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Squaring down with zeros cancellation in generalized systems[☆]

Cristian Oară[a,*], Cristian Flutur[a], Marc Jungers[b,c]

[a] Faculty of Automatic Control and Computers, Politehnica University of Bucharest, Austrului 34, sector 2, RO 024 074, Bucharest, Romania

[b] Université de Lorraine, CRAN, UMR 7039, 2 avenue de la forêt de Haye, Vandœuvre-lès-Nancy Cedex, 54516, France

[c] CNRS, CRAN, UMR 7039, France

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ABSTRACT

Squaring down with simultaneous zeros cancellation by series compensation of a general (possibly polynomial or improper) linear system is investigated. All static and dynamical compensators that spotlight minimal McMillan degree are parameterized. This general result is particularized to get compensators that preserve the L^2 or L^∞ norm of the original system, either in continuous or discrete-time. All results are completely general, numerically sound, and based on general realizations allowing for poles at infinity.

1. Introduction

Given a $p \times m$ linear system described by its transfer function matrix (TFM) $G(\lambda)$ of normal rank r, the first stage in many design schemes in control is to find a pre-compensator $G_{prec}(\lambda)$ and a post-compensator $G_{postc}(\lambda)$ such that the series connection system

$$G_{postc}(\lambda)G(\lambda)G_{prec}(\lambda) \tag{1}$$

is rank compressed (or squared-down) and has all its zeros in a desirable set of the closed complex plane $\overline{\mathbb{C}}$. The desirable set of zeros could be obtained by allocating appropriately the newly introduced zeros through the squaring-down process, while simultaneously canceling the offending ones. This preliminary design stage – called ad-hoc *Squaring-Down with Zeros Cancellation (SDZC)* – has been considered in various forms (see [1]), especially in its squaring-down (SD) variant only (without moving the offending zeros) in *pleiades* of papers, receiving different theoretical solutions [2–8].

Among the abundant applications of the SD technique we mention the almost disturbance decoupling with output feedback and the diagonal decoupling [7,9], the problem of almost zeros [6], the design techniques for MIMO systems based on generalized Nyquist criterion [10], on factorizations of TFMs [11], or on model-matching [9], the "fat plant" case in H^2 and H^∞ control [12], and the industrial design which is based on decentralized controller strategies. Numerically-sound algorithms for SD may be found in [13], while extensions of the SD technique to generalized systems have been considered in [14].

The SD problem has been considered so far in two closely related variants in which the compensators $G_{prec}(\lambda)$ and $G_{postc}(\lambda)$ are required to either have dimensions $m\times r$ and $r\times p$, respectively, or be both square and invertible, while the resulting system (1) is either $G_s(\lambda)$ or $\begin{bmatrix} G_s(\lambda) & 0 \\ 0 & 0 \end{bmatrix}$,with $G_s(\lambda)$ square, invertible and of dimension $r \times r$. The first variant will be dubbed the *reduced* SD problem. The reduced SD problem has been solved by giving a class of static (constant) pre- and post-compensators, or a class of dynamic ones. In the static case, the newly introduced zeros cannot always be placed in desired locations and this brings important limitations in the subsequent design stages, while in the dynamic case the new zeros can be placed completely arbitrary. Nevertheless, all previous results in both variants are partial, missing a parametrization of the class of solutions. Since SD is only a preliminary step used in subsequent stages of designing controllers, it is important to keep all degrees of freedom available by providing formulas for all solutions.

The achievements of this paper are manyfold: (1) to extend the technique of SD to include zeros cancellation (SDZC) for the most general class of linear systems; (2) to give existence conditions

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* Corresponding author.
E-mail addresses: cristian.oara@acse.pub.ro (C. Oară), cristian.flutur@acse.pub.ro (C. Flutur), marc.jungers@univ-lorraine.fr (M. Jungers).

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together with the construction of the static compensators; (3) to parameterize the class of minimal McMillan degree dynamic compensators; (4) to give the subclass of unitary compensators which preserve the L^2 and L^∞-norms of the original system; (5) to give a realization based reliable algorithm to compute the compensators.

The SDZC problem may be solved by a sequence of two dual problems, one to find a postcompensator $G_{postc}(\lambda)$ such that $\widehat{G}(\lambda) := G_{postc}(\lambda)G(\lambda)$ is row compressed and has all its zeros in the desired subset, followed by one to find a precompensator $G_{prec}(\lambda)$ such that $\widehat{G}(\lambda)G_{prec}(\lambda)$ is column (and row) compressed and has its zeros in the given subset. Therefore, we will deal with postcompensation only and denote the intervening compensator by $G_c(\lambda)$. Since solutions are highly nonunique, we add the requirement of minimum McMillan degree and call them *minimal*. Specifically, we consider the following problem.

Squaring down with zeros cancellation (SDZC). Let $G(\lambda)$ be a $p \times m$ system with normal rank r, and a fixed disjoint partition

$$\overline{\mathbb{C}} = \Gamma_g \uplus \Gamma_b, \tag{2}$$

where Γ_g is the "good" (desired) set and Γ_b is its "bad" complement. Find the class of minimal invertible $G_c(\lambda)$ (without zeros in Γ_b) such that

$$G_c(\lambda)G(\lambda) = \begin{bmatrix} G_{sz}(\lambda) \\ 0 \end{bmatrix} \begin{matrix} \}r \\ \}p-r \end{matrix} \tag{3}$$

has all zeros in Γ_g and $G_{sz}(\lambda)$ full row normal rank. If $G_c(\lambda)$ is restricted to be $r \times p$ and $G_c(\lambda)G(\lambda) = G_{sz}(\lambda)$, we dub the problem the *restricted* SDZC.

Remark 1.1. The "bad" zeros of $G(\lambda)$ are canceled by the poles of $G_c(\lambda)$, while some (or all) zeros of $G_c(\lambda)$ may appear in the product (3), depending on possible cancellations between zeros of $G_c(\lambda)$ and poles of $G(\lambda)$. Hence, $G_c(\lambda)$ must have its zeros in Γ_g to ensure that $G_{sz}(\lambda)$ ends up with the same property. Moreover, the minimality of $G_c(\lambda)$ implies that all its poles are actually canceled in (3), and therefore the poles of $G_{sz}(\lambda)$ are among the poles of $G(\lambda)$.

Remark 1.2. In general, the choice of the "good" and the "bad" regions in (2) can be made according to any specific need. However, if series compensation is used as a preliminary step to any type of stabilization scheme (*e.g.*, by feedback) then pole-zero cancellation outside the stability domain is not allowed, and the "bad" region should be restricted to a subset of the stability domain.

Remark 1.3. Any first r rows of a solution $G_c(\lambda)$ in (3) generate a solution to the reduced SDZC. Conversely, any $r \times p$ solution $G_c^r(\lambda)$ to the reduced problem may be augmented to an invertible one $G_c(\lambda) := \begin{bmatrix} G_c^r(\lambda) \\ G_\ell(\lambda) \end{bmatrix}$, where $G_\ell(\lambda)$ is a rational basis of the left null space of $G(\lambda)$. However, in general minimality is not preserved from one solution to the other.

The paper is organized as follows. Generalized dynamical systems are reviewed in Section 2. A key projection of the system pencil is given in Section 3. The class of general solutions to the SDZC problem is given in Section 4 and is particularized in Section 5 to ones preserving the L^2/L^∞ norms. Numerical experiments are shown in Section 6 while conclusions are drawn in Section 7.

2. Preliminaries

By $\mathbb{C}$, $\mathbb{C}_-$, $\mathbb{C}_+$, and $\mathbb{C}_0$ we denote the complex plane, the open left half plane, the open right half plane, and the imaginary axis, respectively. Let $\overline{\mathbb{C}} := \mathbb{C} \cup \{\infty\}$. By $\mathbb{D}$ and $\mathbb{D}_1(0)$ we denote the open unit disk and the unit circle, respectively. $\mathbb{D}_c := \overline{\mathbb{C}} \setminus \overline{\mathbb{D}}$ stands for the exterior of the closed unit disk.

For a matrix A, let A^* be its conjugate transpose. If A is invertible, let A^{-1} be its inverse and $A^{-*} := (A^*)^{-1}$. Denote by $\star$ irrelevant matrix entries. The $m \times n$ matrix polynomial $A - \lambda E$ is called a *pencil*. The pencil is *regular* if it is square and $\det(A - \lambda E) \not\equiv 0$, otherwise it is called *singular*. $\Lambda(A - \lambda E)$ stands for the union of the generalized eigenvalues (finite and infinite, multiplicities counting). The TFM $G(\lambda)$ is unitary in continuous-time (discrete-time) if $G(\lambda)^*G(\lambda) = I$, $\forall\lambda \in \mathbb{C}_0$, $(\forall\lambda \in \mathbb{D}_1(0))$ which are not poles of $G(\lambda)$.

Consider a system given by a generalized state-space realization (see [15])

$$\begin{aligned} Ex'(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) + Du(t), \end{aligned} \tag{4}$$

or by its TFM $G(\lambda)$ (possibly improper or polynomial) to which we have associated a realization (4), *i.e.*,

$$G(\lambda) = \left[\begin{array}{c|c} A - \lambda E & B \\ \hline C & D \end{array}\right] := C(\lambda E - A)^{-1}B + D. \tag{5}$$

Here $'$ denotes either the differential operator or the unit shift, $y \in \mathbb{C}^p$ is the output, $u \in \mathbb{C}^m$ is the input, $x \in \mathbb{C}^n$ is the state, $A - \lambda E$ is regular and all the intervening matrices in (4) have complex elements and appropriate dimensions.

Although (5) is suited to represent any TFM model it has a couple of drawbacks for the problems under investigation. For example, if ∞ is a pole of $G(\lambda)$ then the order n of the realization (5) is strictly greater than the McMillan degree of $G(\lambda)$, while D does not represent the value of $G(\lambda)$ at any particular point. To circumvent this, we will work with a slightly more general type of realizations, called *centered* (originally introduced in [16,17], see also [18]). To define a centered realization fix first a $\lambda_0 \in \overline{\mathbb{C}}$, and further α, β, such that

$$\begin{cases} \alpha = 1, \quad \beta = 0, \quad \text{if } \lambda_0 = \infty, \\ \dfrac{\alpha}{\beta} = \lambda_0, \quad \beta \neq 0, \quad \text{if } \lambda_0 \in \mathbb{C}. \end{cases} \tag{6}$$

A realization centered at λ_0 is a representation of the form

$$G(\lambda) = \left[\begin{array}{c|c} A - \lambda E & B \\ \hline C & D \end{array}\right]_{\lambda_0} := D + C(\lambda E - A)^{-1}B(\alpha - \beta\lambda), \tag{7}$$

where $A - \lambda E$ is regular, $A, E \in \mathbb{C}^{n\times n}$, $B \in \mathbb{C}^{n\times m}$, $C \in \mathbb{C}^{p\times n}$, and $D \in \mathbb{C}^{p\times m}$. In particular, if $\lambda_0 = \infty$ we will drop the index λ_0 from the notation in (7), and get precisely the notation and representation in (5). Therefore, realizations (5) are simply realizations centered at $\lambda_0 = \infty$. The positive integer n is called the *order* of the realization (7). We say that (7) (or the pair $(A - \lambda E, B)$) is *controllable at* $\lambda \in \mathbb{C}$ if rank $\begin{bmatrix} A - \lambda E & B \end{bmatrix} = n$, and is *controllable at* ∞ if rank $\begin{bmatrix} E & B \end{bmatrix} = n$ (this notion of controllability at infinity was introduced in [19], see also [20] for a comprehensive review of various related notions and characterizations). Analogously, (7) (or the pair $(C, A - \lambda E)$) is *observable at a certain* $\lambda \in \overline{\mathbb{C}}$ provided the pair $(A^* - \lambda E^*, C^*)$ is controllable at λ. A realization (or a pair) is called *controllable* (*observable*) provided it is controllable (observable) $\forall\lambda \in \overline{\mathbb{C}}$ and it is called *minimal* if its order is as small as possible among all realizations centered at the given λ_0.

The paramount features of centered realizations are revealed by choosing λ_0 different from any pole of $G(\lambda)$—a choice in force throughout the paper. In this case, one recovers all properties of standard state-space realizations (see [21] for more details). For example, minimality is equivalent to controllability plus observability. A nice feature of centered realizations, making them as handy to manipulate as standard ones, is the easiness in obtaining them. A direct method to obtain a centered minimal realization (at any point $\lambda_0 \in \overline{\mathbb{C}}$) starting from the TFM description