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Anti-disturbance control for nonlinear systems subject to input saturation via disturbance observer[☆]

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ABSTRACT

This paper is concerned with the problem of the disturbance observer based control for a class of continuous-time uncertain systems subject to input saturation and nonlinearity. The input of the system includes two parts, the control input and the disturbance input. The nonlinearity of the system, which satisfies a global Lipschitz condition, is considered as two cases of known nonlinearity and unknown nonlinearity. By virtue of the technique of the disturbance observer based controller, the anti-disturbance controllers are designed respectively with both the polytopic and dead-zone representations of the saturation function, which ensure that the resulting closed-loop systems are asymptotically stable with an estimation of the domain of attraction described by the level set of the Lyapunov function. Further, an iterative optimization method is used to obtain the maximum estimation of the domain of the set of initial states. An example of application design for a flight control system illustrates the effectiveness of the proposed results.

1. Introduction

As one kind of uncertainties, external disturbances are often encountered in industrial control systems and can lead to degradation of the system performance, or even instability. Therefore, the study of control for systems with external disturbance uncertainties is of great importance from both theoretical and engineering points of view [1–12]. With the development of high-precision control, some effective disturbance rejection methods have been proposed, such as feedforward compensation control [1,3,6,7,11–13]. However, a restriction on implementation of those disturbance rejection methods is that most of uncertainties in practical engineering systems cannot be measured. Consequently, the disturbance estimation technique is required in control processes [11].

During the past decades, the problem of disturbance observer based control for nonlinear systems with disturbances has been extensively investigated (see, e.g., [1,3,6,7] and the references therein). In [7], the problem of disturbance observer based control for a class of multi-input multi-output (MIMO) nonlinear systems was studied, where the unknown external disturbances were supposed to be generated by an exogenous system. As further research, a method of composite disturbance observer based control for nonlinear systems with two types of disturbances was proposed in [8], while the problem of H_∞ control was examined in [10]. Recently, in [13] a composite anti-disturbance control methodology was introduced for Markovian jump systems with nonlinearity and multiple disturbances. It is worth noting that the above-mentioned results are obtained without consideration of constraint on the system input.

Input saturation is a ubiquitous problem in control systems due to the physical impossibility of applying unlimited wide input channel. Meanwhile, input saturation has a great negative impact on the performance and stability of a control system. Recently, some results of stability analysis and control synthesis of the systems subject to input saturation have appeared in the literature [14–30]. In [14], a semi-global low-and-high gain design technique for a class of linear systems with input saturation and disturbances was developed, where the external disturbances and the control input used the same channel with saturating constraint. In [31], by expressing the saturated linear feedback on a convex hull of a group of linear feedback laws, the conditions were proposed for determining whether a given ellipsoid was contractively

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invariant for a linear system under a saturated linear feedback, followed by further development of analysis and design methods for the closed-loop system with disturbances. The problem of robust exponential convergence for uncertain discrete-time time-delayed systems with input saturation and disturbances was tackled in [24] by designing a state feedback controller, while a similar problem was investigated in the continuous-time domain in [21]. To the best of the authors' knowledge, most of the existing results about the controller design for the systems subject to input saturation and disturbance uncertainties are based on the feedback control methods, which are not applicable to the control problem of the systems subject to saturated input affected by disturbances. How to solve the control problem for the systems with saturated input including two parts (i.e., control input and disturbance input) is the motivation for this paper.

In this paper, we consider the control problem of a class of nonlinear systems subject to input saturation and external disturbance uncertainties from a new perspective, namely, via disturbance observer based control (DOBC). Two cases of known and unknown nonlinearities satisfying a global Lipschitz condition are investigated, respectively, and both the polytopic and dead-zone representation are used to obtain stabilization criteria. It is assumed that external disturbances satisfy the matching condition, that is, the disturbances enter into the system via the same channel as the control input [2,14]. With disturbance observers being first designed, attention is then focused on constructing the disturbance observer based controllers for the nonlinear uncertain systems subject to input saturation such that there exists an initial condition domain ensuring that for every initial condition from this domain, the composite closed-loop system combined with estimation error of disturbances is asymptotically stable. Moreover, the disturbance observer based controllers can be designed by using an iterative optimization algorithm so as to make an initial domain of attraction as large as possible. Finally, a simulation example for a flight control system demonstrates the efficacy of the proposed method.

The remainder of the paper is organized as follows. In Section 2, we give some preliminaries and a formulation of the problem under consideration. Sections 3 and 4 present the main results of the paper based on polytopic bounding and sector bounding, respectively. In Section 5, a simulation example is provided to illustrate the usefulness of the proposed technique. Finally, concluding remarks are given in Section 6.

2. Preliminaries

2.1. System description

Consider a class of uncertain systems subject to saturating input and nonlinearity as follows:

$$\dot{x}(t) = Ax(t) + Ff(x(t)) + B\mathrm{sat}(u(t) + d(t)), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^m$ is the control input, $d(t) \in \mathbb{R}^m$ is the unknown disturbance input, and A, B and F are known real constant matrices. $\mathrm{sat}(\cdot)$ is the standard saturation function defined by

$$\mathrm{sat}(\cdot) = \begin{bmatrix}\mathrm{sat}(\cdot)_1 & \cdots & \mathrm{sat}(\cdot)_m\end{bmatrix}^T,$$

where $\mathrm{sat}(\cdot)_i = \mathrm{sign}(\cdot)\min(\cdot, 1)$ with $\mathrm{sign}(\cdot)$ being the signum function. By slight abuse of notation, we use $\mathrm{sat}(\cdot)$ to denote both the scalar valued and the vector valued saturation functions, and assume the unity saturation level.

The function $f(x(t)) : \mathbb{R}^n \to \mathbb{R}^l$ stands for the nonlinearity of the system and satisfies the following assumption.

Assumption 1. $f(0) = 0$, and $f(x(t))$ satisfies the global Lipschitz condition with respect to x:

$$\|f(x_1) - f(x_2)\| \le \|U(x_1 - x_2)\|, \quad \forall x_1, x_2 \in \mathbb{R}^n,$$

where U is a known matrix with appropriate dimension.

Remark 1. In [14], the linear system comprising a chain of integrators subject to input saturation was considered. In this paper, we study a class of nonlinear systems described by (1) which can include the system in [14] as a special case, so the system (1) is very general. On the other hand, due to consideration of the input saturation constraint, the system under investigation herein has more practical significance when compared with the system in [7].

Remark 2. In [9], the problem of saturating composite disturbance-observer-based control for discrete-time system was investigated. However, the saturation constraint is added onto the feedback input which is a part of the system input. Different from [9], here we consider that the system is subjected to system input saturation.

In this paper, the disturbance input is assumed to be generated by the following dynamic system

$$\begin{aligned}\dot{\omega}(t) &= W\omega(t),\\ d(t) &= V\omega(t),\end{aligned} \tag{2}$$

where $\omega(t) \in \mathbb{R}^r$ is the disturbance system state, and W and V are known real constant matrices with appropriate dimension.

Remark 3. The disturbances generating by system (2) have been extensively investigated and may contain many types of disturbances in practical engineering (see [7,32] and the references therein). When W is chosen as

$$W = \begin{bmatrix}0 & c\\ -c & 0\end{bmatrix}, \quad c > 0,$$

$d(t)$ is a harmonic disturbance with the frequency c.

In the subsequent study, we consider the system (1) and the disturbance dynamic system (2) under the following assumption.

Assumption 2. (A, B) is controllable and (W, V) is observable.

Lemma 1 ([33]). *Let D, S and F be real matrices of appropriate dimensions with $F^TF \le I$. Then, for any scalar $\lambda > 0$, we have*

$$DFS + (DFS)^T \le \lambda^{-1}DD^T + \lambda S^TS.$$

2.2. Approximations of the saturation function

In this subsection, we will present two classical approximations of the saturation.

2.2.1. Polytopic bounding

Let E be the set of $m \times m$ diagonal matrices with elements being either 1 or 0. There are 2^m elements in E. Suppose that each element of E is labeled as E_i, $i \in Q = \{1, \ldots, 2^m\}$, and denote $E_i^- = I - E_i$. Obviously, E_i^- is also an element of E. Furthermore, for a family of scalars η_i satisfying $0 \le \eta_i \le 1$ and $\sum_{i=1}^{2^m} \eta_i = 1$, it is easy to get that

$$\sum_{i=1}^{2^m} \eta_i(E_i + E_i^-) = I, \quad \forall E_i \in E,\ i \in 1, 2, \ldots, 2^m. \tag{3}$$

For further discussion, we denote an ellipsoid for a positive definite matrix $P \in \mathbb{R}^{n\times n}$ by

$$\Omega(P) = \{x \in \mathbb{R}^n : x^TPx \le 1\} \tag{4}$$