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On the LP formulation in measure spaces of optimal control problems for jump-diffusions

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ABSTRACT

In this short note we formulate a infinite-horizon stochastic optimal control problem for jump-diffusions of Ito–Levy type as a LP problem in a measure space, and prove that the optimal value functions of both problems coincide. The main tools are the dual formulation of the LP primal problem, which is strongly connected to the notion of sub-solution of the partial integro-differential equation of Hamilton–Jacobi–Bellman type associated with the optimal control problem, and the Krylov regularization method for viscosity solutions.

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1. Introduction

This short note revisits the infinite-dimensional linear programming (LP) approach to stochastic optimal control problems. We reformulate the problem of minimizing a infinite-horizon cost functional for controlled jump-diffusions of Ito–Levy type over a set of admissible controls as a linear program in a certain measure space. The linear objective function is the integral of the cost function against the occupation measure of the controlled process. The main challenge in this LP approach is to prove equality of the optimal value functions of the original control problem $V(x)$ and the associated infinite-dimensional linear program $\rho(x)$, and absence of duality gap between the primal and dual programs (strong duality).

Using measure-valued (relaxed) controls, Stockbridge [1] proved the equality $\rho = V$ for ergodic optimal control of Markov processes and existence of optimal controls. Bhatt and Borkar [2] and Kurtz and Stockbridge [3] extended these results to the case of feedback controls for time-inhomogeneous finite horizon and discounted infinite horizon problems. Cho and Stockbridge [4], Taksar [5] and Helmes and Stockbridge [6] obtained similar results for optimal stopping and singular control problems.

More recently, using the dual formulation of the primal LP problem and viscosity solution theory, Buckdahn et al. [7] proved the equality $\rho = V$ in the case of optimal control diffusions with compact state space. Goreac and Serea [8] proved the same result for finite-horizon and optimal stopping problems. In this paper we show that this approach can be easily extended to the jump-diffusion case. We emphasize that our proof does not present any significant innovation as we follow closely the arguments in the proof of Theorem 6.4 in Jakobsen et al. [9]. However, to the best of our knowledge, this is the first paper that deals with the LP approach to stochastic optimal control problems for jump-diffusions.

Let us briefly describe the contents of this paper. In Section 2 we introduce the setting for the optimal control problem of jump-diffusions of Itô–Levy type and formulate the primal LP problem associated with the optimal control problem and its dual. In Section 3 we recall the definition of viscosity solution for partial integro-differential equations and prove the main result using the Krylov regularization and results from Jakobsen et al. [9].

2. Optimal control problem and LP formulation

Let $(\Omega, \mathcal{F}, \mathbf{P})$ be probability space endowed with a filtration $\mathbb{F} = \{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions, and let $\{W_t\}_{t\geq 0}$ be a standard d-dimensional Brownian motion with respect to $\mathbb{F}$. Let $E = \mathbb{R}^N \setminus \{0\}$ and let $\nu(dz)$ be a Levy measure on $\mathcal{B}(E)$, that is, a non-negative σ-measure satisfying

$$\int_E (|z|^2 \wedge 1)\, \nu(dz) < +\infty.$$

Let $N(dz, dt)$ be a homogeneous Poisson random measure with compensator intensity measure $\nu(dz)\, dt$, and let $\tilde{N}(dz, dt)$ denote

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the compensated Poisson random measure

$\tilde{N}(dz, dt) := N(dz, dt) - \nu(dz)\, dt.$

Let U be a compact metric space. For each $\mathbb{F}$-adapted U-valued control process $u = \{u_t\}_{t\geq 0}$ consider the controlled Levy–Itô equation

$$\begin{aligned} dX_t &= b(X_t, u_t)\, dt + \sigma(X_t, u_t)\, dW_t \\ &\quad + \int_E \eta(X_{t-}, u_{t-}, z)\, \tilde{N}(dz, dt) \qquad (2.1) \\ X_0 &= x. \end{aligned}$$

The coefficients $b : \mathbb{R}^N \times U \to \mathbb{R}^N$, $\sigma : \mathbb{R}^N \times U \to \mathbb{R}^{N\times d}$ and $\eta : \mathbb{R}^N \times U \times E \to \mathbb{R}^N$ satisfy conditions (A2) and (A3) below. The class $\mathcal{U}(x)$ of admissible control policies is defined as the set of control processes $u = \{u_t\}_{t\geq 0}$ for which Eq. (2.1) has an unique strong solution $X^{x,u} = \left\{X_t^{x,u}\right\}_{t\geq 0}$.

Let $c > 0$ be a fixed discount rate and $h : \mathbb{R}^N \times U \to (-\infty, +\infty]$ denote the cost-to-go function. Let $\mathcal{J}$ be the infinite-horizon discounted cost functional

$$\mathcal{J}(x, u) := \mathbb{E}\left[\int_0^\infty e^{-ct}\, h(X_t^{x,u}, u_t)\, dt\right].$$

We will use the following norms:

$$|\phi|_0 := \sup_{x\in\mathbb{R}^N} |\phi(x)|, \qquad [\phi]_1 := |D\phi|_0 \quad \text{and}$$

$$|\phi|_1 := |\phi|_1 + [\phi]_1$$

and assume the following conditions:

1. The Levy measure $\nu(dz)$ satisfies

$$\int_{|z|\geq 1} e^{m|z|}\nu(dz) < \infty \qquad \text{(A1)}$$

   for some $m > 0$.
2. There exists $K > 0$ such that for all $u \in U$

$$|b(\cdot, u)|_1 + |\sigma(\cdot, u)|_1 + |c(\cdot, u)|_1 + |h(\cdot, u)|_1 \leq K \qquad \text{(A2)}$$

   and

$$|\eta(\cdot, u, z)|_1 \leq K\left[|z|\, \mathbf{1}_{\{0<|z|<1\}}(z) + e^{m|z|}\mathbf{1}_{\{|z|\geq 1\}}(z)\right]. \qquad \text{(A3)}$$

Condition (A1) is equivalent to the Levy process with Levy measure $\nu(dz)$ having finite moments of all orders, see e.g. Applebaum [10, Section 2.5]. It is satisfied, for instance, by one-dimensional tempered α-stable processes with Levy measure

$$\nu(dz) = \frac{C_1 e^{-\lambda_1 z}}{z^{1+\alpha_1}}\mathbf{1}_{\mathbb{R}_+}(z)\, dz + \frac{C_2 e^{-\lambda_2|z|}}{|z|^{1+\alpha_2}}\mathbf{1}_{\mathbb{R}_-}(z)\, dz$$

with $C_1, C_2 \geq 0$, $\lambda_1, \lambda_2 > 0$ and $\alpha_1, \alpha_2 < 2$. Under conditions (A1)–(A3), for each $u \in \mathcal{U}(x)$ there exists an unique strong solution to Eq. (2.1) and satisfies the following estimate, see e.g. Applebaum [10, Section 6.6]:

$$\mathbb{E}\left[\sup_{t\in[0,T]} \left|X_t^{x,u}\right|^p\right] \leq C(1 + |x|^p) \qquad (2.2)$$

for all $p \geq 2$. The main object of study of this paper is the stochastic optimal control problem

$$V(x) := \inf_{u\in\mathcal{U}(x)} \mathcal{J}(x, u), \quad x \in \mathbb{R}^N \qquad (2.3)$$

and the following linear programming (LP) formulation: for each $x \in \mathbb{R}^N$ and $u \in \mathcal{U}(x)$, denote with $\gamma^{x,u}$ the expected discounted *occupation measure* on $\mathcal{B}(\mathbb{R}^N \times U)$ defined as

$$\gamma^{x,u}(Q) := \mathbb{E}\left[\int_0^\infty e^{-ct}\, 1_Q(X_t^{x,u}, u_t)\, dt\right], \quad Q \in \mathcal{B}(\mathbb{R}^N \times U).$$

Using approximation of h by simple functions, it is easy to prove that the occupation measure $\gamma^{x,u}$ satisfies

$$\mathcal{J}(x, u) = \int_{\mathbb{R}^N\times U} h(y, u)\, \gamma^{x,u}(dy, du).$$

Let $\mathcal{C}^2_{\text{pol}}(\mathbb{R}^N)$ denote the class of $\mathcal{C}^2$-functions $f : \mathbb{R}^N \to \mathbb{R}$ with polynomial growth. For each $u \in U$ fixed, let $A^u + J^u$ denote the partial integro-differential operator

$$A^u f(x) := \langle b(x, u), Df(x)\rangle + \frac{1}{2}\text{Tr}[\sigma(x, u)\sigma(x, u)^* D^2 f(x)],$$

$$\begin{aligned} J^u f(x) := \int_E \Big\{ & f(x + \eta(x, u, z)) \\ & - f(x) - \mathbf{1}_{\{|z|<1\}}\langle \eta(x, u, z), Df(x)\rangle \Big\}\, \nu(dz) \end{aligned}$$

for $f \in \mathcal{C}^2_{\text{pol}}(\mathbb{R}^N)$. Here $Df(x)$ and $D^2 f(x)$ denote the vector and square matrix of first and second-order partial derivatives of f respectively.

Notice that the integral term in the operator J^u is well-defined due to the exponential decay of the Levy measure $\nu(dz)$ at infinity (see Assumption A.1) and the fact that the singularity at $z = 0$ is integrable for any $f \in \mathcal{C}^2(\mathbb{R}^N)$, see e.g. Applebaum [10, Section 3.3].

Using Itô's formula for Levy–Itô processes, Kunita's inequality and estimate (2.2), for any $f \in \mathcal{C}^2_{\text{pol}}(\mathbb{R}^N)$ and $T > 0$, we have

$$\begin{aligned} & \mathbb{E}[e^{-cT} f(X_T^{x,u})] - f(x) \\ & = \mathbb{E}\left[\int_0^T e^{-ct}\left\{[(A + J)f](X_t^{x,u}, u_t) - cf(X_t^{x,u})\right\} dt\right]. \end{aligned}$$

Also from estimate (2.2) we have

$$\lim_{T\to\infty} \mathbb{E}[e^{-cT} f(X_T^{x,u})] = 0.$$

By the dominated convergence theorem, taking the limit as $T \to \infty$ it follows:

$$\mathbb{E}\left[\int_0^\infty e^{-ct}[cf - (A + J)f](X_t^{x,u}, u_t)\, dt\right] = f(x).$$

We have proved that the occupation measure $\gamma^{x,u}$ satisfies the linear constraint,

$$\int_{\mathbb{R}^N\times U} [cf - (A^u + J^u)f](y)\, \gamma^{x,u}(dy, du) = f(x), \quad \forall f \in \mathcal{C}^2_{\text{pol}}(\mathbb{R}^N). \qquad (2.4)$$

This suggests to consider the following LP problem over the vector space $\mathcal{M}_b(\mathbb{R}^N \times U)$ of finite signed measures on $\mathcal{B}(\mathbb{R}^N \times U)$:

$$\begin{aligned} \rho(x) := \inf & \int_{\mathbb{R}^N\times U} h(y, u)\, \mu(dy, du) \\ & \text{subject to } \mu \in \mathcal{M}_b(\mathbb{R}^N \times U),\ \mu \geq 0 \\ & \text{and } \int_{\mathbb{R}^N\times U} [cf - (A^u + J^u)f](y)\, \mu(dy, du) = f(x), \\ & \forall f \in \mathcal{C}^2_{\text{pol}}(\mathbb{R}^N). \qquad (2.5) \end{aligned}$$

Clearly, we have $\rho \leq V$. The main purpose of this note is to prove that in fact equality $\rho = V$ holds.

In order to formulate a LP problem with the linear constraint (2.4), and its dual, we recall briefly some facts and notation concerning infinite-dimensional linear programming. Two topological real vector spaces $\mathcal{X}$, $\mathcal{Y}$ are said to form a *dual pair* if there exists a bilinear form $\langle\cdot,\cdot\rangle : \mathcal{X} \times \mathcal{Y} \to \mathbb{R}$ such that the mappings $\mathcal{X} \ni x \to \langle x, y\rangle \in \mathbb{R}$ for $y \in \mathcal{Y}$ separate points of $\mathcal{X}$ and the mappings $\mathcal{Y} \ni y \to \langle x, y\rangle \in \mathbb{R}$ for $x \in \mathcal{X}$ separate points of $\mathcal{Y}$.

We endow $\mathcal{X}$ with the weak topology $\sigma(\mathcal{X}, \mathcal{Y})$, i.e. the coarsest topology for which the maps $\mathcal{X} \ni x \to \langle x, y\rangle \in \mathbb{R}$ are continuous

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