Systems & Control Letters 85 (2015) 100–108

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Extended state observer for uncertain lower triangular nonlinear systems[☆]

Zhi-Liang Zhao [a], Bao-Zhu Guo [b,c,*]

[a] School of Mathematics and Information Science, Shaanxi Normal University, Xi'an 710062, Shaanxi, PR China

[b] Key Laboratory of System Control, Academy of Mathematics and Systems Science, Academia Sinica, Beijing 100190, PR China

[c] School of Computer Science and Applied Mathematics, University of the Witwatersrand, Wits 2050, Johannesburg, South Africa

ARTICLE INFO

Article history:
Received 20 January 2014
Received in revised form
7 September 2015
Accepted 12 September 2015
Available online 18 October 2015

Keywords:
Extended state observer
Nonlinear systems
Uncertainty

ABSTRACT

The extended state observer (ESO) is a key part of the active disturbance rejection control approach, a new control strategy in dealing with large uncertainty. In this paper, a nonlinear ESO is designed for a kind of lower triangular nonlinear systems with large uncertainty. The uncertainty may come from unmodeled system dynamics and external disturbance. We first investigate a nonlinear ESO with high constant gain and present a practical convergence. Two types of ESO are constructed with explicit error estimations. Secondly, a time varying gain ESO is proposed for reducing peaking value near the initial time caused by constant high gain approach. The numerical simulations are presented to show visually the peaking value reduction. The mechanism of peaking value reduction by time varying gain approach is analyzed.

© 2015 Elsevier B.V. All rights reserved.

1. Introduction

The extended state observer (ESO) is the most important part of the active disturbance rejection control (ADRC), a new control strategy proposed by Han [1] in dealing with the systems with large uncertainty. The key idea of ADRC is that the "total disturbance" which contains the unmolded system dynamics, external disturbance, and even the error in control coefficients away from nominal values is considered as an extended state and is estimated, in real time, through ESO. The "total disturbance" is then canceled in the feedback loop by its estimation. This estimation/cancellation nature of ADRC makes it capable of eliminating the uncertainty before it causes negative effect to control plant and the control energy can thereupon be saved significantly in engineering applications [2]. In the past two decades, numerous engineering applications have witnessed the success and powerfulness of ADRC, for which we refer to [3–6], name just a few.

The idea of ESO can be traced back to [7] where the following nonlinear system with control matched uncertainty is considered.

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = x_3(t), \\ \vdots \\ \dot{x}_n(t) = f(t, x_1(t), x_2(t), \ldots, x_n(t)) + w(t) + u(t), \\ y(t) = x_1(t), \end{cases} \tag{1.1}$$

where $f : \mathbb{R}^{n+1} \to \mathbb{R}$ is usually unknown, $w(t)$ is an external disturbance, $y(t) = x_1(t)$ is the output, and $u(t)$ is the control input. An ESO for system (1.1) is constructed in [7] as follows:

$$\begin{cases} \dot{\hat{x}}_1(t) = \hat{x}_2(t) - \alpha_1 \ell_1(\hat{x}_1(t) - y(t)), \\ \vdots \\ \dot{\hat{x}}_n(t) = \hat{x}_{n+1}(t) - \alpha_n \ell_n(\hat{x}_1(t) - y(t)) + u(t), \\ \dot{\hat{x}}_{n+1}(t) = -\alpha_{n+1} \ell_{n+1}(\hat{x}_1(t) - y(t)), \end{cases} \tag{1.2}$$

where $\hat{x}_i(t)$ $(i = 1, 2, \ldots, n)$ are used to estimate $x_i(t)$ and $\hat{x}_{n+1}(t)$ is used to estimate the "total disturbance": $x_{n+1}(t) \triangleq f(t, \cdot) + w(t)$. The nonlinear functions $\ell_i(\cdot)$ are gain functions and α_i are tuning parameters. The multiple choice of tuning parameters has been

☆ This work was partially supported by the National Natural Science Foundation of China (No. 61403242) and the Fundamental Research Funds for the Central University (No. GK201402003).

* Corresponding author at: Key Laboratory of System Control, Academy of Mathematics and Systems Science, Academia Sinica, Beijing 100190, PR China.
E-mail address: bzguo@iss.ac.cn (B.-Z. Guo).

http://dx.doi.org/10.1016/j.sysconle.2015.09.004
0167-6911/© 2015 Elsevier B.V. All rights reserved.

changed in [8] by constant high gain ε^{-1} with linear gain functions $\ell_i(\cdot)$ as follows:

$$\begin{cases}\dot{\hat{x}}_1(t)=\hat{x}_2(t)+\dfrac{\alpha_1}{\varepsilon}(y(t)-\hat{x}_1(t)),\\ \vdots\\ \dot{\hat{x}}_n(t)=\hat{x}_{n+1}(t)+\dfrac{\alpha_n}{\varepsilon^n}(y(t)-\hat{x}_1(t))+u(t),\\ \dot{\hat{x}}_{n+1}(t)=\dfrac{\alpha_{n+1}}{\varepsilon^{n+1}}(y(t)-\hat{x}_1(t)).\end{cases} \tag{1.3}$$

The ESO with linear gain functions is said to be linear ESO (LESO). A similar idea of LESO (1.3) also can be founded in [9]. However, although the LESO (1.3) takes its advantage of simple turning parameter, it also brings the peaking value problem, slow convergence, and many other problems contrast to fast tracking and small peaking value indicated numerically in [1] for nonlinear ESO (1.2). By taking these points into account, we propose in [10] a nonlinear ESO with constant high gain tuning parameter, which covers the LESO (1.3) as a special case.

We emphasize that ESO is an extension of state observer which estimates only the state by the output of system. We refer a few of them to [11–15] and the references therein.

In this paper, we are concerned with the convergence of nonlinear ESO further for the following lower triangular system which covers system (1.1) as its special case:

$$\begin{cases}\dot{x}_1(t)=x_2(t)+g_1(u(t),x_1(t)),\\ \dot{x}_2(t)=x_3(t)+g_2(u(t),x_1(t),x_2(t)),\\ \vdots\\ \dot{x}_n(t)=f(t,x(t),w(t))+g_n(u(t),x(t)),\\ y=x_1(t),\end{cases} \tag{1.4}$$

where $g_i(\cdot)\in C(\mathbb{R}^{i+m},\mathbb{R})$ is known nonlinear function, $f(t,\cdot)\in C(\mathbb{R}^{n+s+1},\mathbb{R})$ is usually an unknown nonlinear function, $x(t)=(x_1(t),x_2(t),\ldots,x_n(t))$ is the state of system, $u\in\mathbb{R}^m$ is the input (control), $y(t)=x_1(t)$ is the output (measurement), and $w\in C(\overline{\mathbb{R}}^+,\mathbb{R})$ is the external disturbance. The objective of this paper is to design a nonlinear ESO for system (1.4) to estimate both state $x(t)$ and "total disturbance" defined by the extended state of system (1.4) as follows:

$$x_{n+1}(t)\triangleq f(t,x(t),w(t)). \tag{1.5}$$

It is noted that the state observer for lower triangular systems with or without uncertainty has been studied by many researchers and some of them can be found in [11,12,16]. However, in these works, the uncertainty is not estimated. Our study on the uncertainty estimation for quite general $f(t,\cdot)$ and $g_i(\cdot)$ in system (1.4) consists of the major contribution of the paper.

We proceed as follows. In next section, we propose a generalized nonlinear constant high gain ESO. The proof for the practical convergence is presented. We exemplify analytically the proposed ESO by two classes of ESO with an explicit estimation of convergence. In Section 3, we propose a time varying gain nonlinear ESO for system (1.4). This is motivated mainly by the problem caused by the constant high gain: there happens often the notorious peaking value problem in the initial stage. The problem can be solved in some extend by the time varying gain approach. A brief analysis is presented for occurrence of peaking value problem behind. This consists of the second major contribution of the present paper. Section 4 presents some numerical simulations for illustration. In particular, the numerical results demonstrate visually the peaking value reduction by the time varying gain approach.

2. Constant high gain ESO

In this section, we design a constant high gain ESO to recover both state of system (1.4) and its extended state (1.5), which reads as follows:

$$\text{ESO}:\begin{cases}\dot{\hat{x}}_1(t)=\hat{x}_2(t)+\dfrac{1}{r^{n-1}}h_1(r^n(y(t)-\hat{x}_1(t)))\\ \quad+g_1(u,\hat{x}_1(t)),\\ \vdots\\ \dot{\hat{x}}_n(t)=\hat{x}_{n+1}(t)+h_n(r^n(y(t)-\hat{x}_1(t)))\\ \quad+g_n(u,\hat{x}_1(t),\ldots,\hat{x}_n(t)),\\ \dot{\hat{x}}_{n+1}(t)=rh_{n+1}(r^n(y(t)-\hat{x}_1(t))),\end{cases} \tag{2.1}$$

where r is the constant high gain parameter and $h_i\in C(\mathbb{R},\mathbb{R})$, $i=1,2,\ldots,n+1$ the design functions.

To get the convergence of ESO (2.1), some mathematical assumptions are required. The following Assumptions A1 and A2 are on $g_i(\cdot)$ and $f(t,\cdot)$ in system (1.4).

Assumption A1. $g_i:\mathbb{R}^{i+1}\to\mathbb{R}$ satisfies

$$\begin{aligned}&|g_i(u,\nu_1,\ldots,\nu_i)-g_i(u,\tilde{\nu}_1,\ldots,\tilde{\nu}_i)|\\ &\quad\le\Gamma(u)\|(\nu_1-\tilde{\nu}_1,\ldots,\nu_i-\tilde{\nu}_i)\|^{\theta_i},\quad\Gamma\in C(\mathbb{R}^m,\mathbb{R}),\end{aligned} \tag{2.2}$$

where $\theta_i\in((n-i)/(n+1-i),\ 1]$, $i=1,2,\ldots,n$.

The condition (2.2) means that $g_i(\cdot)$, $i=1,2\ldots,n$ are Hölder continuous. For triangular systems, the widely assumed Lipschitz continuity is just a special case of the Hölder continuity with the exponents $\theta_i=1$. Systems with appropriate Hölder continuous functions such as weighted homogeneous functions have merits of finite-time stable, and these kinds of functions can be used for feedback control design, see, for instance, [17].

Assumption A2. $f\in C^1(\mathbb{R}^{n+2},\mathbb{R})$ satisfies

$$\begin{aligned}&|f(t,x,w)|+\left|\frac{\partial f(t,x,w)}{\partial t}\right|+\left|\frac{\partial f(t,x,w)}{\partial x_i}\right|+\left|\frac{\partial f(t,x,w)}{\partial w}\right|\\ &\quad\le\varpi_1(x)+\varpi_2(w),\end{aligned}$$

where $i=1,2,\ldots,n$, $\varpi_1\in C(\mathbb{R}^n,\overline{\mathbb{R}}^+)$, $\varpi_2\in C(\mathbb{R},\overline{\mathbb{R}}^+)$ are two known functions.

The succeeding Assumption A3 is on the control input $u(t)$ and external disturbance $w(t)$.

Assumption A3. $\sup_{t\in[0,\infty)}(|w(t)|+|\dot{w}(t)|+\|u(t)\|)<\infty$.

For many practical systems, since the control is bounded, Assumption A3 is reasonable when ESO is used in fault diagnosis. For ESO based feedback control, the boundedness of control should be analyzed separately. However if the Γ in (2.2) is constant, then the assumption of boundedness for control can be removed.

Assumption A4 is on functions $h_i(\cdot)$ in ESO (2.1). It gives a principle of choosing $h_i(\cdot)$.

Assumption A4. All $h_i\in C(\mathbb{R},\mathbb{R})$ satisfy the following Lyapunov conditions: there exist positive constants $R,N>0$, and continuous, radially unbounded, positive definite functions $\mathcal{V},\mathcal{W}\in C(\mathbb{R}^{n+1},\mathbb{R}^+)$ such that

(1) $\sum_{i=1}^n(\nu_{i+1}-h_i(\nu_1))\frac{\partial\mathcal{V}(\nu)}{\partial\nu_i}-h_{n+1}(\nu_1)\frac{\partial\mathcal{V}(\nu)}{\partial\nu_n}\le-\mathcal{W}(\nu)$, $\forall\ \nu=(\nu_1,\nu_2,\ldots,\nu_{n+1})\in\mathbb{R}^{n+1}$;

(2) $\max_{i=1,\ldots,n}\left\{\|(\nu_1,\ldots,\nu_i)\|^{\theta_i}\left|\frac{\partial\mathcal{V}(\nu)}{\partial\nu_i}\right|\right\}\le N\mathcal{W}(\nu)$, $\left|\frac{\partial\mathcal{V}(\nu)}{\partial\nu_{n+1}}\right|\le N\mathcal{W}(\nu)$, $\nu\in\mathbb{R}^{n+1}$, $\|\nu\|\ge R$.

Download English Version:

https://daneshyari.com/en/article/752009

Download Persian Version:

https://daneshyari.com/article/752009

Daneshyari.com