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State estimation with remote sensors and intermittent transmissions

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ABSTRACT

This paper deals with the problem of estimating the state of a discrete-time linear stochastic dynamical system on the basis of data collected from multiple sensors subject to a limitation on the communication rate from the remote sensor units. The optimal probabilistic measurement-independent strategy for deciding when to transmit estimates from each sensor is derived. Simulation results show that the derived strategy yields certain advantages in terms of worst-case time-averaged performance with respect to periodic strategies when coordination among sensors is not possible.

1. Introduction

This paper deals with the problem of estimating the state of a discrete-time linear stochastic dynamical system

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{w}_k \tag{1}$$

on the basis of measurements collected from multiple sensors

$$\mathbf{y}_k^i = \mathbf{C}^i\mathbf{x}_k + \mathbf{v}_k^i \tag{2}$$

for $i \in \mathcal{S} \triangleq \{1, 2, \ldots, S\}$ subject to a limitation on the communication rate from each remote sensor unit to the state estimation unit. More specifically, the attention will be focused on the use of a sensor network consisting of: S remote *sensing nodes* which collect noisy measurements of the given system, can process them to find filtered estimates and transmit such estimates at a reduced communication rate; a *fusion node* F which receives data from the S sensors and, based on such data, should estimate, in the best possible way, the system's state.

The objective is to devise a *transmission strategy* (TS) with fixed rate that guarantees bounded state covariance and possibly optimal estimation performance, in terms of minimum *Mean Square Error* (MSE), at the node F.

The above scenario reflects the practical situation in which the sensing units and the monitoring unit are remotely located with respect to each other and the communication rate between them is severely limited by energy, band and/or security concerns. This happens, for example, in wireless sensor networks wherein every transmission typically reduces the lifetime of the sensor devices, wireless communication being the major source of energy consumption [1]. Further, a reduction in the sensors' data transmission rate can be crucial in networked control systems in order to reduce the network traffic and hopefully avoid congestion [2].

State estimation under finite communication bandwidth has been thoroughly investigated; see e.g. [3–5]. In the above cited references, the emphasis is on the analysis of the quantization effects due to the encoding of transmitted data into a finite alphabet of symbols as well as on the design of efficient, possibly optimal, coding algorithms. Conversely, following [2,6–8], the present work tackles the issue of communication bandwidth finiteness from a completely different viewpoint. Specifically, it is assumed that infinite-precision data are transmitted over the communication channel[1] while the bandwidth limitation is accomplished by imposing a suitable value of the transmission rate. In this context, the focus will be on the choice of a decentralized TS for deciding which data transmit from the remote sensors to F. It is pointed out that the idea of controlling data transmission in a networked control system so as to achieve a tradeoff between communication and estimation performance is not novel in the literature; see [2,6,7,9–11] and the references therein. Further, a somehow related problem is the so-called

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[1] This assumption, though incompatible with the finite bandwidth, holds in practice provided that quantization errors are negligible with respect to measurement errors.

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sensor scheduling problem, wherein, at every time step, the fusion node selects only a subset of the available sensors to receive the information [12–14].

In this paper, probabilistic decentralized measurement-independent strategies are considered and issues related to the stability and optimality of such strategies are investigated. Specifically, attention is devoted to TS's wherein the time intervals between consecutive transmissions are random variables governed by a finite-state Markov chain. Notice that a similar TS has been analyzed in [15] in the context of a model-based networked control system. Further, in the signal processing literature, the idea of randomly varying the time between consecutive measurements, commonly referred to as *additive random sampling* [16], has been extensively studied in order to overcome the aliasing problem. Finally, additive random sampling is a common practice in Internet flow monitoring to avoid synchronization problems [17].

In the present context, the main motivation for considering this kind of probabilistic TS's stems for the observation that in the multisensor case, supposing that the information provided by different sensors be mutually correlated, the performance of periodic TS's can depend in a crucial way on the phase shift among the sensors and coordination would be needed so as to ensure that sensor transmissions be well distributed over time. For example, when all sensors are identical, performance is optimized when sensor transmissions are uniformly distributed over time, whereas a degradation of the estimation performance at the fusion node is expected when all sensors periodically transmit at the very same time instants. Thus, in a purely decentralized setting wherein sensor coordination is not allowed, the performance of periodic TS's can vary significantly for different realizations. In other words, periodic multisensor TS's induce an undesired "non ergodic" behavior. As will be shown, this effect can be counteracted by considering probabilistic TS's generated via aperiodic Markov chains so as to ensure that ergodicity holds and, thus, the long-run average performance be always realization-independent. In this connection, the main contribution of the present paper lies in showing that probabilistic strategies yield certain advantages with respect to periodic ones when coordination among sensors is not possible and provided that the transmission probabilities be adequately chosen.

Another important issue that should be taken into account is that, when many sensors decide to simultaneously transmit to the fusion node, there might be loss of information due to network overload and/or collisions. Clearly, without coordination among the sensors, this possibility cannot be completely ruled out. However, the proposed aperiodic strategy, thanks to the fact that the phase shift among the sensors is governed by an ergodic Markov chain, makes the simultaneous transmission of many sensors an infrequent event, irrespective of the particular Markov chain realization.

Preliminary work has been carried out in [9] which addressed the single-sensor case and in [10] which formulated the optimal multisensor transmission problem with reference to a generic fusion algorithm and discussed possible suboptimal solutions. The main contributions of the present paper with respect to the above cited references are as follows: the multisensor TS problem is formulated considering *covariance intersection* [18,19] as data fusion rule and exploiting local standard detectability forms so as to deal with possible local undetectability issues; a closed-form is derived for the multisensor TS that minimizes the average MSE at the fusion node; issues concerning the misbehavior of non-ergodic (periodic) multisensor TS's are thoroughly worked out, also via simulation experiments, and provably optimal aperiodic perturbations of the periodic strategy are developed which avoid such a misbehavior with a minimal increase of the average cost.

The notations are quite standard: $\mathbb{Z}_+$ is the set of nonnegative integers; given a square matrix $\mathbf{M}$, $\mathrm{tr}(\mathbf{M})$ and $\mathbf{M}'$ denote its trace and, respectively, transpose; $\mathbb{E}\{\cdot\}$ and $\mathbb{P}\{\cdot\}$ denote the expectation and, respectively, probability operators; finally, given a vector-valued sequence $\{\mathbf{z}_k;\, k = 0, 1, \ldots\}$, $\mathbf{z}_{t_1:t_2}$ stands for its restriction to the discrete time interval $\{t_1, t_1 + 1, \ldots, t_2\}$.

2. Communication strategy

The aim of this section is to formalize the concept of *transmission strategy (TS)* with fixed rate α^i for sensor i. To this end, let us introduce for each sensor i a binary variable c_k^i taking value 1 if sensor i transmits at time k and value 0 otherwise.

The attention is restricted to estimate transmission, assuming that each sensor node i has enough processing capability to update on-line the optimal state estimate $\hat{\mathbf{x}}^i_{k|k}$. Note that, with such a choice, the loss of information due to the finite bandwidth is mitigated as the estimate $\hat{\mathbf{x}}^i_{k|k}$ somehow summarizes the information collected by sensor i up to time k being the center of the local posterior PDF. As far as the decision mechanism is concerned, this can be formally defined as follows.

Definition 1. A decision mechanism with rate $\alpha^i \in (0, 1)$ is any, deterministic or stochastic, mechanism of generating c_k^i such that

$$\lim_{t\to\infty} \frac{1}{t} \sum_{k=1}^{t} c_k^i = \alpha^i. \tag{3}$$

Several decision mechanisms can clearly be devised. In this work, the attention will be restricted to *measurement-independent* strategies that, at time k, decide whether to transmit or not independently of the measurement sequence $\mathbf{y}^i_{0:k}$. More specifically, it is supposed that each c_k^i is chosen according to some probabilistic criterion, possibly adapted only on the basis of the past transmission pattern, so as to ensure the desired communication rate. In this respect, let $n_k^i \geq 0$ denote, at a generic time k, the number of time instants elapsed from the last transmission of sensor i, i.e.

$$c^i_{k-n_k^i} = 1 \quad \text{and} \quad c_k^i = c_{k-1}^i = \cdots = c^i_{k-n_k^i+1} = 0.$$

It should be evident that, in the considered framework, the greater n_k^i the more outdated is the last estimate $\hat{x}_{k-n_k^i|k-n_k^i}$ received at the fusion node F from sensor i. Then it seems reasonable to take into account the value of n_{k-1}^i when choosing whether to transmit or not at the generic time k. In this connection, the conditional probabilities $\mathbb{P}\{c_k^i = 1|n_{k-1}^i = j\}$, $j = 0, 1, \ldots$, can be considered as design parameters in the communication strategy that can be suitably tuned to improve performance (e.g., to reduce the MSE).

Specifically, at the generic time k, c_k^i is chosen to be a Bernoulli random variable with parameter $\varphi^i(n_{k-1}^i)$, i.e., c_k^i takes value 1 with probability $\varphi^i(n_{k-1}^i)$ and value 0 with probability $1 - \varphi^i(n_{k-1}^i)$. Clearly, this corresponds to

$$\mathbb{P}\{c_k^i = 1|n_{k-1}^i = j\} = \varphi^i(j),$$

for $j = 0, 1, \ldots$ and $i = 1, \ldots, S$. The functions $\varphi^i : \mathbb{Z}_+ \to [0, 1]$ must be chosen so that the transmission rate constraint is met for each sensor i.

Throughout the paper, the following notation will be adopted

$$\mathbf{c}_k \triangleq \mathrm{col}\left(c_k^1, \ldots, c_k^S\right), \qquad \mathbf{n}_k \triangleq \mathrm{col}\left(n_k^1, \ldots, n_k^S\right),$$
$$\mathbf{y}_k \triangleq \mathrm{col}\left(\mathbf{y}_k^1, \ldots, \mathbf{y}_k^S\right).$$

3. Fusion algorithm

This section is devoted to the description of the operations that are performed in the sensor nodes $1, \ldots, S$ as well as in the fusion node F in order to recover a fused estimate.