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A switching anti-windup design based on partitioning of the input space[☆]

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ABSTRACT

This paper revisits the problem of enlarging the domain of attraction of a linear system with multiple inputs subject to actuator saturation by designing a switching anti-windup compensator. The closed-loop system consisting of the plant, the controller and the anti-windup compensator is first equivalently formulated as a linear system with input deadzone. We then partition the input space into several regions. In one of these regions, all inputs saturate with the time-derivative of the saturated input being zero. In each of the remaining regions, there is a unique input that does not saturate. The time derivative of the deadzone function associated with the unsaturating input is zero. By utilizing these special properties of the inputs on an existing piecewise Lyapunov function of the augmented state vector containing the deadzone function of inputs, we design a separate anti-windup gain for each region of the input space. The switching from one anti-windup gain to another is activated when the input signals leave one region for another, which can be implemented online since only the measurement of the input signals is required. Simulation results indicate that the proposed approach has the ability to obtain a significantly larger estimate of the domain of attraction than the existing approaches.

1. Introduction

Actuator saturation in control systems is ubiquitous in engineering applications. Actuator saturation causes degradation in performance of a control system, and in a severe case, even loss of stability. Many approaches have been proposed to avoid or reduce these adverse effects of actuator saturation. Some of approaches incorporate actuator saturation into controller design. For example, controllers of nested saturation type were constructed for global stabilization for systems that are not exponentially unstable [1,2], and the low gain feedback approach [3–5] was proposed to tackle control problems in a semi-global framework, also for systems that are not exponentially unstable. Design methods were also proposed for systems that are exponentially unstable [6]. The anti-windup approach, on the other hand, aims to weaken the performance degradation caused by actuator saturation of a control system which has been designed to meet performance specifications in the absence of actuator saturation.

Over the past decades, anti-windup techniques have been extensively studied in the research community. A popular class of anti-windup compensators is those of a linear structure, either static ones [7–11] or dynamic ones [12,9,13,10,14–16]. For example, for open-loop stable plants, an LMI-based anti-windup synthesis for both static and dynamic anti-windup compensators was proposed in [9]. This synthesis method was later generalized to study regional and nonlinear performances of open-loop unstable plants [10], where the regional sector conditions [17] were adopted to handle the saturation function. By using a narrower sector, the authors of [14] derived a dynamic anti-windup compensator with less conservativeness. As a further less conservative treatment of saturation functions, the convex hull representation [6] of saturation functions results in a static anti-windup gain with a lager ellipsoidal invariant set as an estimate of the domain of attraction of the closed-loop system.

When a saturated system with linear anti-windup compensators does not satisfy the specified performance, nonlinear anti-windup compensators, which have the potential to achieve better performance [18–24], is a natural alternative choice. In

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particular, a constructive nonlinear anti-windup design was given in [19] for exponentially unstable linear plants. The resulting anti-windup compensator achieves a very desirable tracking performance. Differently from the single nonlinear anti-windup compensators mentioned above, multiple linear anti-windup compensators form a special class of nonlinear anti-windup compensators. For example, a multi-stage anti-windup compensator, which contains two static anti-windup gains activated by different activation mechanisms, was designed [22] to obtain better tracking performance for open-loop stable plants. Recently, switching strategy has been introduced to construct nonlinear anti-windup compensators. A switching anti-windup design via the use of multiple quadratic Lyapunov functions was proposed in [21], where a set of static anti-windup gains are designed and one anti-windup gain is activated when its associated quadratic Lyapunov function takes the maximal value among a set of quadratic Lyapunov functions. Moreover, a saturation-based switching anti-windup was designed [20] via the partitioning of the convex hull representing saturated linear feedback, and the switching occurs when saturated linear feedback leaves one convex sub-hull for another one. Both of these switching anti-windup designs can achieve larger estimates of the domain of attraction than a single gain anti-windup compensator can.

In this paper, we will propose a new nonlinear anti-windup compensator of a switching structure. This anti-windup compensator consists of a set of static anti-windup gains, each of which is associated with one region in a partition of the input space. Among the regions in the partition, there is one region where all inputs saturate with the time-derivatives of the saturated inputs being zero. In each of the remaining regions, there is a unique unsaturating input, the deadzone function and its time derivative associated with which are zero. These special properties exploited from the partitioning of the input space can be incorporated into the time-derivative of an existing piecewise quadratic Lyapunov function [25] of the augmented state vector consisting of the system state and the deadzone function of the inputs. As a result, a set of matrix conditions with less conservativeness are established for the design of the switching anti-windup gains. Based on these matrix conditions, we formulate an optimization problem for the largest estimate of the domain of attraction of the resulting closed-loop system. To implement the resulting switching anti-windup compensator, one needs to determine which of the anti-windup gains to be activated, that is, to determine which region the input signals fall into. Unlike the switching anti-windup compensator in [20,21], whose implementation requires not only the measurement of states and inputs but also the computation of some nonlinear functions, only the input signals are required for the implementation of this switching anti-windup compensator, since the determination of which regions the inputs locate in involves only the input information. In [26], the idea of partitioning of the input space was presented to exploit special properties of saturation functions, which results in a larger estimate of domain of attraction for saturated systems. The partitioning proposed in this paper is different from that in [26]. It captures the special properties of the deadzone function, and enables us to establish matrix inequality conditions involving matrices of lower dimensions than those in [26].

The remainder of the paper is organized as follows. In Section 2, we introduce a partitioning of the input space, and explore some special properties of saturation/deadzone functions. Based on this partitioning, we formulate the problem of designing a switching anti-windup compensator. In Section 3, by using these properties and an existing piecewise quadratic Lyapunov function [25], we establish sufficient conditions for the existence of switching anti-windup gains so that a level set of this Lyapunov function can be used as an estimate of the domain of attraction of the closed-loop system. An optimization problem is then formulated to maximize such an estimate. Section 4 provides simulation results to illustrate the effectiveness of the proposed approach. Section 5 concludes the paper.

Notation. For a square matrix A, $\mathrm{He}(A) := A + A^{\mathrm{T}}$. For two integers l_1 and $l_2 \geq l_1$, $I[l_1, l_2]$ denotes the set of integers $\{l_1, l_1+1, \ldots, l_2\}$. Let I_m denote the identity matrix of dimension m, and $0_{n\times m}$ the $n \times m$ zero matrix. For a matrix $P \in \mathbf{R}^n$, $P = P^{\mathrm{T}} > 0$, $\mathcal{E}(P) := \{x \in \mathbf{R}^n : x^{\mathrm{T}}Px \leq 1\}$. For a matrix $G \in \mathbf{R}^{m\times n}$, denote the jth row of G as G_j and define $\mathcal{L}(G) = \{v \in \mathbf{R}^n : |G_j v| \leq 1, j = 1, 2, \ldots, m\}$.

2. Preliminaries and problem formulation

Consider a linear system with multiple inputs subject to actuator saturation

$$\begin{cases} \dot{x}_{\mathrm{p}} = A_{\mathrm{p}}x_{\mathrm{p}} + B_{\mathrm{p}}\mathrm{sat}(u), \\ y = C_{\mathrm{p}}x_{\mathrm{p}}, \end{cases} \tag{1}$$

where $x_{\mathrm{p}} \in \mathbf{R}^{n_{\mathrm{p}}}$ is the state, $u \in \mathbf{R}^m$ is the control input, $m \geq 2$, $y \in \mathbf{R}^p$ is the measured output, and $\mathrm{sat} : \mathbf{R}^m \to \mathbf{R}^m$ is a standard saturation function defined as

$$\mathrm{sat}(u) = [\mathrm{sat}(u_1) \quad \mathrm{sat}(u_2) \cdots \mathrm{sat}(u_m)]^{\mathrm{T}},$$
$$\mathrm{sat}(u_j) = \mathrm{sgn}(u_j)\min\{1, |u_j|\}, \quad j = 1, 2, \ldots, m.$$

We assume that a linear dynamic controller of the form

$$\begin{cases} \dot{x}_{\mathrm{c}} = A_{\mathrm{c}}x_{\mathrm{c}} + B_{\mathrm{c}}y, \quad x_{\mathrm{c}} \in \mathbf{R}^{n_{\mathrm{c}}}, \ x_{\mathrm{c}}(0) = 0, \\ u = C_{\mathrm{c}}x_{\mathrm{c}} + D_{\mathrm{c}}y, \end{cases}$$

has been designed that stabilizes system (1) with the desired performances in the absence of actuator saturation. When the actuator saturates, control input delivered to system (1), $\mathrm{sat}(u)$, is different from the intended control input u, causing the performance of the closed-loop system to degrade. To alleviate this degradation, an anti-windup compensator is designed that modifies the controller with a “correction” term $E_{\mathrm{c}}(\mathrm{sat}(u) - u)$ as follows:

$$\begin{cases} \dot{x}_{\mathrm{c}} = A_{\mathrm{c}}x_{\mathrm{c}} + B_{\mathrm{c}}y + E_{\mathrm{c}}(\mathrm{sat}(u) - u), \quad x_{\mathrm{c}} \in \mathbf{R}^{n_{\mathrm{c}}}, \ x_{\mathrm{c}}(0) = 0, \\ u = C_{\mathrm{c}}x_{\mathrm{c}} + D_{\mathrm{c}}y. \end{cases} \tag{2}$$

Under this compensated controller, the closed-loop system can be written in the following compact form

$$\begin{cases} \dot{x} = Ax - B\mathrm{dz}(u), \\ u = Fx, \end{cases} \tag{3}$$

where $x = [x_{\mathrm{p}}^{\mathrm{T}} \ x_{\mathrm{c}}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbf{R}^n$, $n = n_{\mathrm{p}} + n_{\mathrm{c}}$, the deadzone function $\mathrm{dz}(u) = u - \mathrm{sat}(u)$, and

$$A = \begin{bmatrix} A_{\mathrm{p}} + B_{\mathrm{p}}D_{\mathrm{c}}C_{\mathrm{p}} & B_{\mathrm{p}}C_{\mathrm{c}} \\ B_{\mathrm{c}}C_{\mathrm{p}} & A_{\mathrm{c}} \end{bmatrix}, \qquad B = \begin{bmatrix} B_{\mathrm{p}} \\ E_{\mathrm{c}} \end{bmatrix},$$
$$F = \begin{bmatrix} D_{\mathrm{c}}C_{\mathrm{p}} & C_{\mathrm{c}} \end{bmatrix}.$$

To design the anti-windup gain E_{c} such that system (3) has as large a domain of attraction as possible, the quadratic Lyapunov function $V_q(x) = x^{\mathrm{T}}Px$ is widely adopted whose level sets are used as estimates of the domain of attraction of system (3). Recently, a piecewise quadratic Lyapunov function [25], which contains the deadzone functions of the inputs,

$$V(x) = \xi^{\mathrm{T}}P\xi = \begin{bmatrix} x \\ \mathrm{dz}(u) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} P_1 & P_2 \\ P_2^{\mathrm{T}} & P_3 \end{bmatrix} \begin{bmatrix} x \\ \mathrm{dz}(u) \end{bmatrix}, \tag{4}$$

has been developed to arrive at a larger estimate of the domain of attraction and a tighter $\mathcal{L}_2$ gain [25] of linear systems with an algebraic loop and subject to actuator saturation. If we set $P_2 = 0$,