Systems & Control Letters 85 (2015) 23–32

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Robust static output feedback $\mathcal{H}_\infty$ control design for linear systems with polytopic uncertainties

Xiao-Heng Chang [a,b], Ju H. Park [b,*], Jianping Zhou [c]

[a] School of Information Science and Engineering, Wuhan University of Science and Technology, 430081 Wuhan, China
[b] Department of Electrical Engineering, Yeungnam University, 280 Daehak-Ro, Kyongsan 38541, Republic of Korea
[c] School of Computer Science and Technology, Anhui University of Technology, 243032 Ma'anshan, China

ARTICLE INFO

Article history:
Received 27 May 2014
Received in revised form
18 May 2015
Accepted 24 August 2015
Available online 29 September 2015

Keywords:
Linear systems
Polytopic uncertainties
Static output feedback $\mathcal{H}_\infty$ controllers
Linear matrix inequalities (LMIs)

ABSTRACT

This paper investigates the problem of robust static output feedback $\mathcal{H}_\infty$ control for linear systems with polytopic uncertainties. A new method is proposed for robust static output feedback $\mathcal{H}_\infty$ controller design. The proposed design method is applicable for general uncertain systems, without the need to impose any constraints on system matrices. The corresponding design conditions are presented in the form of linear matrix inequalities (LMIs). One of the advantages of the new method lies in its less conservatism. The proposed design method is also applicable to both continuous-time and discrete-time systems. The performance of the method is compared with other methods based on several examples.

1. Introduction

Polytopic uncertainty is one of the important types of parametric uncertainty in robust control theory and practical situations where the set of system parameters is a convex polyhedron. The nominal system is located at the center of this polyhedron. Robust analysis and control design problems for linear systems with polytopic uncertainties have been studied extensively in the past decades, and many remarkable results have been obtained [1–8]. On the other hand, in control theory and practice, static output feedback control is very useful and widely adopted in practice since it can be easily implemented with low cost. Robust static output feedback controller design for linear systems with polytopic uncertainties has been investigated by many researchers. In general, the design problem of such controllers can be represented as a bilinear matrix inequality (BMI) problem, which is nonconvex and NP-hard [9,10]. Various numerical algorithms based on the linear matrix inequality (LMI) technique have been widely applied to design robust static output feedback controllers. Two-step methods for designing output feedback controllers have been proposed [11–15]. The first step in these methods is to obtain a state feedback controller gain, while the second step is to obtain an output feedback controller gain. The methods imply that the design is dependent on the state feedback controller gain obtained in the first step. In [16], an iterative LMI (ILMI) approach was presented to obtain the gains of static output feedback controllers. The approach is important for solving the BLMI problem and has been widely used to handle coupling constraints.

Robust static output feedback $\mathcal{H}_\infty$ controller design problems have been extensively discussed in recent years for linear systems with polytopic uncertainties in both the continuous-time and discrete-time contexts using LMI-based convex conditions. A linear parameter-dependent stabilization method for designing static output feedback $\mathcal{H}_\infty$ controllers was proposed [17]. In another study [18], a descriptor approach with non-strict inequality (semi-definite matrix inequality) was used to study the output feedback $\mathcal{H}_\infty$ control problem of continuous-time systems with polytopic uncertainties. Another method inserted an equality-constrained condition for the Lyapunov matrix and derived LMI-based conditions for solving the static output feedback controller design problem of linear systems [19], which result can be extended to design $\mathcal{H}_\infty$ controllers for uncertain linear systems. Another method introduced a slack variable with sub-triangle structure and proposed LMI-based conditions for designing robust static output feedback $\mathcal{H}_\infty$ controllers for linear systems with time-invariant uncertainties [20]. Sufficient conditions for designing static output feedback $\mathcal{H}_\infty$ controllers were given based on the properties of the null space of output matrices and by introducing parameter-independent slack variables with a lower-triangular structure [21].

* Corresponding author.
E-mail addresses: changxiaoheng@sina.com (X.-H. Chang), jessie@ynu.ac.kr (J.H. Park), jpzhou2010@yahoo.com (J. Zhou).

http://dx.doi.org/10.1016/j.sysconle.2015.08.007
0167-6911/

Recently, new LMI-based conditions with a line search over a scalar variable for designing robust static output feedback $\mathcal{H}_\infty$ controllers were proposed [22], where the uncertain output matrix of the considered system is allowed to be of non-full row rank. By assuming that the system input matrix is of full column rank and introducing an additional positive definite matrix, LMI-based design conditions were presented for output feedback $\mathcal{H}_\infty$ control of linear discrete-time systems [23]. In order to obtain LMI-based conditions for designing static output feedback $\mathcal{H}_\infty$ controllers, we have to impose some constraints on system matrices, that is, the above mentioned results are limited and cannot be applied to general uncertain systems. In addition, it should be mentioned that the problems of multi-objective output feedback control were considered in [24,25], where the controllers are designed via the technique of linearizing change of variables. However, the technique cannot lead to a solution of output feedback controller for systems with polytopic uncertainty. This is due to the fact that to linearize the matrix inequality, the introduced new variables will have to be vertex-dependent and involve the controller parameters to be sought, which implies that the required controller parameters cannot be computed from the introduced variables [4].

This paper addresses the robust static output feedback $\mathcal{H}_\infty$ controller design problem for linear systems with polytopic uncertainties. The focus is on designing a static output feedback controller for a general uncertain system, which guarantees a prescribed $\mathcal{H}_\infty$ performance level for the closed-loop system. A new method is proposed to derive sufficient conditions for robust static output feedback $\mathcal{H}_\infty$ controller design, which can be described by a set of LMIs. The method can effectively solve the BMI problem in the literature for robust static output feedback $\mathcal{H}_\infty$ controls, in which the aforementioned constraints imposed on the system matrices have been avoided. Besides wide applications, this new method is less conservative. Numerical examples are provided to illustrate the feasibility of the proposed design method.

The remainder of this paper is organized as follows. Section 2 presents a new LMI-based condition for designing robust static output feedback $\mathcal{H}_\infty$ controllers for uncertain continuous-time systems. Section 3 generalizes the results of Section 2 to the $\mathcal{H}_\infty$ controller design of discrete-time systems. Section 4 demonstrates examples and compares the proposed results of this paper with previous approaches.

Notations. In symmetric block matrices, the symbol $(*)$ represents a term that is induced by symmetry. I is the identity matrix with appropriate dimensions. $diag\{\cdots\}$ denotes a diagonal matrix. $L_2[0,\ \infty)$ $\big(l_2[0,\ \infty)\big)$ is the space of square-integrable (summable) vector functions over $[0,\ \infty)$. $diag\{\cdots\}$ indicates a block-diagonal matrix. The notation $He(A) = A + A^T$ will also be used, and the notation $\mathcal{F}_{m\times n}$ indicates that $\mathcal{F} \in \mathcal{R}^{m\times n}$.

2. Robust static output feedback $\mathcal{H}_\infty$ control of continuous-time systems

Consider a linear continuous-time system with time-invariant polytopic uncertainties described by state-space equations

$$\begin{aligned} \dot{x}(t) &= A(\alpha)x(t) + B(\alpha)u(t) + E(\alpha)w(t),\\ z(t) &= C_1(\alpha)x(t) + D(\alpha)u(t) + F(\alpha)w(t),\\ y(t) &= C_2(\alpha)x(t) + H(\alpha)w(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathcal{R}^n$ is the state variable, $u(t) \in \mathcal{R}^m$ is the control input, $w(t) \in \mathcal{R}^f$ is an arbitrary noise signal in $L_2[0,\ \infty)$, $z(t) \in \mathcal{R}^q$ is the controlled output variable, and $y(t) \in \mathcal{R}^p$ is the measurement output. The matrices $A(\alpha),\ B(\alpha),\ E(\alpha),\ C_1(\alpha), D(\alpha),\ F(\alpha)$, $C_2(\alpha)$, and $H(\alpha)$ are constant matrices of appropriate dimensions and belong to the following uncertainty polytope [1]:

$$\begin{aligned} \Omega &= \Big\{\Big[A(\alpha),\ B(\alpha),\ E(\alpha),\ C_1(\alpha),\ D(\alpha),\ F(\alpha),\ C_2(\alpha),\ H(\alpha)\Big]\\ &= \sum_{i=1}^{\mu}\alpha_i\Big[A_i,\ B_i,\ E_i,\ C_{1i},\ D_i,\ F_i,\ C_{2i},\ H_i\Big],\ \sum_{i=1}^{\mu}\alpha_i = 1,\ \alpha_i \ge 0\Big\}. \end{aligned} \tag{2}$$

For system (1), the following static output feedback controller is exploited:

$$u(t) = Ky(t) = K\big(C_2(\alpha)x(t) + H(\alpha)w(t)\big), \tag{3}$$

where K is the controller gain matrix with appropriate dimensions to be determined later.

Combining (1) and (3), the closed-loop system is obtained as

$$\begin{aligned} \dot{x}(t) &= A(\alpha)x(t) + B(\alpha)K\big(C_2(\alpha)x(t) + H(\alpha)w(t)\big)\\ &\quad + E(\alpha)w(t),\\ z(t) &= C_1(\alpha)x(t) + D(\alpha)K\big(C_2(\alpha)x(t) + H(\alpha)w(t)\big)\\ &\quad + F(\alpha)w(t). \end{aligned} \tag{4}$$

For the continuous-time system (1), the objective of $\mathcal{H}_\infty$ control is to find an asymptotically stable $\mathcal{H}_\infty$ controller in the form of (3) such that the following two conditions are satisfied [26]:

(1) The closed-loop system (4) is asymptotically stable when $w(t) = 0$.
(2) The closed-loop system (4) has a prescribed level γ of $\mathcal{H}_\infty$ noise attenuation, i.e. under the zero initial condition $x(0) = 0$, the inequality $\int_0^\infty z^T(t)z(t)dt \le \gamma^2\int_0^\infty w^T(t)w(t)dt$ is satisfied for any nonzero $w(t) \in L_2[0,\ \infty)$.

Remark 1. In order to obtain LMI-based conditions for designing static output feedback $\mathcal{H}_\infty$ controllers, previous approaches [19–22] have to impose some constraints on the system matrices. These works require the output matrix $C_2(\alpha)$ to be fixed; i.e., $C_2(\alpha) = C_2$ (C_2 is of full row rank) and $H(\alpha) = 0$. In [22], a new design approach was given for continuous-time linear polytopic systems, where the output matrix $C_2(\alpha)$ is not required to be of full row rank and fixed. However, the results were obtained based on another constraints of $C_1^T(\alpha)D(\alpha) = 0$, $F(\alpha) = 0$, and $H(\alpha) = 0$. In our study, the constraints on the system matrices have been avoided, so our results are more suitable for general systems.

For convenience of comparison with previous studies, the conditions for static output feedback $\mathcal{H}_\infty$ controller design are given as follows.

Lemma 1 (*See [19]*). *Consider the closed-loop system (4) with $C_2(\alpha) = C_2$ (C_2 is of full row rank) and $H(\alpha) = 0$. For a given scalar $\gamma > 0$, if there exist matrices $Q > 0$, Y, and N satisfying the following conditions*

$$\begin{bmatrix} A_iQ + QA_i^T + B_iYC_2 + C_2^TY^TB_i^T & * & *\\ E_i^T & -\gamma^2 I & *\\ C_{1i}Q + D_iYC_2 & F_i & -I \end{bmatrix} < 0,\quad i = 1,\ 2,\ \ldots,\ \mu, \tag{5}$$

$$C_2Q = NC_2, \tag{6}$$

then the system is asymptotically stable with the $\mathcal{H}_\infty$ performance γ.

Lemma 2 (*See [22]*). *Consider the closed-loop system (4) with $C_1^T(\alpha)$ $D(\alpha) = 0$, $F(\alpha) = 0$, and $H(\alpha) = 0$. Given a scalar $\gamma > 0$, for a known scalar parameter β, if there exist matrices V, U, and $Q_j > 0$, $j = 1,\ 2,\ \ldots,\ \mu$ satisfying the following LMIs:*

$$\Omega_{ii} < 0,\quad i = 1,\ 2,\ \ldots,\ \mu, \tag{7}$$

$$\Omega_{ij} + \Omega_{ji} < 0,\quad i < j,\ i,\ j = 1,\ 2,\ \ldots,\ \mu, \tag{8}$$