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A generalized negative imaginary lemma and Riccati-based static state-feedback negative imaginary synthesis

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In this paper, we present a generalized negative imaginary lemma based on a generalized negative imaginary system definition. Then, an algebraic Riccati equation method is given to determine if a system is negative imaginary. Also, a state feedback control procedure is presented that stabilizes an uncertain system and leads to the satisfaction of the negative imaginary property. The controller synthesis procedure is based on the proposed negative imaginary lemma. Using this procedure, the closed-loop system can be guaranteed to be robustly stable against any strict negative imaginary uncertainty, such as in the case of unmodeled spill-over dynamics in a lightly damped flexible structure. A numerical example is presented to illustrate the usefulness of the results.

1. Introduction

Negative imaginary (NI) systems theory has had an important impact in modeling and controlling flexible structure systems [1–5] This theory was introduced by Lanzon and Petersen in [1,2] for the robust control of flexible structures with force actuators combined with colocated position or acceleration sensors. The NI property arises in many practical systems. For example, this property arises when considering the transfer function from a force actuator to a corresponding colocated position sensor (for instance, a piezoelectric sensor) in a lightly damped structure [6,2]. Another area where the underlying system dynamics are frequently NI is in nano-positioning systems; see e.g., [7–9,5]. Also, the positive-position feedback control scheme in [6,10] can be considered using the NI framework. Furthermore, other control methodologies in the literature such as integral resonant control (IRC) [11] and resonant feedback control [12] fit into the NI framework and their stability robustness properties can be explained by NI systems theory.

Lanzon and Petersen studied the stability robustness of interconnected NI systems in [1,2]. They have shown that a necessary and sufficient condition for the internal stability of a positive-feedback interconnection of an NI system $G(s)$ and a strictly negative imaginary (SNI) system $\bar{G}(s)$ is given by the condition.

$$\lambda_{\max}(G(0)\bar{G}(0)) < 1, \tag{1}$$

where the notation $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue of a matrix with only real eigenvalues. The results provided in [1,2] have been used in a number of practical applications. For instance, a positive position feedback control scheme based on the NI stability result provided in [1,2] is used to design a novel compensation method for a coupled fuselage-rotor mode of a rotary wing unmanned aerial vehicle in [13]. In [14,15], the NI stability result is applied to nanopositioning in an atomic force microscope. An identification algorithm which enforces an NI constraint is proposed in [3] for estimating model parameters, following which an integral resonant controller is designed for damping vibrations in flexible structures. In addition, it is shown in [16] that the class of linear systems having NI transfer function matrices is closely related to the class of linear port Hamiltonian systems. In [5], an IRC scheme based on the stability results provided in [1,2] is used to design an active vibration control system for the mitigation of human-induced vibrations in light-weight civil engineering structures.

In the papers [1,2], the authors consider systems with poles in the open left half of the complex plane. These results have been extended in [17] to include NI systems with poles in the closed left half of the complex plane, except at the origin. Also, further extensions to NI systems theory include the study of NI controller

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synthesis [18], connections between NI systems analysis and μ-analysis [19], and conditions for robust stability analysis of mixed NI and bounded-real classes of uncertainties [20].

An important class of NI systems corresponds to flexible systems with free body motion. For instance, these systems arise in areas such as rotating flexible spacecraft [21], rotary cranes [22], robotics and flexible link manipulators [11,12], and dual-stage hard disk drives [23,24]. These flexible structure systems with free body motion lead to dynamical models including poles at the origin. In the papers [25,26], the notion of NI systems was extended to allow for up to two poles at the origin and corresponding stability results were presented. This extension allows for flexible structures with free body motion. Also, a negative imaginary definition for transfer functions which are not necessarily rational is given in [27].

In this paper, we first consider the generalized definition of NI systems presented in [26] and provide a corresponding generalized NI lemma. Also, we present an algebraic Riccati equation (ARE) method to find if a given system satisfies the generalized NI property. Then, we use these analysis methods in order to derive a state feedback controller synthesis method that yields a closed-loop system which satisfies the generalized NI property. The robustness of this control strategy follows from the NI stability results in [25,26]. One advantage of presenting the NI lemma in the ARE format relates to how simple and well understood the ARE is in modern control theory including H_2 and H_∞ control [28]. The analytical properties of AREs and efficient numerical algorithms for solving the AREs have been very well developed [29].

A preliminary conference version of the stabilization result presented in this paper was presented in [30]. In this paper, we extend the work in [30] to derive a controller that stabilizes an uncertain system when full state feedback is available. Unlike the results in [30], the resulting closed-loop system is asymptotically stable. Also, we present proofs of the results, which were not included in [30] and a new illustrative example corresponding to a practical atomic force microscope (AFM) system is developed in this paper.

2. Preliminaries

In this section, we present the definition for SNI systems. Also, we introduce some technical results which will be used in deriving the main results of this paper.

The following defines SNI systems [1]:

Definition 1. A square transfer function matrix $G(s)$ is SNI if the following conditions are satisfied:

1. $G(s)$ has no pole in $Re[s] \geq 0$;
2. For all $\omega > 0$, $j\,(G(j\omega) - G(j\omega)^*) > 0$.

Now, consider the following LTI system:

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{2}$$

$$y(t) = Cx(t) + Du(t) \tag{3}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{m\times n}$, and $D \in \mathbb{R}^{m\times m}$.

The following lemma will be used in deriving an SNI lemma.

Lemma 1 (*[31]*). *Suppose the transfer function matrix $G(s)$ has a minimal realization $\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$. Also, suppose A has no pure imaginary eigenvalues for $\omega_0 > 0$ and $\lambda \in \mathbb{R}$ is not an eigenvalue of $\frac{CB+B^TC^T}{2} > 0$. Then, λ is an eigenvalue of $H(j\omega_0) = \frac{1}{2}j\omega_0(G(j\omega_0) - G(j\omega_0)^*)$ if and only if $j\omega_0$ is an eigenvalue of the matrix*

$$N_\lambda = \begin{bmatrix} A + BR_\lambda^{-1}CA & BR_\lambda^{-1}B^T \\ -A^TC^TR_\lambda^{-1}CA & -A^T - A^TC^TR_\lambda^{-1}B^T \end{bmatrix},$$

where $R_\lambda = 2\lambda I - CB - B^TC^T$.

Now, consider the following theorem which characterizes the SNI property based on the spectrum of a corresponding Hamiltonian matrix.

Theorem 1. *Suppose the transfer function matrix $G(s)$ has a minimal realization $\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$ such that $CB + B^TC^T > 0$. Then, $G(s)$ is SNI if and only if the following conditions are satisfied:*

1. *A is a Hurwitz matrix, and $D = D^T$;*
2. *The Hamiltonian matrix*

$$N_0 = \begin{bmatrix} A - BR^{-1}CA & -BR^{-1}B^T \\ A^TC^TR^{-1}CA & -A^T + A^TC^TR^{-1}B^T \end{bmatrix}$$

has no eigenvalues at $s = j\omega$ for any $\omega > 0$. Here, $R = (CB + B^TC^T)$.

Proof. Let

$$H(j\omega) = \frac{1}{2}j\omega(G(j\omega) - G(j\omega)^*).$$

Suppose that $G(s)$ is SNI. It follows that A is a Hurwitz matrix. Also, $j(G(j\omega) - G(j\omega)^*) > 0$ for all $\omega > 0$, which implies that $H(j\omega) > 0$ for all $\omega > 0$. Then, $\lambda = 0$ is not an eigenvalue of $H(j\omega)$ for any $\omega > 0$. It follows from Lemma 1 that $j\omega$ is not an eigenvalue of N_0 for any $\omega > 0$. Note that N_0 is equivalent to N_λ in Lemma 1 with $\lambda = 0$.

Conversely, suppose that N_0 has no eigenvalues $s = j\omega$ with $\omega > 0$ and A is a Hurwitz matrix. It follows from Lemma 1 that $\lambda = 0$ is not an eigenvalue of $H(j\omega)$ for all $\omega > 0$. Since the eigenvalues of $H(j\omega)$ are continuous functions of ω, this implies that either $H(j\omega) > 0$ for all $\omega > 0$ or $H(j\omega) < 0$ for all $\omega > 0$. However, using the fact that $CB+B^TC^T > 0$, it follows that $\lim_{\omega\to 0} H(j\omega) > 0$, which implies $j(G(j\omega) - G(j\omega)^*) > 0$ for all $\omega > 0$. Hence $G(s)$ is SNI, since A is a Hurwitz matrix. □

3. Generalized negative imaginary systems

We now recall a generalized definition of NI systems which allows for free body dynamics.

Definition 2 (*[25,26]*). A square transfer function matrix $G(s)$ is NI if the following conditions are satisfied:

1. $G(s)$ has no pole in $Re[s] > 0$.
2. For all $\omega > 0$ such that $s = j\omega$ is not a pole of $G(s)$,

$$j\left(G(j\omega) - G(j\omega)^*\right) \geq 0. \tag{4}$$

3. If $s = j\omega_0$ with $\omega_0 > 0$ is a pole of $G(s)$, then it is a simple pole and the residue matrix $K = \lim_{s\longrightarrow j\omega_0}(s - j\omega_0)jG(s)$ is Hermitian and positive semidefinite.
4. If $s = 0$ is a pole of $G(s)$, then $\lim_{s\longrightarrow 0} s^kG(s) = 0$ for all $k \geq 3$ and $\lim_{s\longrightarrow 0} s^2G(s)$ is Hermitian and positive semidefinite.

Unlike the NI definition presented in [17], Definition 2 allows for (up to two) poles at the origin. In the next section, we will present a generalized NI lemma corresponding to this definition.

3.1. Generalized negative imaginary lemma

Here, we present an NI lemma that allows for systems with free body motion to be included in the NI framework.