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Global stable tracking control of underactuated ships with input saturation

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ABSTRACT

In this paper, tracking control of underactuated ship in the presence of input saturation is addressed. By dividing the tracking error dynamic system into a cascade of two subsystems, the torques in surge and yaw axes are designed separately using the backstepping technique. More specifically, we design the yaw axis torque in such a way that its corresponding subsystem is finite time stable, which makes it to be de-coupled from the second subsystem after a finite time. This enables us to design the torque in the surge axis independently. It is shown that the closed-loop system is stable and the mean-square tracking errors can be made arbitrarily small by choosing design parameters. Simulation results also verify the effectiveness of the proposed scheme.

1. Introduction

The control of underactuated systems with nonholonomic constraints has received vast attention over the past few decades. Typical examples of such systems include nonholonomic mobile robots, underactuated ships, underwater vehicles and VCTOL aircrafts et al. There has been significant interest in tracking control of underactuated ships as evidenced by various control schemes proposed for such a problem. In [1], two effective control schemes are presented to solve the global tracking control problem for underactuated ships with Lyapunov method by exploiting the inherent cascade interconnected structure of the ship dynamics. In [2] by dividing the ship dynamics into two linear subsystems a global tracking controller is proposed. In [3] a continuous time-varying tracking controller is designed to yield global uniformly ultimately bounded tracking by using a transformation of the ship tracking system into a skew-symmetric form and designing a time-varying dynamic oscillator. In [4] the method developed for chain-form system was adopted for underactuated ships through a coordinate transformation to steer both the position variables and the course angle of the ship providing exponential stability of the reference trajectory. In [5] a single controller is designed to achieve stabilization and tracking of underactuated ships simultaneously. In [6] a controller that achieves practical stabilization of arbitrary reference trajectories for underactuated ships using the transverse function approach is investigated. Subsequent related works on the tracking control of underactuated ships include, but are not limited to, [7–11] and many references therein.

In practice, an underactuated ship is ultimately driven by a motor which can only provide limited amount of torque. Thus the magnitude of the control signal is always constrained. Under such a constraint, the controllability of the system is seriously affected. Thus it often severely limits system performance, giving rise to undesirable inaccuracy or leading to instability. This brings great challenges to the design and analysis of controllers. Therefore, the development of a control scheme for an underactuated ship with input saturation has been an extremely difficult task, yet with practical interest and theoretical significance. In [12] a tracking control method is presented to address such an issue by employing the linearization and dynamic surface control, only ensuring bounded tracking errors. Other than this one, there is no other literature dealing with this problem as far as we know, due to its difficulty.

In this paper, we solve this problem by proposing a state-feedback control scheme. First the tracking error dynamic system is divided into a cascade of two subsystems so that the torques in surge and yaw axes can be designed separately based on the two subsystems. A new finite-time control scheme is proposed to design the torque in yaw which enables the two subsystems to be de-coupled within finite time. After decoupling, a new

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backstepping scheme similar to that in [13] is proposed for the surge subsystem. The designed controllers for the two subsystems are analyzed, respectively. Then the overall system is studied. It is shown that the closed-loop system is stable and the mean-square tracking errors can be made arbitrarily small by adjusting design parameters. To illustrate the effectiveness of the controller, simulation studies are conducted.

The remaining part of the paper is organized as follows. In Section 2, we present the system model of underactuated ships and formulate the tracking control problem. In Section 3, we develop control laws for the surge force and yaw force. By studying the resulting closed loop system, our main results are established. In Section 4, simulation results are presented. Finally, some brief concluding remarks are given in Section 5. In addition, necessary preliminaries required for our design and analysis on finite-time stability are presented in the Appendix.

2. Problem formation

2.1. Underactuated ship model

Similar to [1], we consider the ship as shown in Fig. 1. The only two propellers are the torques in surge and yaw axes. The motion of the ship dynamics is described by the following differential equations:

$$\begin{aligned} \dot{x} &= u\cos\phi - v\sin\phi \\ \dot{y} &= u\sin\phi + v\cos\phi \\ \dot{\phi} &= r \end{aligned} \tag{1}$$

$$\begin{aligned} \dot{u} &= \frac{m_2}{m_1}vr - \frac{d_1}{m_1}u + \frac{1}{m_1}Sat_u(\tau_u) \\ \dot{v} &= -\frac{m_1}{m_2}ur - \frac{d_2}{m_2}v \\ \dot{r} &= \frac{m_1 - m_2}{m_3}uv - \frac{d_3}{m_3}r + \frac{1}{m_3}Sat_r(\tau_r) \end{aligned} \tag{2}$$

where (x, y) denotes the position of the ship surface in X and Y directions, respectively, ϕ denotes the heading angle of the ship, u, v and r are the surge, sway and yaw velocities, respectively. τ_u and τ_r represent the torques in surge and yaw axis, respectively, which are considered as actual control inputs. Since there is no torque introduced in the sway direction, the system modeled by (1)–(2) is underactuated. The positive constants m_j denote the ship inertia including added mass, and d_j, $1 \leq j \leq 3$ represent the hydrodynamic damping coefficients. The saturation function is defined as follows:

$$Sat_\star(x) = \begin{cases} x & \text{if } |x| < \tau_{\star\max} \\ \tau_{\star\max} & \text{if } |x| \geq \tau_{\star\max} \end{cases} \tag{3}$$

where $\star$ denotes u or r respectively and $\tau_{u\max}$ and $\tau_{r\max}$ are their respective saturation limits.

Our control objective is formally stated as follows.

• *Control objective*: Design the control inputs τ_u and τ_r to force the ship, as modeled in (1)–(2) subject to input saturation, to track a prescribed path, which is denoted by $\Omega = (x_d, y_d, \phi_d, u_d, r_d)$ generated by the dynamic equations of a virtual underactuated ship:

$$\begin{aligned} \dot{x}_d &= u_d\cos(\phi_d) - v_d\sin(\phi_d) \\ \dot{y}_d &= u_d\sin(\phi_d) + v_d\cos(\phi_d) \\ \dot{\phi}_d &= r_d \\ \dot{v}_d &= -\frac{m_1}{m_2}u_d r_d - \frac{d_2}{m_2}v_d. \end{aligned} \tag{4}$$

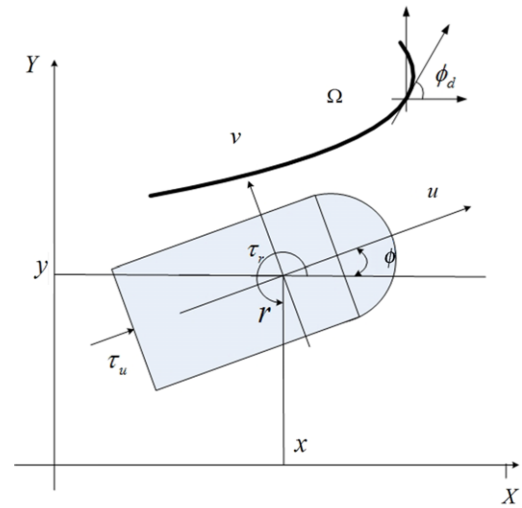

Fig. 1. The coordinates of an underacted ship.

2.2. Variable transformation

To facilitate the control design, first the path-following errors $\{x_e, y_e, \phi_e\}$ are defined, based on standard approach in tracking control of underactuated mechanical systems such as [14]

$$\begin{bmatrix} x_e \\ y_e \\ \phi_e \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - x_d \\ y - y_d \\ \phi - \phi_d \end{bmatrix}. \tag{5}$$

Differentiating both sides of (5) along (1) results in the kinematic error dynamics:

$$\begin{aligned} \dot{x}_e &= u_e - u_d\Big(\cos(\phi_e) - 1\Big) - v_d\sin(\phi_e) + r_e y_e + r_d y_e \\ \dot{y}_e &= v_e - v_d\Big(\cos(\phi_e) - 1\Big) + u_d\sin(\phi_e) - r_e x_e - r_d x_e \\ \dot{\phi}_e &= r_e \\ \dot{u}_e &= \frac{m_2}{m_1}vr - \frac{d_1}{m_1}u + \frac{1}{m_1}Sat_u(\tau_u) - \dot{u}_d \\ \dot{v}_e &= -\frac{m_1}{m_2}(ur - u_d r_d) - \frac{d_2}{m_2}v_e \\ \dot{r}_e &= \frac{m_1 - m_2}{m_3}uv - \frac{d_3}{m_3}r + \frac{1}{m_3}Sat_r(\tau_r) - \dot{r}_d \end{aligned} \tag{6}$$

where $u_e = u - u_d$, $v_e = v - v_d$ and $r_e = r - r_d$.

To achieve the control objective, the following assumption is imposed.

Assumption 1. (a) There is a constant $\delta_r > 0$ such that, for any time period (t_0, t),

$$\int_{t_0}^{t} r_d^2(\tau)d\tau \geq \delta_r(t - t_0).$$

(b) There exists a positive constant Δ such that

$$\frac{|m_1 - m_2|}{2d_1\alpha}\tau_{u\max}^2 + \Delta \leq \tau_{r\max} \tag{7}$$

where $\alpha = \min\{\frac{d_1}{m_1}, \frac{2d_2}{m_2}\}$. Furthermore, r_d is chosen such that $|m_d\dot{r}_d + d_3 r_d| < \Delta$.

Remark 1. • Assumption (a) is a PE condition on the reference angular velocity r_d which is commonly required, see for example [1,2] et al.