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Stability analysis of systems with time-varying delay via relaxed integral inequalities

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ABSTRACT

This paper investigates the stability of linear systems with a time-varying delay. The key problem concerned is how to effectively estimate single integral term with time-varying delay information appearing in the derivative of Lyapunov–Krasovskii functional. Two novel integral inequalities are developed in this paper for this estimation task. Compared with the frequently used inequalities based on the combination of Wirtinger-based inequality (or Auxiliary function-based inequality) and reciprocally convex lemma, the proposed ones can provide smaller bounding gap without requiring any extra slack matrix. Four stability criteria are established by applying those inequalities. Based on three numerical examples, the advantages of the proposed inequalities are illustrated through the comparison of maximal admissible delay bounds provided by different criteria.

1. Introduction

Time-varying delays are frequently introduced into control loops during implementing of practical control systems through communication networks [1]. The stability, as the basic requirement of control systems, may be destroyed due to the presence of time delays. Hence, the stability analysis of systems with time-varying delays has been becoming a hot topic in the past few decades [2–6].

The important objective of stability analysis is to find the maximal admissible delay region such that time-delay system remains stable for the time-varying delay within this region [7]. The determination of such region requires suitable stability criteria. Benefit from the advantages of wide applications and easy extension of Lyapunov–Krasovskii functional (LKF) method and the convenient tractability of the linear matrix inequality (LMI), the delay-dependent stability criterion derived in the framework of the LKF and the LMI is the most effective criterion to provide admissible region of the time-varying delay [8].

In order to obtain delay-dependent criteria via the LKF method, the following double integral term is usually applied in the LKF [9]:

$$V_r(t) = \int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R\dot{x}(s)dsd\theta \tag{1}$$

where $R > 0$ is the Lyapunov matrix to be determined, h is upper bound of time-varying delay (Note that this paper discusses the time-varying delay with zero low bound, i.e., $0 \leq d(t) \leq h$), and $x(t)$ is the system state. Then its derivative includes the following single integral terms with time-varying delay information:

$$\mathcal{S}(t) := -\int_{t-d(t)}^{t} \dot{x}^T(s)R\dot{x}(s)ds - \int_{t-h}^{t-d(t)} \dot{x}^T(s)R\dot{x}(s)ds. \tag{2}$$

In order to obtain the LMI-based criterion, a challenging problem is how to find the upper bound of $\mathcal{S}(t)$ [9].

Before 2004, model transformations, together with Park or Moon inequality [10], were generally applied to handle $\mathcal{S}(t)$ [11,12]. The model transformation may result in additional dynamics and the inequality-based cross term bounding leads to conservatism [13]. The free-weighting-matrix (FWM) approach was proposed in 2004 to overcome those drawbacks [14,15]. However, the second single integral term, $\int_{t-h}^{t-d(t)} \dot{x}^T(s)R\dot{x}(s)ds$, was ignored based on the above methods. Later, the improved FWM approaches [16–18] without ignoring such term were developed and used to be the most popular method for studying of different

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time-delay systems [19–21]. However, the drawback of the FWM-based method is that many slack matrices bring heavy computation complexity, and it is a bit difficult to judge how to introduce slack matrices reasonably [22].

An alternative type of method that estimates $\mathscr{S}(t)$ using bounding inequalities is applied to avoid introducing too many slack matrices. The estimation of $\mathscr{S}(t)$ based on this type of method includes two key steps, namely, (1) two integral terms in $\mathscr{S}(t)$ are estimated respectively via suitable bounding inequalities; and (2) the $d(t)$ with the form, $\frac{1}{d(t)}$ and $\frac{1}{h-d(t)}$, appearing in the transformed quadratic terms is handled via suitable techniques. For the first step, Jensen inequality [23] is commonly used in the early researches. Later, some tighter inequalities, such as Wirtinger-based inequality [9] and auxiliary function based inequality [24], are developed to improve the results. Recently, Bessel–Legendre inequality, which contains the above ones as spacial cases, further increases the estimation accuracy [25]. For the second step, the simplest treatment is to directly replace $d(t)$ with its bounds [26], while the enlargement leads to obvious conservatism. Another way for this task is to use the convex combination method [27] after moving the $d(t)$ in the denominator to the numerator of the quadratic terms via some FWM-based inequalities [8,28], simple enlargement treatment [29,30], and vector-redefined method [31], but it usually requires the introducing of many slack matrices and/or the enlargement treatment. The reciprocally convex lemma [32] directly handling the $d(t)$ in the denominator is the most effective method since it leads to least conservatism while only introduces a few slack matrices.

Due to the characteristic of few slack matrices introducing and small conservatism, the combination of the bounding inequality and the reciprocally convex lemma is becoming the most popular framework for estimating $\mathscr{S}(t)$ during the investigation of the systems with time-varying delay. To the best knowledge of the authors, most current researches following this framework still focus on the development of new bounding inequalities for the aforementioned first step task [33–36]. However, there is no reported research that discusses the tighter estimation of $\mathscr{S}(t)$ considering two steps together. This motivates the present research.

This paper develops two relaxed integral inequalities to estimate $\mathscr{S}(t)$ by considering two integral terms together, instead of the two-step estimation method applied in the existing work. The first (or second) proposed inequality is tighter than the one, obtained via the combination of the Wirtinger-based inequality (or the auxiliary function based inequality) and the reciprocally convex lemma, without requiring any extra slack matrix. Four stability criteria of a linear system with a time-varying delay are established by applying those inequalities. Finally, three numerical examples are given to illustrate the effective of the proposed inequalities and the corresponding criteria.

The reminder of paper is organized as follows. Section 2 gives problem formulation and preliminaries. In Section 3, two novel inequalities are given and the comparison with the commonly used ones is discussed. Section 4 gives several new stability criteria of a linear system with a time-varying delay. Section 5 illustrates the advantages of the proposed method via numerical examples. Conclusions are given in Section 6.

Notations. Throughout this paper, the superscripts T and -1 mean the transpose and the inverse of a matrix, respectively; $\mathscr{R}^n$ denotes the n-dimensional Euclidean space; $\|\cdot\|$ refers to the Euclidean vector norm; $P > 0$ (≥ 0) means P is a real symmetric and positive-definite (semi-positive-definite) matrix; I and 0 stand for the identity matrix and the zero-matrix, respectively; $diag\{\cdot\}$ denotes the block-diagonal matrix; and symmetric term in the symmetric matrix is denoted by $*$.

2. Problem formulation and preliminaries

Consider the following linear system with a time-varying delay:

$$\begin{cases}\dot{x}(t) = Ax(t) + A_d x(t-d(t)), \quad t \geq 0\\ x(t) = \phi(t), \quad t \in [-h, 0]\end{cases} \tag{3}$$

where $x(t) \in \mathscr{R}^n$ is the system state, A and A_d are the system matrices, the initial condition $\phi(t)$ is a continuously differentiable function, and $d(t)$ is the time-varying delay satisfying

$$0 \leq d(t) \leq h \tag{4}$$

and

$$\mu_1 \leq \dot{d}(t) \leq \mu_2 \tag{5}$$

where h, μ_1, and μ_2 are constant.

This paper aims to derive new delay-dependent stability criteria for analyzing the stability of system (3). In this paper, the key problem to be concerned during the criterion-deriving is how to estimate the following single integral term with time-varying delay information:

$$\mathscr{S}(t) = -\int_{t-d(t)}^{t} \dot{x}^T(s)R\dot{x}(s)ds - \int_{t-h}^{t-d(t)} \dot{x}^T(s)R\dot{x}(s)ds. \tag{6}$$

This paper will develop two new inequalities for the above estimation task.

The Wirtinger-based integral inequality [9] and the auxiliary function based inequality [24] to be used are given in the following lemma, shown as inequalities (7) and (8), respectively.

Lemma 1 *([9,24]). For symmetric matrix $R > 0$, scalars a and b with $a < b$, and vector ω such that the integration concerned is well defined, the following inequalities hold*

$$(b-a)\int_a^b \dot{\omega}^T(s)R\dot{\omega}(s)ds \geq \chi_1^T R\chi_1 + 3\chi_2^T R\chi_2 \tag{7}$$

$$(b-a)\int_a^b \dot{\omega}^T(s)R\dot{\omega}(s)ds \geq \chi_1^T R\chi_1 + 3\chi_2^T R\chi_2 + 5\chi_3^T R\chi_3 \tag{8}$$

where

$$\chi_1 = \omega(b) - \omega(a)$$

$$\chi_2 = \omega(b) + \omega(a) - \frac{2}{b-a}\int_a^b \omega(s)ds$$

$$\chi_3 = \omega(b) - \omega(a) + \frac{6}{b-a}\int_a^b \omega(s)ds - \frac{12}{(b-a)^2}\int_a^b\int_s^b \omega(u)duds.$$

The reciprocally convex lemma proposed in [32] is reformulated as the following simple form [9].

Lemma 2 *([32,9]). For vectors β_1 and β_2, real scalar $\alpha \in (0,1)$, symmetric matrix $R > 0$, and any matrix S satisfying $\begin{bmatrix} R & S\\ * & R\end{bmatrix} \geq 0$, the following inequality holds*

$$\frac{1}{\alpha}\beta_1^T R\beta_1 + \frac{1}{1-\alpha}\beta_2^T R\beta_2 \geq \begin{bmatrix}\beta_1\\ \beta_2\end{bmatrix}^T \begin{bmatrix} R & S\\ * & R\end{bmatrix}\begin{bmatrix}\beta_1\\ \beta_2\end{bmatrix}. \tag{9}$$

3. New inequalities for estimating $\mathscr{S}(t)$

This section discusses the methods of estimating $\mathscr{S}(t)$. The commonly used method based on the bounding inequality and the