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Finite-time formation control for linear multi-agent systems: A motion planning approach

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ABSTRACT

This paper investigates the problem of finite-time formation control for multi-agent systems with general linear dynamics. First of all, the considered formation problem is converted into the motion planning problem, where the systems are steered from initial positions to the desired terminal configurations. Then, by using Pontryagin maximum principle, an optimal formation control law is developed for multi-agent systems based on some invertible conditions. With the designed control law, the multi-agent systems can achieve the desired formation in finite time, where the formation configurations and the settling time can be specified in advance according to task requirements. Meanwhile, a performance index is guaranteed to be optimal. Further, it is proved that the formation problem concerned is solved if and only if the linear systems are controllable. Finally, a possible application of the proposed control law to spacecraft formation flying with circular and near-circular reference orbit is illustrated.

1. Introduction

In the past two decades, the coordination problem of multi-agent systems has been considerably studied due to its broad applications in such areas as spacecraft formation flying and sampling, distributed sensor networks, and automated highway systems, to name just a few [1–3]. One critical issue arising from multi-agent systems is to develop control laws that enable all agents to reach a desired formation, which is known as the formation problem.

Consensus algorithms, as an interesting topic of the coordination control, can be applied to tackle formation problems by appropriately choosing information states on which consensus is reached [4]. Consensus of multi-agent systems with various dynamics has been well studied over the recent decades. Early seminal works for systems with first-order dynamics have been launched by Olfati-Saber and Murray [5], and Ren and Beard [6], to name a few. Further, in [7,8], consensus for systems with second-order dynamics and high-order integrator dynamics has also been extensively investigated. Nevertheless, most actual multi-agent systems have more complex physical dynamics. Motivated by this observation, consensus for systems with general linear dynamics also receives dramatic attention. Corresponding results can be found in [9–12]. Besides, consensus for multiple Euler–Lagrange systems and nonlinear systems is investigated in [13,14], respectively. To sum up, a common feature in the works mentioned above is that the desired consensus or formation is achieved when the time variable tends to infinity. Nevertheless, in many practical applications, it is more desirable for systems to achieve consensus or formation with a fast convergence rate.

As an important performance indicator of consensus protocols, convergence rate is a hot research topic in the area of consensus problems. For the above mentioned asymptotic control laws, the convergence rate is at best exponential with infinite settling time [15]. Finite-time consensus algorithms, by contrast, are more desirable in real practice. Besides a faster convergence rate, the closed-loop systems with finite-time control laws usually demonstrate better disturbance rejection properties [16]. Finite-time consensus problems for multi-agent systems are first introduced in [17]. Then, several kinds of finite-time consensus protocols have been developed for systems with first-order dynamics in [18,19]. In particular, the finite-time algorithms are developed to deal with the time-invariant and time-variant formation problems in [19]. Extensions to the second-order multi-agent systems are also studied in [15,20–22]. In [23], the consensus problems for systems with multiple second-order integrators and coupled harmonic oscillators are addressed. Furthermore, the finite-time con-

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sensus for systems with multiple rigid bodies is also investigated by many researchers. Corresponding results can be found in [24–27].

This paper studies the finite-time formation problem of multi-agent systems with general linear dynamics, which may also be considered as the linearized model of a nonlinear network. A new framework is introduced, which converts the finite-time formation problem of multi-agent systems into the motion planning problem. Then, based on Pontryagin maximum principle, an optimal control law is proposed for the multi-agent systems to achieve the desired formation in finite time. The contributions of this paper are three-fold. First, it is the first time that the finite-time formation control law is proposed for systems with general linear dynamics, as far as we know. Compared with the case of first-order and second-order dynamics, general linear systems are more common in practical applications. Second, unlike the settling time conservatively estimated according to the designed Lyapunov function, controller parameters, and the information of the communication topology [23,21], in this paper, the settling time can be precisely specified in advance according to task requirements, which is more meaningful and is consistent with practical needs. Last but not the least, the proposed control laws are applied to spacecraft formation flying with circular and near-circular reference orbit. With such control laws, the specified formation is achieved in finite time, and simultaneously, the entire energy expenditure is minimized, which is crucial for spacecraft with limited fuel.

The remaining part of this paper is organized as follows. The problem formulation is given in Section 2. Main theoretical results are provided in Section 3. In Section 4, an application of the proposed control laws to spacecraft formation flying is reported to illustrate the theoretical results. Concluding remarks are finally given in Section 5.

2. Problem formulation

Consider a group of N agents with general linear dynamics. The dynamics of the ith agent is described by

$$\dot{x}_i = Ax_i + Bu_i,$$
$$x_i(t_0) = x_i^0, \quad i = 1, \ldots, N, \tag{1}$$

where $x_i \in \mathbb{R}^n$ is the state, $u_i \in \mathbb{R}^m$ its control input, and t_0 its initial time. A and B are constant real matrices with compatible dimensions.

Now for the given initial states $x_i^0,\ i = 1, \ldots, N$, and formation configurations $D_i \in \mathbb{R}^n,\ i = 1, \ldots, N-1$, determined by formation tasks, the objective of this paper is to find control inputs $u_i,\ i = 1, \ldots, N$, such that

$$x_1(t_f) - x_i(t_f) = D_{i-1}, \quad i = 2, \ldots, N, \tag{2}$$

where $t_f > t_0$ is the terminal time which is given by formation task in advance, meanwhile, to minimize the following cost function:

$$J = \frac{1}{2}\int_{t_0}^{t_f} \sum_{i=1}^{N} \left(u_i^T(t) u_i(t)\right) dt. \tag{3}$$

The above problem is referred to as *finite-time optimal formation control*. When $D_i = \mathbf{0},\ i = 1, \ldots, N-1$, the corresponding problem is referred to as *finite-time optimal consensus control*. Note that the latter is the special case of the former. Thus, we shall focus on the formation control problem.

Remark 1. Generally, formation configurations are given by $x_i(t_f) - x_j(t_f) = D_{ij},\ i, j = 1, \ldots, N$. For such formation configurations, there are some redundancies. For example, for the formation configurations $\{D_{12}, D_{13}, D_{23}\}$, there must exist the relation $D_{12} - D_{13} = D_{32}$. Remove these redundancies, the formation configurations considered in this paper are equivalent to general ones.

3. Main results

The problem of finite-time optimal formation control is an optimal control problem. Therefore, the next task is to solve the corresponding optimal control problem.

In order to using Pontryagin maximum principle [28], the Hamiltonian for this problem can be constructed as follows:

$$H = -\frac{1}{2}\sum_{i=1}^{N} u_i(t)^T u_i(t) + \sum_{i=1}^{N} p_i^T \left(Ax_i + Bu_i\right), \tag{4}$$

where $p_i \in \mathbb{R}^n$ is the co-state (Lagrangian multiplier). Then, the corresponding Hamiltonian system can be written as

$$\dot{x}_i = \frac{\partial H}{\partial p_i} = Ax_i + Bu_i, \tag{5}$$

$$\dot{p}_i = -\frac{\partial H}{\partial x_i} = -A^T p_i, \quad i = 1, \ldots, N. \tag{6}$$

According to Pontryagin maximum principle, the optimal control u_i satisfies the necessary condition that

$$\frac{\partial H}{\partial u_i} = -u_i + B^T p_i = 0, \quad i = 1, \ldots, N. \tag{7}$$

Since these equations have the unique solutions, the above condition is also sufficient. Then, it follows from (7) that

$$u_i = B^T p_i, \quad i = 1, \ldots, N. \tag{8}$$

Let $x = (x_1^T, \ldots, x_N^T)^T$, $p = (p_1^T, \ldots, p_N^T)^T$ and $u = (u_1^T, \ldots, u_N^T)^T$. Then, the Hamiltonian system (5), (6) and the control (8) can be rewritten in a matrix form as

$$\dot{x} = (I_N \otimes A)x + (I_N \otimes B)u, \tag{9}$$

$$\dot{p} = -(I_N \otimes A^T)p, \tag{10}$$

$$u = (I_N \otimes B^T)p. \tag{11}$$

Similarly, it follows from the given formation configurations (2) that

$$\begin{bmatrix}\mathbf{1}_{N-1} & -I_{N-1}\end{bmatrix} \otimes I_n \cdot x(t_f) = D, \tag{12}$$

where $\mathbf{1}_{N-1}$ denotes the column vector with all entries equal to one, and $D = (D_1^T, \ldots, D_{N-1}^T)^T$. Substituting (11) into Hamiltonian system (9), (10) gives

$$\begin{bmatrix}\dot{x}\\ \dot{p}\end{bmatrix} = \begin{bmatrix} I_N \otimes A & I_N \otimes BB^T \\ 0 & -(I_N \otimes A^T)\end{bmatrix}\begin{bmatrix}x\\ p\end{bmatrix}. \tag{13}$$

By integrating the above equations from t_0 to t, one gets

$$\begin{bmatrix}x(t)\\ p(t)\end{bmatrix} = \exp\left((t - t_0)\begin{bmatrix} I_N \otimes A & I_N \otimes BB^T \\ 0 & -(I_N \otimes A^T)\end{bmatrix}\right)\begin{bmatrix}x_0\\ p_0\end{bmatrix} \tag{14}$$

where $x_0 = ({x_1^0}^T, \ldots, {x_N^0}^T)^T$ and $p_0 \in \mathbb{R}^{Nn}$ is the initial value for p. Thus, the determination of the optimal control (8) is boiled down to finding the initial value p_0. In order to solve p_0, a lemma related to the transversality condition corresponding to the formation configurations (2) is given.

Lemma 1. *If* $x_1(t_f) - x_i(t_f) = D_{i-1},\ i = 2, \ldots, N$, *then*

$$\mathbf{1}_N^T \otimes I_n \cdot p(t_f) = 0. \tag{15}$$