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New methods for mode-independent robust control of Markov jump linear systems☆

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ABSTRACT

This paper treats the H_2 and H_∞ controls of linear systems with Markov jump disturbances, via new design methods based on linear matrix inequalities (LMIs). The proposed techniques are especially tailored to the scenario where the jump process cannot be measured, and apply to homogeneous Markov chains of any structure. In the scenario of polytopic uncertainty affecting the system matrices, new uncertainty-dependent methods are introduced for the design of robust controllers. Several numerical examples illustrate situations where the proposed techniques are less conservative than the ones found in the literature.

1. Introduction

This paper is concerned with linear systems whose switching is governed by a homogeneous Markov chain with finite state space. This class is commonly referred to in the specialized literature as Markov Jump Linear Systems (MJLS for short) (see, e.g., [1–4]), and has been associated to dynamical systems which are subject to abrupt changes (e.g., *failure*) in their dynamics. Our focus is in the H_2 and H_∞ control problems, in the scenario in which the controllers have *no access to the jump process*. We shall be particularly interested in the case in which the class of admissible control policies considered is of reduced complexity, in the sense that the control gain is assumed independent of the unknown Markov chain (also known as the *operation mode* of the system). In this case, the control problem is also dubbed in the specialized literature as the *mode-independent case* (see, e.g. [5–8]). The difficulties here assume the form of *non-convexity* properties, and the methods for tackling them are all somewhat conservative or inefficient from the computational point of view (i.e., not solvable in polynomial time, see, e.g., [9]). An exception to the rule is the case in which the Markov chain reduces to a generalized Bernoulli process, the so-called *Bernoulli jump case*. Nudged by a favorable structure, in conjunction with recent interest in its applications, the Bernoulli jump case has been fleshed out by an increasing amount of literature on this subject. In this setup, for instance, Costa and Fragoso [10] derived criteria for mean square stability which are easier to check than those related to the more general MJLS case. An H_∞ control scenario has been the subject of Seiler and Sengupta [11], and the obtained results were subsequently applied to networked control systems. Ref. [12] also treated this framework, and addressed robustness issues. Exact parameterizations for H_2 and H_∞ controllers were derived in [6]. The study of the Bernoulli jump case has been of particular interest to the *networked control systems* literature, in which the jump process models packet dropout in a communication link (see, for instance, in [13–16]).

If the transition probabilities of the Markov chain do not follow the delicate Bernoulli structure then, as far as the authors are aware, all existing methods in the literature are somewhat *conservative*. In [17], for instance, mode-independent stabilization is tackled via a *common Lyapunov function*, a technique which is non-conservative only in some rare cases reported in the switched systems literature (see [18]). Ref. [19] also treated the mode-independent stabilization problem, but its correlation with the current paper is restricted to the scenario of polytopic uncertainty. The paper do Val et al. [20] introduced a more general setup, dubbed *cluster observations*, in which the controller is allowed to switch within a set of operation modes that do not necessarily match those of the to-be-controlled system. More recent results regarding cluster observations can also be found, for instance, in [21,22]. In the recent paper Oliveira et al. [7], the mode-independent H_2 control is addressed via a two-step procedure, which involves the computation of a mode-dependent mean square stabilizing gain, and then feeding the obtained result to an LMI problem which yields the desired controller. The technique is

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then subsequently applied to the control of a DC (direct current) motor, and the numerical results outperform [20]. A closely related reference is [8], which addressed the linear quadratic control of MJLS via a certain time-varying controller which is independent of the jump process.

1.1. About this paper

In this paper, we make further foray in the design of controllers for the case in which we have no access of the Markov chain. We propose new conditions for H_2 and H_∞ control. Out of the bent which wends most of the technique dealing with the mode-independent case in the specialized literature, which are mere particularization of mode-dependent techniques, we design new tools which are firmly associated to the mode-independent setup and, in fact, it is unclear by now how they could be extended to the mode-dependent case. A glimpse of some distinguishing aspects of the proposed approach are as follows.

- Differently from what occurs in [23,21,20], the proposed results have the desirable feature of not being conservative in the Bernoulli jump scenario. Although this also occurs in, e.g., [6], the distinction here is that the proposed results apply to Markov chains with general transition probabilities (i.e., they are not restricted to the Bernoulli structure).
- In the mixed H_2/H_∞ control setup, the results are expressed via slack variables which allow for some degree of separation between the H_2 and H_∞ variables. The class of systems is also more general than the one treated in [24].
- The results are amenable to a further degree of relaxation, via the technique developed in [19]. In the scenario of polytopic uncertainty, this yields an *uncertainty-dependent* design of reduced conservatism. (Following de Souza [19], "uncertainty-dependent" here means that some of the decision variables are functions of the polytope vertices, which of course is more general – hence less conservative – than the simpler case in which the variables are not allowed to depend on the vertices—as in e.g. [17].)

The basic definitions and preliminary results which are necessary to our development are enclosed in Section 2. Section 3 features the first main result of the paper (Theorem 1), which consists of new H_2/H_∞ designs via LMIs, followed by a brief discussion of the Bernoulli jump case (Proposition 1). The scenario of polytopic uncertainty is then treated in Section 4. The main result here comes in the form of Theorem 2, which features a new LMI-based design for robust mode-independent controllers. Closing the paper, we present in Section 5 several numerical examples which illustrate the potentials of the proposed results, along with a brief conclusion in Section 6.

1.2. Notation

We denote by $(\Omega, \mathfrak{F}, \mathfrak{F}_k, \mathbb{P})$ a complete stochastic basis carrying an increasing filtration $\mathfrak{F}_k \subset \mathfrak{F}$ on $k \in \{0, 1, 2, \dots\}$, and by $\mathbb{E}$ the usual mathematical expectation. The notation $\ell_2^n(\Omega, \mathfrak{F}, \mathbb{P})$ is adopted to represent the space of all discrete-time $\mathfrak{F}_k$-adapted processes of finite energy, i.e., those of the form $w = \{(w(k), \mathfrak{F}_k),\ k = 0, 1, 2, \dots\}$ with $\|w\|_{\ell_2}^2 \triangleq \sum_{k=0}^{\infty} \mathbb{E}[\|w(k)\|^2] < \infty$. Also, $\mathrm{Her}(G) \triangleq G + G'$ for any $G \in \mathbb{R}^{n\times n}$, with G' standing for the transpose of G. For $X = (X_1, \dots, X_N)$, we also define, for later use, the operations

$$\mathcal{E}_i(X) \triangleq \sum_{j=1}^{N} p_{ij} X_j, \qquad \mathcal{E}(X) \triangleq \sum_{j=1}^{N} p_j X_j. \tag{1}$$

2. Preliminaries

Consider a homogeneous Markov chain $\theta = \{\theta(k),\ k = 0, 1, 2, \dots\}$ in the stochastic basis $(\Omega, \mathfrak{F}, \mathfrak{F}_k, \mathbb{P})$, with finite state space $\mathcal{S} = \{1, \dots, N\}$ and such that, whenever $\mathbb{P}(\theta(k) = i) > 0$:

$$\mathbb{P}\big(\theta(k+1) = j \,\big|\, \theta(k) = i\big) = p_{ij}, \tag{2}$$

for given $p_{ij} \geq 0$ such that $\sum_{j=1}^{N} p_{ij} = 1$ for each $i \in \mathcal{S}$ (i.e., $[p_{ij}] \in \mathbb{R}^{N\times N}$ is the transition matrix of the Markov chain). The initial distribution of θ is denoted as follows:

$$\nu = (\nu_1, \dots, \nu_N), \qquad \nu_i \triangleq \mathbb{P}(\theta(0) = i), \quad i \in \mathcal{S}. \tag{3}$$

The subject matter of this paper is the design of static state-feedback controllers of the form

$$u(k) = Kx(k), \quad K \in \mathbb{R}^{n_u \times n} \tag{4}$$

for the control system

$$\begin{cases} x(k+1) &= A_{\theta(k)} x(k) + B_{\theta(k)} u(k) + J_{\theta(k)} w(k) + E_{\theta(k)} \varpi(k) \\ z(k) &= C_{\theta(k)} x(k) + D_{\theta(k)} u(k) \\ \zeta(k) &= F_{\theta(k)} x(k) + G_{\theta(k)} u(k) + H_{\theta(k)} \varpi(k) \end{cases} \tag{5}$$

where $x = \{x(k) \in \mathbb{R}^n,\ k = 0, 1, 2, \dots\}$ is the measured state; $w = \{w(k) \in \mathbb{R}^{n_w},\ k = 0, 1, 2, \dots\}$ and $z = \{z(k) \in \mathbb{R}^{n_z},\ k = 0, 1, 2, \dots\}$ are H_2 inputs and outputs; and $\varpi = \{\varpi(k) \in \mathbb{R}^{n_\varpi},\ k = 0, 1, 2, \dots\}$, $\zeta = \{\zeta(k) \in \mathbb{R}^{n_\zeta},\ k = 0, 1, 2, \dots\}$ are H_∞ inputs and outputs.

Remark 1. The control system treated in [24] is a particular case of (5), because separate channels for H_2 and H_∞ performance are considered, and because J and E are allowed to depend on θ in (5).

The combination of (5) and (4) yields the closed-loop system

$$\mathcal{G}_K = \begin{cases} x(k+1) &= \widehat{A}_{\theta(k)} x(k) + J_{\theta(k)} w(k) + E_{\theta(k)} \varpi(k) \\ z(k) &= \widehat{C}_{\theta(k)} x(k) \\ \zeta(k) &= \widehat{F}_{\theta(k)} x(k) + H_{\theta(k)} \varpi(k) \end{cases} \tag{6}$$

with

$$\widehat{A}_i = A_i + B_i K, \qquad \widehat{C}_i = C_i + D_i K, \qquad \widehat{F}_i = F_i + G_i K, \tag{7}$$

whose analysis is the subject of the remainder of this section.

Definition 1 (*See [1, Chapter 3]*)**.** The closed-loop system (6) is termed internally mean square stable (internally MSS) whenever

$$w \equiv 0, \qquad \varpi \equiv 0 \Longrightarrow \lim_{k\to\infty} \mathbb{E}\left[\|x(k)\|^2\right] = 0 \quad \forall x_0, \theta_0. \tag{8}$$

We define the corresponding set of all internally mean square stabilizing controllers as

$$\mathcal{K} = \left\{K \in \mathbb{R}^{n_u \times n},\ (8) \text{ is satisfied}\right\}. \tag{9}$$

We shall borrow from [1, Definition 4.7] and [24, Definition 4] the following definitions for the H_2 and H_∞ norms of the system (6).

Definition 2 (H_2 *Norm*)**.** Whenever system (6) is internally MSS (i.e., $K \in \mathcal{K}$), we define its H_2 norm as

$$\|\mathcal{G}_K\|_2 = \left(\sum_{i=1}^{n_w} \sum_{j=1}^{N} \nu_j \sum_{k=0}^{\infty} \mathbb{E}\left[\left\|z_0^i(k)\right\|^2\right]\right)^{1/2} \tag{10}$$

where z_0^i stands for the H_2 output of system (6) when $x_0 = 0$, $\varpi \equiv 0$, and the H_2 input

$$w_i(k) = \begin{cases} \mathfrak{e}_i, & k = 0, \\ 0, & k = 1, 2, \dots, \end{cases}$$

is applied, with $\mathfrak{e}_i$ standing for the ith vector of the canonical basis in $\mathbb{R}^{n_w}$.