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Stability analysis of large-scale distributed networked control systems with random communication delays: A switched system approach

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ABSTRACT

In this paper, we consider the stability analysis of large-scale distributed networked control systems with random communication delays. The stability analysis is performed in the switched system framework, particularly as the Markov jump linear system. There have been considerable research on stability analysis of the Markov jump systems. However, these methods are not applicable to large-scale systems because large numbers of subsystems result in extremely large number of switching modes. To circumvent this scalability issue, we propose a new reduced mode model for stability analysis, which is computationally scalable. We also consider the case in which the transition probabilities for the Markov jump process contain uncertainties. We provide a new method that estimates bounds for uncertain Markov transition probability matrix to guarantee the system stability. Numerical example verifies the computational efficiency of the proposed methods.

1. Introduction

A networked control system (NCS) is a system that is controlled over a communication network. Recently, NCSs have attracted considerable research interest due to emerging networked control applications. For example, NCSs are broadly used in applications including traffic monitoring, networked autonomous mobile agents, chemical plants, sensor networks, and distributed software systems in cloud computing architectures. Due to the communication network, communication delays or communication losses may occur, resulting in performance degradation or even instability. Therefore, it has led various researchers to analyze NCSs associated with communication delays [1–8]. Particularly in [6], the NCS with communication delays was analyzed by adopting the switched system [7–11], which refers to the dynamical system consists of a family of subsystems and a switching logic governing switching between subsystems.

In this paper, we study large-scale distributed networked control system (DNCS), which denotes NCS with a large number of spatially distributed subsystems (or agents). For such large-scale systems, our primary goal is to analyze system stability when *random communication delays* exist. Typically, the system behavior with random communication delays have been widely modeled as *Markov jump linear system* (MJLS) [6,12–16] where the switching sequence is governed by a Markovian process. Since stability has been one of the major concerns, considerable effort has been on stability analysis of MJLS [17,10,18–20]. However, these results are only applicable to the systems with a small number of switching modes. Large-scale DNCSs, in which we are particularly interested, give rise to an extremely large number of switching modes. Thus, previous conditions developed for the stability analysis of MJLSs cannot be evaluated for large-scale DNCSs as they are not *computationally tractable*. Although the literature [21] recently investigated the switched system that circumvents computation issues associated with a large number of switching modes, it is developed for independent and identically distributed (i.i.d.) switching. We consider Markovian switching in this paper. In addition, we are also interested in large-scale DNCSs where the transition probabilities are inaccurately known [20,22,23]. This can happen because in practice it is difficult to accurately estimate the Markov transition probability matrix that models the random communication delays.

This paper provides two key contributions to analyze the stability of large-scale DNCSs with random communication delays. Firstly, we guarantee the mean square stability of such systems by introducing a reduced mode model. We prove that the mean square stability for individual switched system implies a necessary and sufficient stability condition for the entire DNCS. This drastically reduces the number of modes necessary for analysis.

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Secondly, we present a new method to estimate the bound for uncertain Markov transition probability matrix, in order to guarantee the system stability. These results enable us to analyze large-scale systems in a computationally tractable manner.

Rest of this paper is organized as follows. We introduce the problem for the large-scale DNCS in Section 2. Section 3 presents the switched system framework for the stability analysis with communication delays. In Section 4, we propose the reduced mode model to efficiently analyze stability. Section 5 quantifies the stability region and bounds for uncertain Markov transition probability matrix. This is followed by the application of the proposed method to an example system in Section 6, and we conclude the paper with Section 7.

Notation. The set of real numbers is denoted by $\mathbb{R}$. The symbols $\|\cdot\|$ and $\|\cdot\|_\infty$ stand for the Euclidean and infinity norm, respectively. The symbol $\#(\cdot)$ denotes the cardinality—the total number of elements in the given set. In addition, the symbols $\mathrm{tr}(\cdot)$, $\rho(\cdot)$, $\otimes$, and $\mathrm{diag}(\cdot)$ represent trace operator, spectral radius, Kronecker product, and block diagonal matrix operator, respectively.

2. Problem formulation

2.1. Distributed networked control system with no delays

Consider a discrete-time dynamics of each agent in the DNCS, given by:

$$x_i(k+1) = \sum_{j\in\mathcal{N}_i} A_{ij}x_j(k), \quad i = 1, 2, \ldots, N, \tag{1}$$

where k is a discrete-time index, N is the total number of agents (subsystems), $x_i \in \mathbb{R}^n$ is a state for the ith agent, $\mathcal{N}_i$ is a set of neighbors for x_i including the agent x_i itself, and $A_{ij} \in \mathbb{R}^{n\times n}$ is a time-invariant system matrix that represents the linear interconnections between agents. Note that we have $A_{ij} = 0$ if there is no interconnection between the agents i and j.

To represent the entire systems dynamics, we define the state $x(k) \in \mathbb{R}^{Nn\times Nn}$ as $x(k) \triangleq [x_1(k)^\top, x_2(k)^\top, \ldots, x_N(k)^\top]^\top$. Then, the system dynamics of the DNCS is given as

$$x(k+1) = \mathcal{A}x(k), \tag{2}$$

with the following definition for the matrix $\mathcal{A} \in \mathbb{R}^{Nn\times Nn}$

$$\mathcal{A} \triangleq \begin{bmatrix} A_{11} & A_{12} & A_{13} & \cdots & A_{1N} \\ A_{21} & A_{22} & A_{23} & \cdots & A_{2N} \\ A_{31} & A_{32} & A_{33} & \cdots & A_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{N1} & A_{N2} & A_{N3} & \cdots & A_{NN} \end{bmatrix},$$

$$A_{ij} = \begin{cases} 0, & \text{if no connection between the agents } i \text{ and } j, \\ A_{ij}, & \text{otherwise.} \end{cases}$$

For the discrete-time system in (2), it is well known that the system is stable if and only if the condition $\rho(\mathcal{A}) < 1$ is satisfied. Throughout the paper we assume that the system without communication delays, defined in (2), is stable. Then, we address stability in the presence of *random communication delays*. We remind the reader that N is very large.

2.2. DNCS with communication delays

Often, network communication between agents encounter time delays or packet losses while sending and receiving data. We denote the symbol τ as random communication delays and assume that τ has a discrete value bounded by $0 \le \tau \le \tau_d < \infty$, where τ_d is a finite-valued maximum delay. Then, the dynamics for the agent i with communication delays can be expressed as:

$$x_i(k+1) = \sum_{j\in\mathcal{N}_i} A_{ij}x_j(k^*), \quad i = 1, 2, \ldots, N, \tag{3}$$

where $k^* \triangleq k - \tau$. Note that we have no communication delays when $i = j$ because there is no communication in this case.

The random communication delay, represented by the term k^*, forms a stochastic process. To analyze the stability of the DNCS, we define an augmented state $X(k)$ as $X(k) \triangleq [x(k)^\top, x(k-1)^\top, \ldots, x(k-\tau_d)^\top]^\top \in \mathbb{R}^{Nnq\times Nnq}$, where $q \triangleq \tau_d + 1$. Then, the dynamics for the entire system is given by

$$X(k+1) = W(k)X(k), \tag{4}$$

where $W(k) \triangleq \begin{bmatrix} \tilde{A}_1(k) & \tilde{A}_2(k) & \cdots & \tilde{A}_{q-1}(k) & \tilde{A}_q(k) \\ I & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix} \in \mathbb{R}^{Nnq\times Nnq}$,

the matrix I denotes an identity matrix with proper dimensions, and the time-varying matrices $\tilde{A}_j(k) \in \mathbb{R}^{Nn\times Nn}$, $j = 1, 2, \ldots, q$, model the randomness in the communication delays between neighboring agents.

3. Switched system approach

Without loss of generality, the dynamics of the large-scale DNCS with communication delays in (4) can be transformed into a switched system framework as follows:

$$x(k+1) = W_{\sigma(k)}x(k), \qquad \sigma(k) \in \{1, 2, \ldots, m\}, \tag{5}$$

where $W_{\sigma(k)}$ is the time-invariant matrix, representing communication delays in agents, $\{\sigma(k)\}$ is the switching sequence, and m is the total number of switching modes. When the switching sequence $\{\sigma(k)\}$ is stochastic, (5) is referred to as a stochastic switched linear system or a stochastic jump linear system, according to the literature [7]. For the stochastic switched linear system, the switching sequence $\{\sigma(k)\}$ is governed by the mode-occupation switching probability $\pi(k) = [\pi_1(k), \pi_2(k), \ldots, \pi_m(k)]$, where π_i is a fraction number, satisfying $\sum_{i=1}^m \pi_i = 1$ and $0 \le \pi_i \le 1$, $\forall i$. In this case, each π_i denotes the modal probability corresponding to each mode dynamics W_i. In order to properly describe the behavior of random communication delays, it is necessary to adopt a certain switching logic, which is used to update the switching probability $\pi(k)$. For this purpose, the MJLS framework has been widely employed [12–16]. Thus, we make the following assumption in our analysis.

- Assumption: Consider the stochastic jump linear system (5) with the switching probability $\pi(k) = [\pi_1(k), \pi_2(k), \ldots, \pi_m(k)]$. Then, $\pi(k)$ is updated by the Markovian process given by $\pi(k+1) = \pi(k)P$, where $P \in \mathbb{R}^{m\times m}$ is the Markov transition probability matrix.

Since the MJLS is a family of the stochastic switched linear system, various stability notions can be defined [10]. In this paper, we will consider the mean square stability condition, defined below.

Definition 3.1 (*Definition 1.1 in [11]*)**.** The MJLS is said to be mean square stable if for any initial condition x_0 and arbitrary initial probability distribution $\pi(0)$, $\lim_{k\to\infty} \mathbb{E}\left[\|x(k, x_0)\|^2\right] = 0$.

The total number of switching modes m depends on the size q and N. Since the communication delays take place independently while receiving and sending the data for each agent, m is calculated by counting all possible scenarios to distribute every matrices $A_{ij} \in \mathbb{R}^{n\times n}$ for $i \neq j$ in the block matrix $\mathcal{A} \in \mathbb{R}^{Nn\times Nn}$ given in (2), into