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Estimation of solutions of observable nonlinear systems with disturbances

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ABSTRACT

A family of continuous-time observable nonlinear systems with input and output is considered. A new technique of estimation of the state variables is proposed. It relies on the use of past values of the output, as done to construct some observers which converge in finite time, and on a recent technical result pertaining to the theory of the monotone systems. It applies to systems with additive disturbances and disturbances in the output. The nonlinear terms are not supposed to be globally Lipschitz, but it is requested that they depend only on the input and output variables.

1. Introduction

The problem of estimating the value of the solutions of a system when some variables are not accessible by measurements is a fundamental problem, which has been addressed by many techniques in many contributions. Traditional state estimators, such as for instance the Luenberger observer [1], are very popular. They compute point estimates of the state from input–output data, possibly supplemented by an estimate of the dispersion of the possible values of the state around this point estimate. Guaranteed state estimators [2,3], also known as set-membership estimators [4,5], compute sets guaranteed to contain the actual value of the state if some hypotheses on the state perturbation and measurement noise are satisfied. Guaranteed state estimation can be traced back to the seminal work of F.C. Schweppe [6]. His idea was recursively to compute ellipsoids guaranteed to contain the actual state (of course, other types of containers than ellipsoids could and have been used). In the last two decades, a new technique of guaranteed state estimations has been proposed. It is based on tools called interval observers and is developed and applied in many studies, see, for instance, [7–11] and the references therein. Typically, interval observers bound the actual state between two functions which take advantage of the solutions of two deterministic and possibly coupled dynamical systems. A key feature of the interval observers is that they can be applied only when an approximate knowledge of the initial conditions (in terms of an upper and a lower bound) is known. By contrast with all the mentioned results, some observers make it possible to determine the value of the solutions of continuous-time systems in finite time. This is the case in particular of the observers designed in [12–14] [15,16] which converge to the solutions in finite time. Some papers present finite-time convergent observers for nonlinear systems that are linearizable up to output injection. This is the case of [15,16]. All the above works consider systems without additive disturbances in the measurements. Recently finite time observers were designed for a class of nonlinear systems with unknown inputs [17]. The latter results are confined to the case where the number of unknown inputs is not greater than the number of outputs.

The aim of the present work is to propose a new approach of guaranteed state estimations for estimating state variables of nonlinear systems in the case where no approximate knowledge of the initial conditions is known and the systems have no monotonicity property. It is based on formulas incorporating past values of the input and the output of the studied system, which are reminiscent of the formulas proposed in [12,17,15] and uses a recent technical result stated and proved in [18], which makes it possible to express a function without monotonicity property as another function (with domain of definition of dimension larger than the

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domain of the considered function) which is increasing with respect to some of its variables and decreasing with respect to others. We consider the systems for which two types of bounded deterministic time-varying disturbances are present: in the dynamics and in the output. Usually, these disturbances are present in applications; notice in particular that in general the measures are inaccurate. The results we obtain are of two types: some of the formulas of estimation we propose are functions of the past values of the input and the output only and have terms with distributed delays and others take advantage of dynamic extensions and have terms with pointwise constant delays only. In the absence of unknown uncertainties, the formulas we exhibit provide with the exact values of the solutions, after a finite time interval. When unknown disturbances are present and upper and lower bounded by known constant vectors, then, the formulas we propose give, after a finite time interval, upper and lower bounds for each component of the solutions, as interval observers do. The difference between these bounds is bounded as long as the disturbances are bounded. Finally, it is worth mentioning that we consider systems with nonlinear terms which are not supposed to be globally Lipschitz, but it is required that they depend only on the inputs and outputs.

The paper is organized as follows. The family of systems studied is presented in Section 2. The main results are stated and proved in Sections 3 and 4. They are illustrated through examples in Section 5. Concluding remarks are given in Section 6.

Notation, definitions and prerequisites

The notation will be simplified whenever no confusion can arise from the context. Any $k \times n$ matrix, whose entries are all 0 is simply denoted 0. The Euclidean norm of vectors of any dimension and the induced norm of matrices of any dimension are denoted $|\cdot|$. All the inequalities must be understood *componentwise* (partial order of $\mathbb{R}^r$) *i.e.* $v_a = (v_{a1}, \ldots, v_{ar})^\top \in \mathbb{R}^r$ and $v_b = (v_{b1}, \ldots, v_{br})^\top \in \mathbb{R}^r$ are such that $v_a \leq v_b$ if and only if, for all $i \in \{1, \ldots, r\}$, $v_{ai} \leq v_{bi}$. A symmetric matrix $M \in \mathbb{R}^{n\times n}$ is positive (resp. negative) semidefinite if for all vectors $v \in \mathbb{R}^n$, $v^\top M v \geq 0$ (resp. $v^\top M v \leq 0$). Then we denote $M \succeq 0$ (resp. $M \preceq 0$). A matrix $M \in \mathbb{R}^{n\times n}$ is said to be Schur stable if its spectral radius is smaller than 1. For two matrices $M = (m_{ij}) \in \mathbb{R}^{r\times s}$ and $N = (n_{ij}) \in \mathbb{R}^{r\times s}$ of same dimension, $\max\{M, N\}$ is the matrix where each entry is $\max\{m_{ij}, n_{ij}\}$. For a matrix $M \in \mathbb{R}^{r\times s}$, $M^+ = \max\{M, 0\}$, $M^- = \max\{-M, 0\}$. A matrix $M \in \mathbb{R}^{r\times s}$ is said to be *nonnegative* if $M^+ = M$. A sequence (u_i) is *nonnegative* if for all integer k, u_k is nonnegative. If a matrix M is Metzler, then for all $t \geq 0$, $e^{Mt} \geq 0$. For any continuous function $\varphi : [-\tau, \infty) \to \mathbb{R}^n$ and all $t \geq 0$, we define φ_t by $\varphi_t(m) = \varphi(t + m)$ for all $m \in [-\tau, 0]$, i.e., $\varphi_t \in C_{\text{in}}$ is the translation operator.

2. Family of studied systems

Throughout the paper, we consider the nonlinear system

$$\begin{cases} \dot{x}(t) = F(x(t), u(t), \epsilon_2(t)) \\ y(t) = Cx(t) + \epsilon_1(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, and $C \in \mathbb{R}^{q\times n}$, $y(t) \in \mathbb{R}^q$ is the output, $u(t) \in \mathbb{R}^p$ is a possible known input, F is a nonlinear function of class C^1, and $\epsilon_1 : [0, +\infty) \to \mathbb{R}^q$ and $\epsilon_2 : [0, +\infty) \to \mathbb{R}^m$ are disturbances, which are supposed to be piecewise continuous and bounded.

We introduce the following assumption:

Assumption A. The function F is such that there exist a matrix $A \in \mathbb{R}^{n\times n}$ and a function f of class C^1 such that, for all $x \in \mathbb{R}^n$, $u \in \mathbb{R}^p$, $\varepsilon \in \mathbb{R}^m$

$$F(x, u, \varepsilon) = Ax + f(Cx, u, \varepsilon) \tag{2}$$

and the pair (A, C) is observable.

We will also use the following assumption:

Assumption B. There are known constant vectors $\overline{\epsilon}_1 \in \mathbb{R}^q$, $\underline{\epsilon}_1 \in \mathbb{R}^q$ and $\overline{\epsilon}_2 \in \mathbb{R}^n$, $\underline{\epsilon}_2 \in \mathbb{R}^n$ such that, for $i = 1, 2$, and for all $t \geq 0$, the inequalities

$$\underline{\epsilon}_i \leq \epsilon_i(t) \leq \overline{\epsilon}_i \tag{3}$$

are satisfied.

Discussion of the assumptions

• Notice that, along the trajectories of (1), $f(Cx(t), u(t), \epsilon_2(t)) = f(y(t) - \epsilon_1(t), u(t), \epsilon_2(t))$. Thus Assumption A implies that in the system (1), f can be seen as a function which depends only on y, u, ϵ_1 and ϵ_2. Therefore, the family of systems (1) satisfying Assumption A belongs to the family of the systems affine in the unmeasured part of the state. For these systems, many constructions of asymptotic observers (see for instance [19]) and interval observers (see for instance [7,20]) have been proposed.

• All the results of our paper can be extended straightforwardly to the case where the function f depends on t explicitly. For the sake of simplicity, we restrict ourselves to time-invariant systems.

• The decomposition (2) of the function F is not unique. In particular, let us notice for later use that when Assumption A is satisfied, a matrix A with real negative eigenvalues can always be selected.

• We prove in Appendix B that, under Assumption A, for any selected matrix A, there is a matrix $L \in \mathbb{R}^{n\times q}$ such that the matrix

$$H = A + LC \in \mathbb{R}^{n\times n} \tag{4}$$

is Hurwitz and there is a constant $\tau > 0$ such that the matrix

$$e^{-\tau H} - e^{-\tau A} \in \mathbb{R}^{n\times n} \tag{5}$$

is invertible. See also [12], where it is proved that L can be chosen such that $e^{-\tau H} - e^{-\tau A}$ is invertible for arbitrarily small constants $\tau > 0$.

• We will use the following notation:

$$E_\tau = \left(e^{-\tau H} - e^{-\tau A}\right)^{-1}. \tag{6}$$

• Assumption B is realistic and is frequently satisfied in practice. Moreover, it can be relaxed by allowing the bounds $\overline{\epsilon}_i$, $\underline{\epsilon}_i$ to depend on t. However, for the sake of simplicity, we restrict ourselves to the case where they are constant.

3. Exact estimation

The results of this section provide with exact estimations of the solutions in finite time, but they can be applied only when the functions ϵ_1 and ϵ_2 are known. Moreover, it is important to keep in mind that since Assumption A does not imply that the function f is globally Lipschitz, the finite escape time-phenomenon may occur even if $u(t)$ is a bounded function. The results of the present section owe a great deal to the contributions [12,15]. However, they are very different from those of [12], which are devoted to linear systems without functions ϵ_1 and ϵ_2. The results in [15] are also concerned with systems without functions ϵ_1 and ϵ_2, but they apply to systems (1) when they satisfy Assumption A.

3.1. Exact estimation, direct approach

Let us state and prove the following result: