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A parameter estimation approach to state observation of nonlinear systems

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ABSTRACT

A novel approach to the problem of partial state estimation of nonlinear systems is proposed. The main idea is to translate the state estimation problem into one of estimation of *constant, unknown parameters* related to the systems initial conditions. The class of systems for which the method is applicable is identified via two assumptions related to the transformability of the system into a suitable cascaded form and our ability to estimate the unknown parameters. The first condition involves the solvability of a partial differential equation while the second one requires some persistency of excitation-like conditions. The proposed observer is shown to be applicable to position estimation of a class of electromechanical systems, for the reconstruction of the state of power converters and for speed observation of a class of mechanical systems.

1. Introduction

The problem of designing observers for nonlinear systems has received a lot of attention due to its importance in practical applications, where some of the states may not be available for measurement. The interested reader is referred to [1,2] for a recent review of the literature.

In this paper a new framework for constructing globally convergent (reduced-order) observers for a well-defined class of nonlinear systems is presented. Instrumental to this development is to formulate the observer design problem as a problem of *parameter estimation*, which represents the *initial conditions* of the unknown part of the state. This new family of observers are called parameter estimation-based observers (PEBO). The class of systems for which PEBO is applicable is identified via two assumptions. The first one characterises, via the solvability of a partial differential equation (PDE), systems for which there exists a partial change of coordinates that assigns a particular cascaded structure to the system that permits to obtain a classical *regression form* involving only measurable quantities and the unknown parameter. The second assumption pertains to our ability to consistently estimate this unknown parameter that, in general, may enter nonlinearly in the regression form. For *linear* regression forms, which may be obtained via over-parameterisation of the nonlinear regression, many well-established parameter estimation algorithms are available and the second assumption can be replaced by the well-known persistency of excitation (PE) condition [3,4]. The latter condition is related with the "regular input" or "universal input" assumptions imposed in standard observer designs [1,2,5].

It should be underscored that, in contrast with the classical observer design method based on linearisation up to output injection [6], where the PDE to be solved imposes stringent conditions on the system, this is not the case for our PDE. The proposed PEBO also compares favourably with Kazantzis–Kravaris–Luenberger observers [7] in the following sense. Although both observers require an *injectivity* condition, in our observer this is imposed only on the *partial* change of coordinates mapping while in the Kazantzis–Kravaris–Luenberger observers the stronger requirement of injectivity of the full-state change of coordinates is needed. As is well-known [8] ensuring the latter injectivity property is the main stumbling block for the application of this kind of observers.

The method is shown to be applicable for position estimation of a class of electromechanical systems. This class contains, as a particular case, the interesting example of permanent magnet synchronous motors (PMSM) that have been widely studied in the control and drives literature—see [9,10] and references therein. It also allows us to design observers for a class of power converters under more realistic measurement assumptions than the existing results obtained with other observer design techniques. Finally, it generates simple speed observers for mechanical systems that

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are partially linearisable via change of coordinates (PLvCC)—a practically important class that has been thoroughly studied in [11].

The remaining of the paper is organised as follows. Section 2 presents the problem formulation and main result. Section 3 is devoted to a discussion of the results. The case of linear time-invariant (LTI) systems is treated in Section 4. Section 5 illustrates the application of the technique to three physical examples. The paper is wrapped-up with concluding remarks and future research directions in Section 6.

2. Problem formulation and main result

In this section the (partial state) observer problem addressed in the paper, and the approach that we propose to solve it, are presented. The class of systems for which the PEBO design technique is applicable is identified via two assumptions. The first one, given in Section 2.2, characterises systems for which there exists a partial change of coordinates that assigns a particular cascaded structure to the system that permits to reformulate the state observation problem as a problem of parameter estimation. The second assumption, given in Section 2.3, pertains to our ability to consistently estimate this unknown parameter.

2.1. Partial state observer design problem

Consider the dynamical system

$$\begin{aligned} \dot{x} &= f_x(x, y, u) \\ \dot{y} &= f_y(x, y, u), \end{aligned} \tag{1}$$

where $f_x : \mathbb{R}^{n_x} \times \mathbb{R}^{n_y} \times \mathbb{R}^m \to \mathbb{R}^{n_x}$ and $f_y : \mathbb{R}^{n_x} \times \mathbb{R}^{n_y} \times \mathbb{R}^m \to \mathbb{R}^{n_y}$ are smooth mappings.[1] Assume that the input signal vector $u : \mathbb{R}_+ \to \mathbb{R}^m$ is such that all trajectories of the system are bounded. Find, if possible, mappings $F : \mathbb{R}^{n_\xi} \times \mathbb{R}^{n_y} \times \mathbb{R}^m \to \mathbb{R}^{n_\xi}$ and $G : \mathbb{R}^{n_\xi} \times \mathbb{R}^{n_y} \times \mathbb{R}^m \to \mathbb{R}^{n_x}$, for some positive integer n_ξ, such that the (partial state) observer

$$\begin{aligned} \dot{\xi} &= F(\xi, y, u) \\ \hat{x} &= G(\xi, y, u), \end{aligned} \tag{2}$$

ensures that ξ is bounded and

$$\lim_{t\to\infty} \left|\hat{x}(t) - x(t)\right| = 0, \tag{3}$$

for all initial conditions $(x(0), y(0), \xi(0)) \in \mathbb{R}^{n_x+n_y+n_\xi}$ and a well defined class of input signals $u \in \mathcal{U}$.

It is important to underscore that, in contrast with the usual observer problem formulation, a provision regarding the input signal is added. This additional qualifier is needed because the observation problem will be recast in terms of parameter estimation whose solution requires "sufficiently exciting" signals. See Section 1.2 in [2] for a thorough discussion of the role of the input in the observation problem. Also, note that we have written the system dynamics including the output y as part of the state, this is done, of course, without loss of generality.

2.2. System re-parameterisation

Assumption 1. There exists three mappings

$$\begin{aligned} \phi &: \mathbb{R}^{n_x} \times \mathbb{R}^{n_y} \to \mathbb{R}^{n_z} \\ \phi^{\mathrm{L}} &: \mathbb{R}^{n_z} \times \mathbb{R}^{n_y} \to \mathbb{R}^{n_x} \\ h &: \mathbb{R}^{n_y} \times \mathbb{R}^m \to \mathbb{R}^{n_z}, \end{aligned}$$

with $n_z \geq n_x$, verifying the following conditions.

(i) (Left invertibility of $\phi(\cdot,\cdot)$ with respect to its first argument)

$$\phi^{\mathrm{L}}(\phi(x, y), y) = x, \quad \forall x \in \mathbb{R}^{n_z}, \ \forall y \in \mathbb{R}^{n_y}.$$

(ii) (Transformability into cascade form)

$$\frac{\partial \phi}{\partial x} f_x(x, y, u) + \frac{\partial \phi}{\partial y} f_y(x, y, u) = h(y, u). \quad \square \tag{4}$$

An immediate corollary of (ii) in Assumption 1 is that the partial change of coordinates

$$z = \phi(x, y), \tag{5}$$

ensures

$$\dot{z} = h(y, u). \tag{6}$$

Moreover, the left invertibility condition (i) ensures that the partial state x can be recovered from z and y, that is,

$$x = \phi^{\mathrm{L}}(z, y). \tag{7}$$

The cascade structure of the system is given in Fig. 1.

Proposition 1. *Consider the system (1) verifying Assumption 1. Define the dynamic extension*

$$\dot{\chi} = h(y, u), \tag{8}$$

with $\chi(0) \in \mathbb{R}^{n_z}$. We can compute a mapping $\Phi : \mathbb{R}^{n_z} \times \mathbb{R}^{n_y} \times \mathbb{R}^m \times \mathbb{R}^{n_z} \to \mathbb{R}^{n_y}$ such that

$$\dot{y} = \Phi(\chi, y, u, \theta) \tag{9}$$

$$x = \phi^{\mathrm{L}}(\chi + \theta, y), \tag{10}$$

where $\theta \in \mathbb{R}^{n_z}$ is a vector of constant, *unknown parameters.*

Proof. From (6) and (8) we get $\dot{z} = \dot{\chi}$. Hence, integrating this equation yields

$$z(t) = \chi(t) + \theta, \tag{11}$$

where

$$\theta := z(0) - \chi(0). \tag{12}$$

Replacing (11) in (7) yields (10). Finally, the regression model (9) is obtained replacing (10) in (1) to get

$$f_y(\phi^{\mathrm{L}}(\chi + \theta, y), y, u) =: \Phi(\chi, y, u, \theta), \tag{13}$$

completing the proof. □

2.3. Consistent parameter estimation

An immediate consequence of Proposition 1 is that the problem of observation of the unmeasurable state x is translated into a standard *parameter estimation* problem for the regression model (9) with the observed state generated by

$$\hat{x} = \phi^{\mathrm{L}}(\chi + \hat{\theta}, y), \tag{14}$$

where $\hat{\theta} : \mathbb{R}_+ \to \mathbb{R}^{n_z}$ is an *on-line estimate* of the constant vector θ. Therefore, to complete the PEBO design it is necessary to ensure the existence of a consistent estimator for the unknown parameter θ. Towards this end, the assumption below is introduced.

Assumption 2. There exists two mappings

$$\begin{aligned} H &: \mathbb{R}^{n_z} \times \mathbb{R}^{n_\zeta} \times \mathbb{R}^{n_y} \times \mathbb{R}^m \to \mathbb{R}^{n_\zeta} \\ N &: \mathbb{R}^{n_z} \times \mathbb{R}^{n_\zeta} \times \mathbb{R}^{n_y} \times \mathbb{R}^m \to \mathbb{R}^{n_z}, \end{aligned}$$

[1] Throughout the paper it is assumed that all mappings are sufficiently smooth.