Systems & Control Letters 77 (2015) 1–10

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

The obstacle avoidance motion planning problem for autonomous vehicles: A low-demanding receding horizon control scheme

Giuseppe Franzè *, Walter Lucia

DIMES - Università degli Studi della Calabria, Rende (CS), 87036, Italy

ARTICLE INFO

Article history:
Received 6 August 2013
Received in revised form
26 August 2014
Accepted 18 December 2014
Available online 22 January 2015

Keywords:
Obstacle avoidance
Set-theoretic approach
Receding horizon control
Constraint satisfaction

ABSTRACT

The paper addresses the obstacle avoidance motion planning problem for ground vehicles operating in uncertain environments. By resorting to set-theoretic ideas, a receding horizon control algorithm is proposed for robots modelled by linear time-invariant (LTI) systems subject to input and state constraints and disturbance effects. Sequences of inner ellipsoidal approximations of the exact one-step controllable sets are pre-computed for all the possible obstacle scenarios and then on-line exploited to determine the more adequate control action to be applied to the robot in a receding horizon fashion. The resulting framework guarantees Uniformly Ultimate Boundedness and constraints fulfilment regardless of any obstacle scenario occurrence.

1. Introduction

The problem of motion planning and control for autonomous mobile vehicles deals with finding appropriate command inputs such that the resulting vehicle trajectory satisfies the requirements of a specified task [1]. For most real mobile robotics applications, a basic requirement is the capability to safely operate in dynamic and a priori unknown environments [2]. Despite the extensive research, this problem still represents a relevant challenge because of unavoidable uncertainties in the operating scenario, inherent deficiencies in perception abilities and computational capabilities of the robot and restrictions on the vehicle mobility due to nonholonomic kinematic constraints, limited control ranges and under-actuations, see e.g. [3]. Avoidance of collisions with moving obstacles is a key component of the safe navigation whose typical objective is to reach a target through the obstacle-free part of the environment, see [4,5] and references therein. In this respect, it is well-known that methods based on navigation functions are not capable to properly take care of any physical constraints (see e.g. the specific schemes for navigation of autonomous vehicles [6,7] and air traffic management [8]) and this represents a *non-negligible* drawback because it makes restrictive such approaches [9].

In particular, during the last decade much attention has been devoted to exploit the possibility of extending road-map and potential functions methods to the case of dynamic obstacle scenarios [10–14]. In [10] Probabilistic Roadmap Methods (PRM) are considered with the aim to overcome the assumption of static environments properly of this class of strategies. A straightforward and high-demanding solution consists in updating the roadmap after an obstacle has changed its position. Therefore, the proposed algorithm creates a robust roadmap in the preprocessing phase by using the observation that the behaviour of the moving obstacles is often not unconstrained but restricted to pre-specified areas. In [11], the so-called Partial Motion Planner (PMP) mechanism is designed so that uncertainties arising from planning within dynamic environments can be handled. The main idea is to pre-calculate admissible state trajectories by using the Inevitable Collision States (ICS) framework that, though it is capable to generate safe paths, is subject to high computational burdens which could lead to violate the real-time constraints under which the robot must take a decision. By considering dynamic objects characterized by piecewise constant velocities, an explicit kinematic model of the robot is considered in [12]: the family of feasible trajectories and their corresponding steering controls are derived in a closed form. In [13], the path planning problem under nonholonomic constraints is addressed by using the so-called *Follow the Gap Method* (FGM). There by calculating a gap array around the robot, the appropriate gap is selected, the best heading vector through the gap derived and the final angle to the target point computed. Along similar lines is the contribution in [14] where a hybrid approach using *a-priori* knowledge of the environment guarantees that the autonomous vehicle cannot be trapped in deadlocks.

All these contributions share as a common denominator the fact that the path following obstacle avoidance problem is par-

* Corresponding author.
E-mail addresses: franze@dimes.unical.it (G. Franzè), wlucia@dimes.unical.it (W. Lucia).

http://dx.doi.org/10.1016/j.sysconle.2014.12.007

tially tackled: the control unit design is either leaved completely out (e.g. [13,14]) or it is obtained by considering specific kinematic vehicle descriptions (see [12]).

Moving from this analysis, we are here interested to consider constrained Receding Horizon Control schemes which are an extremely appealing methodology for dealing with the obstacle avoidance motion planning problem in virtue of their intrinsic capability to generate at each time instant feasible trajectories that allow to safely reach a given goal, see to cite a few the recent contributions [15–18]. In [15] the dynamic window approach (DWA) navigation scheme is recast within a continuous time nonlinear model control predictive (MPC) framework by resorting to the ideas developed in [19]. In particular the algorithm is based on the jointly use of a model-based optimization scheme and a convergence-oriented potential field method. Although interesting, the approach suffers of unavoidable high computational burdens mainly due to the MPC phase that it is slightly mitigated by splitting the dissipative controls into two subsets with piecewise controls. The authors of [16] consider the collision avoidance as well as navigation towards the destination using a MPC approach. The related on-line optimization is solved by means of nonlinear programming, while a local path planning generator makes use of the so-called *distance* and parallax methods. The main handicap of this method relies on the heavy computational loads pertaining to the computation of the predictive controller action. In this contribution, this difficulty is attacked by considering a very large sensing range that allows to have available sufficient time for solving the underline constrained optimization problem. Along similar lines is also [17] and the references therein. More relevant for our purposes, although limited to a single static obstacle configuration, are instead the developments of [18], where algorithms for the computation of the set of states that can be robustly steered in a finite number of steps via state feedback control to a given target set while avoiding pre-specified zones or obstacles are achieved by exploiting polyhedral algebra concepts. There, it is shown how such regions are necessary to adequately deal with the obstacle avoidance problem even if the computational burdens may become prohibitive in some situations, e.g. when the system is subject to bounded disturbances.

In the sequel, we shall consider the class of dynamic environments defined as follows:
The obstacle locations on the working area are known, but at each time instant it is unpredictable which is the current obstacle configuration. The consequence of such a formulation is that the resulting set-up gives rise to a certain degree of uncertainty that if not properly treated can lead to collisions during the vehicle navigation.

To deal with this problem, we develop a novel discrete-time receding horizon control (RHC) strategy based on set-theoretic ideas so that the prescribed saturation and geometric constraints are always fulfilled despite of any obstacle scenario occurrence. The key motivation supporting such an approach relies on the capabilities of the RHC philosophy combined both with set-invariance concepts and the ellipsoidal calculus to guarantee control performance and computational load savings under constraints satisfaction and disturbances effect attenuation requirements, see [20].

Then, the main ingredients of the proposed strategy can be summarized as follows:

- Compute a stabilizing state feedback law and a robust positively invariant ellipsoidal set centred at the goal location;
- Enlarge the set of initial states that, according to the obstacle scenario configurations, can be steered to the target in a finite number of steps;
- At each sampling time, an on-line receding horizon strategy is obtained by deriving the smallest ellipsoidal set complying with the current obstacle configuration. The control move is computed by minimizing a performance index such that the one-step ahead state prediction belongs to the successor set.

A relevant feature of this scheme is the capability to ensure that there exists at each time instant a feasible solution complying with the time-varying obstacle configuration prescriptions. Then, a second important merit relies on the needed computational resources that are significantly modest because the command input computation prescribes at most the solution of a Quadratic Programming (QP) problem under linear constraints.

Finally, the theoretical results are illustrated by means of a simulation campaign on a point mobile robot model whose navigation within a planar environment is limited by the occurrence of moving obstacles.

2. Preliminaries and notations

Throughout the paper, we consider autonomous vehicles described by discrete-time LTI systems

$$x(t+1) = \Phi x(t) + Gu(t) + G_w w(t) \tag{1}$$

where $t \in \mathbb{Z}_+ := \{0, 1, \ldots\}$, $x(t) \in \mathbb{R}^n$ denotes the plant state, $u(t) \in \mathbb{R}^m$ the control input and $w(t) \in \mathcal{W} \subset \mathbb{R}^w$, $\forall t \in \mathbb{Z}_+$, an exogenous bounded disturbance. Moreover, the model (1) is subject to the following set-membership state and input constraints:

$$u(t) \in \mathcal{U}, \quad \forall t \geq 0, \tag{2}$$

$$x(t) \in \mathcal{X}, \quad \forall t \geq 0, \tag{3}$$

with $\mathcal{U}$, $\mathcal{X}$ compact subsets of $\mathbb{R}^m$ and $\mathbb{R}^n$, respectively.

Definition 1. A set $\mathcal{T} \subseteq \mathbb{R}^n$ is robustly positively invariant for (1) if there exists a control law $u(t) \in \mathcal{U}$ such that once the closed-loop solution $x_{CL}(t)$ enters inside that set at any given time t_0, it remains in it for all future instants, i.e. $x_{CL}(t_0) \in \mathcal{T} \rightarrow x_{CL}(t) \in \mathcal{T}$, $\forall w(t) \in \mathcal{W}$, $\forall t \geq t_0$.

Given the system (1), it is possible to determine the sets of states i-step controllable to $\mathcal{T}$ via the following recursion (see [21]):

$$\begin{aligned} \mathcal{T}_0 &:= \mathcal{T} \\ \mathcal{T}_i &:= \{x \in \mathcal{X} : \exists\, u \in \mathcal{U} : \Phi x + Gu + G_w w \in \mathcal{T}_{i-1}, \forall w \in \mathcal{W}\}, \\ &= \{x \in \mathcal{X} : \exists\, u \in \mathcal{U} : \Phi x + Gu \in \tilde{\mathcal{T}}_{i-1}\} \end{aligned} \tag{4}$$

where $In[\cdot]$ denotes the inner ellipsoidal approximation, whereas $\tilde{\mathcal{T}}_{i-1} := In[\mathcal{T}_{i-1} \sim G_w \mathcal{W}]$ is defined as $In[\{x \in \mathcal{T}_{i-1} : x + w \in \mathcal{T}_{i-1}, \forall w \in G_w \mathcal{W}\}]$ and $\sim$ is known as the *P-difference* operator [22]. Moreover, $\mathcal{T}_0$ is known as the terminal region and $\mathcal{T}_i$ is the set of states that can be steered into $\mathcal{T}_{i-1}$ using a single control move.

Definition 2. Let S be a neighbourhood of the origin. The closed-loop trajectory of (1) is said to be Uniformly Ultimate Bounded in S if for all $\mu > 0$ there exist $T(\mu) > 0$ and $u(t) \in \mathcal{U}$ such that, for every $\|x(0)\| \leq \mu$, $x_{CL}(t) \in S$ for all $t \geq T(\mu)$.

Definition 3. Given a set $W \subset \mathbb{R}^n$ and a point $x \in \mathbb{R}^n$, the distance is defined as:

$$dist(x, W) = \inf_{w \in W} \|x - w\|_*,$$

where $*$ is any relevant norm.

Definition 4. Given two sets $W, R \subset \mathbb{R}^n$ the distance is defined as:

$$dist(W, R) = \inf\{\|w - r\|_* \mid w \in W,\ r \in R\}.$$

Definition 5. An oriented graph is an ordered pair $\mathbf{G} = (\mathbf{V}, \mathbf{E})$ such that

- $\mathbf{V}$ is the vertex set;
- $\mathbf{E}$ is a subset of ordered pairs of $\mathbf{V}$ known as the edge set, i.e.

$$\mathbf{E} \subset \{\{u, v\} \mid u, v \in \mathbf{V}\}.$$