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Combination of sign inverting and delay scheduling control concepts for multiple-delay dynamics

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ABSTRACT

In this paper we treat a novel combination of two controversial control concepts, Sign Inverting Control (SIC) and Delay Scheduling (DS), for systems with multiple independent and large delays. SIC suggests the inversion of the control polarity and DS prolongs the existing delays. The combined scheme functions with a single requirement that, the union of the control schemes provides a larger stable operating region than each of its components does, in the domain of the delays. The critical knowledge that is needed to execute such a unified control strategy is the crisp description of these stable regions for each time-delayed control scheme. This need can be fulfilled using the recent Cluster Treatment of Characteristic Roots (CTCR) paradigm, which establishes the stable regions exhaustively and non-conservatively. The resulting options in selecting operating modes render more robust control performance against much larger delay variations than each of the schemes. We also investigate the disturbance rejection speeds within these enlarged stable regions in order to improve the control performance even further. Such multi-faceted paradoxical combinations provide previously-unexplored tools to control designers. Experimental validations of these novel concepts are presented on a simple setup with a single-axis manipulator.

1. Introduction and motivation

The understanding of time-delayed dynamics has matured continuously over the past few decades as many researchers expressed interest in this field [1–5]. The main source of notoriety in time-delayed systems (TDS) is the introduction of transcendental terms to the characteristic equation, resulting in infinitely many roots (a.k.a. *infinite dimensionality*). A major focus of the cutting-edge research has been on the development of tools and methods that enable stability analysis of these systems. Such efforts resulted in various numerical methods [6–11], and an analytical procedure of the authors' research group, Cluster Treatment of Characteristic Roots (CTCR) [12,13]. CTCR is, in fact, a paradigm that imparts a method to assess stability of linear time invariant (LTI) time-delay systems. It starts from the exhaustive determination of stability boundaries in the domain of the delays. Contrary to frequent misconception, the CTCR paradigm is transparent to the methodology which evaluates these hypersurfaces. A broad range of clever procedures in the literature [2,4,13–15] can determine them, and more advanced methods are still evolving. Using this information, CTCR produces a crisp (i.e. non-conservative) and exhaustive declaration of stable regions in the domain of the delay(s). We name this stability-based partitioning in the delay space the "*stability map*" of the system.

In addition to stability analysis, the authors' group spent considerable effort on the control synthesis for TDS. Such studies resulted in the development of several concepts including *Delay Scheduling Control*(*DS*) and *Sign Inverting Control*(*SIC*). Earlier development of DS is discussed in several publications leading to [16] which handle multiple-delay cases with experimental validations. The article [17], on the other hand, is the only archival document on SIC. It presents the preliminary development on the concept which treats the class of dynamics with a single delay only. The present paper is prepared with two additional contributions in mind:

(i) It is the first treatise of SIC under multiple independent and large delays;
(ii) It is the only attempt on the combination of SIC and DS techniques.

For both SIC and DS operations, as well as for the stability paradigm CTCR, an important attribute is the "large delays". By "large" we mean the delays encountered in the operation are in the order of magnitude of the period of the fastest controlled dynamics. Say, for a desired trajectory which has 10 Hz as the highest

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frequency content, this study is focusing on control feedback delays in the order of 10^{-1} s (sec). The practical implication of this point is that small delays (such as a few sampling periods) are not of concern. On the contrary, this line of study investigates cases which bring much longer sensing and actuation delays, characterized as "feedback delays".

Sign inverting control idea originates from a very favorable and practical suggestion: in SIC the controller inverts the polarity of the "nominal" control actuation (e.g., a servo motor receiving a −V input voltage instead of +V). This action is based on the expectation that stable operating regions of a TDS may be expanded considerably by simply reversing the sign of the feedback gains. The mathematical implication of this concept is more intriguing, as we will describe in the later sections of the paper. If the resulting stability maps for both nominal and SIC strategies are known crisply, the control designer will have much larger choice of delay selections to make without jeopardizing stability. In other words, by inverting the sign of the feedback control, stronger delay robustness may be achieved. The selection rules of the control strategy for different delay compositions are the main questions we attempt to answer in this paper. We present several angles of approach within the text to resolve this nontrivial problem.

Remark 1. SIC is the simplest control alteration scheme one can design. It has no amplification and/or dynamics in the feedback line, but a simple change of sign (or polarity in the control actuator output). As such it is very attractive. But the stability repercussions of this change have to be studied carefully. That is the underlying contribution of this paper.

Delay scheduling control is another control concept which is based on deliberately increasing delays such that control performance is improved [16]. The highlight of this paradoxical scheme is to schedule the delays by *prolonging* them further. This is a counter-intuitive proposition but it is the only game we can impose on delays. They cannot be reduced beyond what they are at the present, due to causality of the dynamic events; however, they can be prolonged further by the controller. For instance, additional "hold buffers" can be artificially introduced in the feedback line of the controlled system. Owing to the complex infinite dimensionality of the delayed dynamics such manipulations on the delays may create some improved characteristics also. For instance, disturbance rejection speeds may be improved by these variations in delays. As we will discuss later, this scheme has intriguing characteristics which complement the SIC logic. Therefore the study includes DS along with the main theme of the work, SIC. We will return to DS further in the experimental validation section.

It is important to note that both methods (SIC and DS) require a precise knowledge of the stable operating regions in the delay space, i.e., the *stability maps*. These regions (also known as 'stability pockets'), can be exhaustively obtained using the CTCR [12,13], bringing this paradigm in the heart of the discussion.

A general class of linear time-invariant time-delayed systems (LTI-TDS) is considered with two independent delays in this study:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \bar{\mathbf{B}}\mathbf{u} \tag{1}$$

where $\mathbf{x}(n \times 1)$ is the state vector, $\mathbf{u}(m \times 1), m \leq n$ is the control input, $\mathbf{A}$ and $\bar{\mathbf{B}}$ are matrices of appropriate dimensions. The conventional full-state feedback control logic is taken as $\mathbf{u} = -\mathbf{K}_1\mathbf{x}(t-\tau_1) - \mathbf{K}_2\mathbf{x}(t-\tau_2)$ where, $\mathbf{K}_1, \mathbf{K}_2 (m \times n)$ are the feedback gain matrices and τ_1, τ_2 are the delays occurring in the feedback lines. The dynamics of the system becomes:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_1\mathbf{x}(t-\tau_1) + \mathbf{B}_2\mathbf{x}(t-\tau_2) \tag{2}$$

where $(\tau_1, \tau_2) \in \Re^{2+}$, $\mathbf{B}_1 = -\bar{\mathbf{B}}\mathbf{K}_1$, $\mathbf{B}_2 = -\bar{\mathbf{B}}\mathbf{K}_2$. The characteristic equation for this system is:

$$CE(s, \tau_1, \tau_2) = \det\left(s\mathbf{I} - \mathbf{A} - \sum_{i=1}^{2} \mathbf{B}_i\, e^{-\tau_i s}\right). \tag{3}$$

It is well known that this system represents a globally asymptotically stable dynamics when all of its infinite spectra lie in the left-half of the complex plane. For a given selection of "nominal" feedback control structure, $\mathbf{B}_1$ and $\mathbf{B}_2$, the CTCR paradigm provides a non-conservative and exhaustive stability picture in the domain of the delays, τ_1 and τ_2. The same exercise is conducted for SIC (i.e., for $-\mathbf{B}_1$ and $-\mathbf{B}_2$ instead of $\mathbf{B}_1, \mathbf{B}_2$). The reasoning for the sign inversion and its consequences are detailed in the following sections of the paper. If one knows the stable regions for both feedback structures $(\mathbf{B}_1, \mathbf{B}_2)$ and $(-\mathbf{B}_1, -\mathbf{B}_2)$, a mixed control logic can be designed using the two schemes over a much larger choice of delay compositions (τ_1, τ_2).

The structure of the paper is as follows. Section 2 presents a detailed treatment of SIC for multiple-delay systems. Descriptions and results of an experimental validation exercise that demonstrates this control scheme are provided under Section 3. Section 4 offers concluding remarks. For completeness of the discussions we present a brief overview of CTCR, borrowing from [18] in Appendix.

2. New control strategy: "sign inverting control (SIC)"

This is the first attempt in deploying SIC on systems which have multiple, large and independent delays. The core concept of SIC was introduced earlier on a system with a single time delay [17]. Sign inverting control alters only the *sign* of the feedback control with the intention that the stable operating regions in the delay domain may be expanded beyond what is possible with the starting control law. In order to discriminate the two control laws, we are referring to the starting logic as the "*nominal control law*", and the other one after inversion, "*SIC*".

For clarity, we wish to declare three crucial requirements for SIC to be viable in a lemma:

Lemma 1. *Let the sets of delay compositions, (τ_1, τ_2), for which the original LTI-MTDS with nominal control in (2) and the SIC applied system are asymptotically stable, be denoted by $\mathcal{N}$ and $\mathcal{S}$, respectively. In order to have a feasible deployment of SIC logic, the following three conditions are necessary and sufficient.*

(i) $\mathcal{N} \neq \emptyset$;
(ii) $\mathcal{S} \neq \emptyset$;
(iii) $(\mathcal{N} \cup \mathcal{S}) \backslash (\mathcal{N} \cap \mathcal{S}) \neq \emptyset$ *(i.e. the symmetric difference of $\mathcal{N}$ and $\mathcal{S}$ are nonempty) and* $\mathcal{S} \not\subset \mathcal{N}$.

In the contrary case if any one of these conditions is not satisfied, SIC becomes infeasible.

Proof. The condition (i) is a trivial starting point for the controlled operation. If a selected nominal control scheme does not have any stable operating delay compositions (no stability pockets in the delay space) it cannot be used for the given time-delayed feedback structure to start with. The condition (ii) is proven in a similar manner as in (i). In case the SIC logic does not have any stable operating delay compositions it cannot be selected as a viable option to use. In (iii), the notation $\mathcal{N} \cup \mathcal{S}$ denotes the delay compositions that render either nominal system or SIC applied system stable and $\mathcal{N} \cap \mathcal{S}$ denotes those that guarantee the stability for both systems. If (iii) is false, on the other hand, it means that $\mathcal{S} \equiv \mathcal{N}$ or $\mathcal{S} \subset \mathcal{N}$. For either case, SIC logic does not have advantages over nominal control from the perspective of delay robustness. Thus, SIC becomes infeasible. When $\mathcal{N} \subset \mathcal{S}$, however, SIC still offers an advantage. Sufficiency is the natural consequence of Remark 1. If a delay composition causes instability for the nominal (or SIC) control logic, while the companion rule, i.e., SIC (or the nominal) logic renders stability for the same delays, this would be sufficient to make the change in control law. ◊

In order to verify if these three conditions hold for a given set of nominal and SIC control regimes one needs a method for exhaustive and non-conservative declarations of stability for