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Quasi-ISS/ISDS observers for interconnected systems and applications

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ABSTRACT

This paper considers interconnected nonlinear dynamical systems and studies observers for such systems. For single systems the notion of quasi-input-to-state dynamical stability (quasi-ISDS) for reduced-order observers is introduced and observers are investigated using error Lyapunov functions. It combines the main advantage of ISDS over input-to-state stability (ISS), namely the memory fading effect, with reduced-order observers to obtain quantitative information about the state estimate error. Considering interconnections quasi-ISS/ISDS reduced-order observers for each subsystem are derived, where suitable error Lyapunov functions for the subsystems are used. Furthermore, a quasi-ISS/ISDS reduced-order observer for the whole system is designed under a small-gain condition, where the observers for the subsystems are used. As an application, we prove that quantized output feedback stabilization for each subsystem and the overall system is achievable, when the systems possess a quasi-ISS/ISDS reduced-order observer and a state feedback law that yields ISS/ISDS for each subsystem and therefor the overall system with respect to measurement errors. Using dynamic quantizers it is shown that under the mentioned conditions asymptotic stability can be achieved for each subsystem and for the whole system.

1. Introduction

This paper is motivated by the work [1], where the notion of quasi-input-to-state stable (quasi-ISS) reduced order observers was introduced, by the works [2,3], where the stability property input-to-state dynamical stability (ISDS) was established and by the work [4], where the ISDS property for interconnected systems was investigated. The aim of this paper is to show the advantage of the ISDS-like observers over the ISS-like ones and to show that for large scale systems that can be decomposed into an interconnection of several subsystems the observers can be designed in a decentralized manner.

In practice, observers are used for systems, where the state or parts of the state cannot be measured due to uneconomic measurement costs or physical circumstances like high temperatures, where no measurement equipment is available, for example. We are interested in the topic under which conditions the designed observer guarantees that the state estimation error is stable. The used stability properties for observers are based on ISS, introduced in [5] and ISDS, and are called quasi-ISS and quasi-ISDS, respectively.

ISDS, which is equivalent to the ISS property, has some advantages over ISS. One of these advantages is the so-called memory fading effect. Fading memory estimates were first studied in [6] and further studied for example in [7]. It is known for ISS (ISDS) systems that the influence of the "older" signals on the current state is essentially smaller than the influence of the recent ones. However, the ISS estimation of trajectories does not take this into account. The advantage of the ISDS estimation is that it takes this dissipative property into account. In particular, if the input tends to zero, then the ISDS estimate will tend to zero, whereas the ISS estimate depends on the supremum norm of the input, moreover the fading rate is given by the ISDS.

This motivates the introduction of the quasi-ISDS property for observers, where the approaches of reduced-order observers and the ISDS property are combined and which have the advantage that the recent disturbance of the measurement is taken into account. We investigate under which conditions a quasi-ISDS reduced order observer can be derived for single nonlinear systems, where error Lyapunov functions (see [8,9]) are used. The design of observers in the context of this paper was investigated in [10,9,11,12,1], for example, and remarks on the equivalence of full order and reduced order observers can be found in [13].

Investigating interconnected systems it turns out that the ISS and ISDS properties can be studied in a decentralized way, provided that a small-gain condition is satisfied, see [14,15,4,16,17].

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This allows to decompose a large system into an interconnection of several subsystems and to study their stability properties separately. Examples of such large-scale interconnections can be found in [15] in the ISS framework and in [4] in the ISDS framework. In this paper we show that if quasi-ISS/ISDS reduced-order observers for each subsystem of an interconnected system are given, then an observer for the whole system can be obtained under a small-gain condition.

Furthermore, the problem of stabilization of systems is investigated and we apply the presented approach to quantized output feedback stabilization for single and interconnected systems. The goal of stabilizing a system is an important problem in applications. Many approaches were performed during the last years and the design of stabilizing feedback laws is a popular research area, which is linked up with many applications. The stabilization using output feedback quantization was investigated in [18–21,9,22,11,1], for example. A quantizer is a device, which converts a real-valued signal into a piecewise constant signal. It may affect the process output or may also affect the control input.

Adapting the quantizer with a so-called zoom variable leads to dynamic quantizers, which have the advantage that asymptotic stability for single and interconnected systems can be achieved under certain conditions.

The paper is organized as follows: Section 2 contains some basic notions. The quasi-ISDS property is introduced in Section 3, where as the first main result of this paper a quasi-ISDS observer for single systems is presented. In Section 4 we consider interconnected systems and as the second result we derived quasi-ISS/ISDS reduced-order observers for each subsystem. Furthermore, this section contains as the third main result the design of a quasi-ISS/ISDS observer for the whole system using a small-gain condition. The quantized output feedback stabilization and the dynamic quantizers for interconnections can be found in Section 5. Finally, Section 6 concludes the paper and gives an outlook for future research activities.

2. Preliminaries

By x^T we denote the transposition of a vector $x \in \mathbb{R}^n$, $n \in \mathbb{N}$, furthermore $\mathbb{R}_+ := [0, \infty)$ and $\mathbb{R}^n_+$ denotes the positive orthant $\{x \in \mathbb{R}^n : x \geq 0\}$, where we use the standard partial order for $x, y \in \mathbb{R}^n$ given by

$$x \geq y \Leftrightarrow x_i \geq y_i, \quad i = 1, \dots, \quad \text{and} \quad x \ngeq y \Leftrightarrow \exists i : x_i < y_i.$$

The relation $x > y$ for vectors is defined in the same way.

We denote the standard Euclidean norm in $\mathbb{R}^n$ by $|\cdot|$ and the supremum norm over an interval $[a, b]$, $a \leq b$ of a function f by $\|f\|_{[a,b]}$.

We consider general nonlinear systems of the form

$$\dot{x} = f(x, u), \qquad y = h(x), \tag{1}$$

where $x \in \mathbb{R}^N$ is the state, $u \in \mathbb{R}^M$ is an essentially bounded measurable control input, $y \in \mathbb{R}^P$ is the output, function $f : \mathbb{R}^N \times \mathbb{R}^M \to \mathbb{R}^N$ is locally Lipschitz in x uniformly in u and function $h : \mathbb{R}^N \to \mathbb{R}^P$ is continuously differentiable with locally Lipschitz derivative (called a C^1_L function). In addition, it is assumed that $f(0, 0) = 0$ and $h(0) = 0$ holds. Note that these considerations guarantee that a unique solution of system (1) exists.

A state observer for the system (1) is of the form

$$\dot{\hat{\xi}} = F(\bar{y}, \hat{\xi}, u), \qquad \hat{x} = H(\bar{y}, \hat{\xi}, u), \tag{2}$$

where $\hat{\xi} \in \mathbb{R}^L$ is the observer state, $\hat{x} \in \mathbb{R}^N$ is the estimate of the system state x and $\bar{y} \in \mathbb{R}^P$ is the measurement of y that may be disturbed by $d \in \mathbb{R}^P$: $\bar{y} = y + d$, where d is measurable and essentially bounded function. The function $F : \mathbb{R}^P \times \mathbb{R}^L \times \mathbb{R}^M \to \mathbb{R}^L$ is locally Lipschitz in $\bar{y}$ and $\hat{\xi}$ uniformly in u and function $H : \mathbb{R}^P \times \mathbb{R}^L \times \mathbb{R}^M \to \mathbb{R}^N$ is continuously differentiable with locally Lipschitz derivative (called a C^1_L function). In addition, it is assumed that $F(0, 0, 0) = 0$ and $H(0, 0, 0) = 0$ holds.

We denote the state estimation error by

$$\tilde{x} = \hat{x} - x.$$

For the next sections we need the following sets of comparison functions:

Definition 2.1. We define the following classes of functions:

$$\mathcal{K} := \{\gamma : \mathbb{R}_+ \to \mathbb{R}_+ |\, \gamma \text{ is continuous}, \gamma(0) = 0 \text{ and strictly increasing}\}$$

$$\mathcal{K}_\infty := \{\gamma \in \mathcal{K} |\, \gamma \text{ is unbounded}\}$$

$$\mathcal{L} := \left\{\gamma : \mathbb{R}_+ \to \mathbb{R}_+ |\, \gamma \text{ is continuous and strictly decreasing with } \lim_{t\to\infty} \gamma(t) = 0\right\}$$

$$\mathcal{KL} := \{\beta : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+ |\, \beta \text{ is continuous}, \beta(\cdot, t) \in \mathcal{K},\ \beta(r, \cdot) \in \mathcal{L},\ \forall t, r \geq 0\}$$

$$\mathcal{KLD} := \{\mu \in \mathcal{KL} |\, \mu(r, t + s) = \mu(\mu(r, t), s), \forall r, t, s \geq 0\}.$$

3. Quasi-ISDS observers for single systems

In this section, we introduce quasi-ISDS observers and give a motivating example for the introduction. Then, we show that the reduced-order observer designed in Theorem 1 in [1] is a quasi-ISDS observer provided that an error ISDS Lyapunov function exists.

We recall the definition of quasi-ISS observers from [1]:

Definition 3.1. System (2) is called a quasi-ISS observer for the system (1), if there exists a function $\tilde{\beta} \in \mathcal{KL}$ and for each $K > 0$, there exists a function $\tilde{\gamma}^{\text{ISS}}_K \in \mathcal{K}_\infty$ s.t.

$$|\tilde{x}(t)| \leq \max\{\tilde{\beta}(|\tilde{x}_0|, t), \tilde{\gamma}^{\text{ISS}}_K(\|d\|_{[0,t]})\}, \tag{3}$$

whenever $\|u\|_{[0,t]} \leq K$ and $\|x\|_{[0,t]} \leq K$.

Now we define quasi-ISDS observers.

Definition 3.2. System (2) is called a quasi-ISDS observer for the system (1), if there exist functions $\tilde{\mu} \in \mathcal{KLD}$, $\tilde{\eta} \in \mathcal{K}_\infty$ and for each $K > 0$ a function $\tilde{\gamma}^{\text{ISDS}}_K \in \mathcal{K}_\infty$ such that

$$|\tilde{x}(t)| \leq \max\{\tilde{\mu}(\tilde{\eta}(|\tilde{x}_0|), t), \operatorname*{ess\,sup}_{\tau\in[0,t]} \tilde{\mu}(\tilde{\gamma}^{\text{ISDS}}_K(|d(\tau)|), t - \tau)\}, \tag{4}$$

whenever $\|u\|_{[0,t]} \leq K$ and $\|x\|_{[0,t]} \leq K$.

Function $\tilde{\mu}$ called decay rate, describes the fading memory effect. Namely it shows how fast the influence of the "older" values of the disturbance d on the state $\tilde{x}$ decays with time. This is an important advantage over the ISS property (3), where the estimation of $|\tilde{x}|$ depends on d via $\|d\|_{[0,t]}$, which is nondecreasing in time. The notion of ISDS was introduced in [2]. In [4] the advantages of ISDS over ISS were discussed.

The motivation of the introduction of quasi-ISDS observers will be illustrated by the following example.

Example 3.3. Consider the system as in the Example 1 in [1]

$$\dot{x} = -x + x^2 u, \qquad y = x, \tag{5}$$

where $\dot{\hat{x}} = -\hat{x} + y^2 u$ is an observer. We consider the perturbed measurement $\bar{y} = y + d$, with $d = e^{-t\frac{1}{10}}$. Then, the error dynamics becomes

$$\dot{\tilde{x}} = -\tilde{x} + 2xud + ud^2.$$