach that differs from the previous in several distinctive ways: the Lyapunov matrix is non-diagonal (when it exists), the terminal set is time varying and moves within the maximal constraint admissible invariant set of the overall system. These features are possible under appropriate assumptions and additional computations. The implementation of the proposed approach is easiest when the network is strongly connected or when a central collector is used. When this is not the case, additional linear programming (LP) problems are needed. Fortunately, these computations can be speeded up significantly using preprocessing.

This work does not address the algorithmic details for the numerical determination of the non block-diagonal Lyapunov matrix or the consensus algorithm of the DMPC problem as they are standard in the literature, see for example [4,12].

The rest of the paper is organized as follows. This section ends with the notations needed, followed by the next section on the review of preliminary results of and additional notations for the DMPC problem. Section 3 discusses the choice of the distributed stage and terminal cost functions and the decomposition of the terminal set, including the solutions of a series of LP problems. The feasibility and stability of the overall system is shown in Section 4.

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Section 5 describes preprocessing steps that result in significant saving in the computations of the series of LP. Several numerical examples, including one for which a diagonal Lyapunov matrix does not exist, are provided Section 6. The last section concludes the work.

The notations used in this paper are as follows. Non-negative and positive integer sets are indicated by $\mathbb{Z}_0^+$ and $\mathbb{Z}^+$ respectively with $\mathbb{Z}^M := \{1, 2, \dots, M\}$ and $\mathbb{Z}_L^M := \{L, L+1, \dots, M\}$, $M \geq L$, $M, L \in \mathbb{Z}_0^+$. Similarly, $\mathbb{R}_0^+$ and $\mathbb{R}^+$ refer respectively to the sets of non-negative and positive real number. I_n is an $n \times n$ identity matrix and $int(\cdot)$ refers to the interior of a set. Given $\sigma > 0$ and $X \subset \mathbb{R}^n$ with $0 \in int(X)$, $\sigma X = \{\sigma x : x \in X\}$. The p-norm of $x \in \mathbb{R}^n$ is $\|x\|_p$ while $\|x\|_Q^2 = x^T Q x$ for $Q \succ 0$. For a square matrix Q, $Q \succ (\succeq) 0$ means Q is positive definite (semi-definite). Given a set of vectors, $c^i \in \mathbb{R}^n$, $i \in \mathbb{Z}^M$, the collection of vectors, $(c^1, c^2, \dots, c^M)$ also refers to the stack vector of $[(c^1)^T (c^2)^T \cdots (c^n)^T]^T \in \mathbb{R}^{Mn}$ for notational simplicity. Let $\Omega \subset Z^M$ be an index set, $|\Omega|$ is its cardinality and $c^\Omega := \{c^i : i \in \Omega\}$ is the collection of vectors (or stacked vector) of c^i with $i \in \Omega$. Several representations of the states and controls are needed: $x^i(t)$, $u^i(t)$ refer to the state and control of the ith system at time t; x_k^i, u_k^i are the kth predicted state and control of the ith system; $x = (x^1, x^2, \dots, x^M)$, $u = (u^1, u^2, \dots, u^M)$ are the collections of x^i and u^i over the M systems; boldface $\boldsymbol{x}^i = (x_0^i, x_1^i, \dots, x_N^i)$, $\boldsymbol{u}^i = (u_0^i, u_1^i, \dots, u_{N-1}^i)$ are the collections of the N predicted states and predicted controls over the horizon (of length N) for the ith system; in situation where the reference to time is needed, x_k^i, u_k^i can be written as $x_{k|t}^i$ and $u_{k|t}^i$. Hence, $x_{0|t}^i = x^i(t)$ and $u_{0|t}^i = u^i(t)$. Additional notations are introduced as required in the text.

2. Preliminaries

Combining all the M systems of (1), the overall system is

$$x(t+1) = Ax(t) + Bu(t), \quad t \in \mathbb{Z}_0^+ \tag{3}$$

$$x(t) \in X, \quad u(t) \in U \tag{4}$$

where $x = (x^1, x^2, \dots, x^M) \in \mathbb{R}^n$, $u = (u^1, u^2, \dots, u^M) \in \mathbb{R}^m$ are the overall states and controls of the full system with $n = \sum_{i \in \mathbb{Z}^M} n_i$ and $m = \sum_{i \in \mathbb{Z}^M} m_i$. Also, $A \in \mathbb{R}^{n \times n}$ is a block matrix with its (i, j) block being $A^{ij} \in \mathbb{R}^{n_i \times n_j}$ and $B \in \mathbb{R}^{n \times m}$ is a block diagonal matrix with blocks $\{B^1, B^2, \dots, B^M\}$ and $B^i \in \mathbb{R}^{n_i \times m_i}$. The constraint sets of X and U are

$$X := X^1 \times X^2 \times \cdots \times X^M, \qquad U := U^1 \times U^2 \times \cdots \times U^M. \tag{5}$$

The connection among the systems is static and can be represented as a network with its structure captured in a set of pairwise indices,

$$D := \{(i, j) : A^{ij} \neq 0\}, \tag{6}$$

indicating adjacency relationship among the M systems. The connection among the M systems is assumed to be arbitrary and, hence, A is not symmetric. However, the scheme proposed in this work requires the states of system i be communicated to all its neighbors. For this reason, define the set of neighbors of i, including i, as

$$\Omega_i := \{j : (i, j) \in D \text{ or } (j, i) \in D\} \cup \{i\}. \tag{7}$$

In general, $|\Omega_i| < M$. When $|\Omega_i| = M$ for all $i \in \mathbb{Z}^M$, the network is strongly connected in the sense that each system is a neighbor of every other system. Several other variables, sets and states can be defined based on Ω_i and its complement:

$$n_{\Omega_i} := \sum_{j \in \Omega_i} n_j, \qquad n_{\overline{\Omega}_i} := n - n_{\Omega_i}, \qquad \overline{\Omega}_i := \mathbb{Z}^M \setminus \Omega_i, \tag{8}$$

$$x^{\Omega_i} := \{x^j : j \in \Omega_i\} \in \mathbb{R}^{n_{\Omega_i}}, \qquad x^{\overline{\Omega}_i} := \{x^j : j \in \overline{\Omega}_i\} \in \mathbb{R}^{n_{\overline{\Omega}_i}}. \tag{9}$$

The variables u^i, x^i, x^{Ω_i} and $x^{\overline{\Omega}_i}$ can be extracted from u and x respectively from

$$u^i := F^i u, \qquad x^i := S^i x, \qquad x^{\Omega_i} = E^i x, \qquad x^{\overline{\Omega}_i} = \overline{E}^i x \tag{10}$$

where $F^i \in \{0, 1\}^{m_i \times m}$, $S^i \in \{0, 1\}^{n_i \times n}$, $E^i \in \{0, 1\}^{n_{\Omega_i} \times n}$ and $\overline{E}^i \in \{0, 1\}^{n_{\overline{\Omega}_i} \times n}$ are the appropriate selection matrices. From (10) and the fact that $[(E^i)^T (\overline{E}^i)^T]^T$ is a permutation matrix,

$$A^{ij} = S^i A (S^j)^T, \qquad \begin{bmatrix} x^{\Omega_i} \\ x^{\overline{\Omega}_i} \end{bmatrix} = \begin{bmatrix} E^i \\ \overline{E}^i \end{bmatrix} x,$$
$$x = \begin{bmatrix} E^i \\ \overline{E}^i \end{bmatrix}^{-1} \begin{bmatrix} x^{\Omega_i} \\ x^{\overline{\Omega}_i} \end{bmatrix} := H^i x^{\Omega_i} + \overline{H}^i x^{\overline{\Omega}_i} \tag{11}$$

where $E^i H^i = I_{n_{\Omega_i}}$, $\overline{E}^i \overline{H}^i = I_{n_{\overline{\Omega}_i}}$, $E^i \overline{H}^i = 0$, $\overline{E}^i H^i = 0$.

Assumptions of the system, needed in the sequel, are given below.

A1. The sets X^i and U^i, $i \in \mathbb{Z}^M$ are polytopes and contain the origin in their respective interiors.
A2. There is no delay or loss of information during communication between system i and all its neighbors.
A3. Matrices A and B are known to all systems.
A4. The set of M systems (or nodes) with edges defined by (7) forms an undirected and connected graph.

Both A1 and A2 are mild assumptions and are standard requirement in DMPC. Assumption A3 is needed as the models of the overall system are used to estimate the size of the terminal set at time t by system i. Assumption A3 may be hard to be satisfied when the network consists of heterogeneous systems. But in the typical case where most systems are similar or are members of only a few distinctly different classes of system, A3 is not a strong assumption. Assumption A4 defines the scope of the systems considered in this work. Suppose A4 is violated and the set of M systems has 2 or more connected components, then the approach described hereafter can be applied to them individually.

2.1. Central MPC

As a comparison for DMPC, a centralized MPC (CMPC) problem is needed. The CMPC assumes that the system given by (3) is solved via a single online finite horizon optimization problem of the form

$$V_N^*(x) = \min_{\boldsymbol{u}} V_N(\boldsymbol{x}, \boldsymbol{u}) := \min_{\boldsymbol{u}} \sum_{k=0}^{N-1} l(x_k, u_k) + l_f(x_N) \tag{12a}$$

s.t. $x_{k+1} = Ax_k + Bu_k, x_k \in X, u_k \in U, x_N \in X_f, x_0 = x, \ k \in \mathbb{Z}_0^{N-1}$

where N is the prediction horizon, $\boldsymbol{x} := \{x_0 \ x_1 \ \cdots \ x_N\}$, $\boldsymbol{u} := \{u_0 \ u_1 \ \cdots \ u_{N-1}\}$ are the predicted states and inputs respectively, X and U are those given by (5) and X_f is an appropriate terminal set. In this setting, CMPC is like a standard MPC problem without any constraints introduced by Ω_i and has the stage and the terminal costs being

$$l(x_k, u_k) = \|x_k\|_Q^2 + \|u_k\|_R^2, \qquad l_f(x_N) = \|x_N\|_P^2 \tag{13}$$

for some appropriate matrices $Q, R, P \succ 0$ and a scalar $\delta > 0$ that satisfy

$$(A + BK)^T P (A + BK) - P \preceq -(Q + K^T R K) - \delta I_n \tag{14}$$

for some stabilizing K. In addition, $l_f : X_f \to \mathbb{R}_0$ is defined on

$$X_f := \{x \in \mathbb{R}^n | Gx \leq \mathbf{1}_L\}. \tag{15}$$