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Stability analysis of complex-valued nonlinear delay differential systems

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ABSTRACT

Since a quantum system, which is one of the foci of ongoing research, is a classical example of a complex-valued system, in this paper, the issue of asymptotic stability of solutions to complex-valued nonlinear delay differential systems is addressed. By taking advantage of the theory of matrix measure, the exponential stability criteria of a complex-valued nonlinear delay system are established, which not only improve some known results in literature, but also greatly reduce the complexity of analysis and computation. As an application, the exponential stability conditions of 2-dimensional real-valued time-varying delay systems are derived, the conditions are easier to verify in comparison with known results. The effectiveness of the main results are illustrated by some numerical examples.

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1. Introduction

Stability of stationary solutions of dynamic systems, which concerns the long-term, qualitative properties of systems, is one of the fundamental issues in differential equation theory and applications. It has attracted an increasing research interest within the control community, for example [1–6] and references therein. Recently, some researchers have studied the stability of real differential systems using matrix measures [7–10]. However, the common setting adopted in the aforementioned works is always the real number field, namely, the study objectives are real-valued differential systems. The study objective in this paper are complex-valued differential systems. Complex-valued differential systems also have many potential applications in science and engineering. For example, quantum systems, which are one of the foci of ongoing research [11–14], are classical complex-valued differential systems. Another important example of complex-valued differential systems are complex-valued neural networks. Complex-valued neural networks have been found highly useful in extending the scope of applications in optoelectronics, filtering, imaging, speech synthesis, computer vision, remote sensing, quantum devices, spatio-temporal analysis of physiological neural devices and systems, and artificial neural information processing [15–17]. In fact, equations of many classical systems except quantum systems and complex-valued neural networks, such as the Ginzburg–Landau equation [18], the Orr–Sommerfeld equation [19], the complex Riccati equation [20] and the complex Lorenz equation [21], are considered in the complex field. Hence, it is significative and important to study the properties of complex-valued differential systems.

Generally speaking, complex-valued differential systems are more complex and difficult than real-valued differential systems. The usual method analyzing complex-valued differential systems is to separate them into a real part and an imaginary part, and then to recast them into an equivalent real-valued differential system, see [16,22,21,23] and references therein. But this method encounters two problems. One is that the dimension of the real-valued system is double that of complex-valued system, which leads to difficulties in analysis. The other one is that this method needs an explicit separation of a complex-valued function $f(t, z)$ into its real part and imaginary part; however, this separation is not always expressible in an analytical form. An efficient way to analyze a complex-valued system is to retain the complex nature of the system and consider its properties on $\mathbb{C}^n$ [24–27].

To the best of our knowledge, there have been few reports about the analysis and synthesis of complex-valued delay differential systems except [28–30,23,31,32], and there is no result so far about the stability of general complex-valued nonlinear delay differential systems with n $(n > 1)$ dimension. In this paper, the exponential stability of the zero solution of the following complex-valued delay differential equation

$$\begin{aligned}\dot{z}(t) &= A(t)z(t) + B(t)\bar{z}(t) + C(t)z(t-\tau)\\ &\quad + D(t)\bar{z}(t-\tau) + h(t, z(t), z(t-\tau))\end{aligned} \quad (1)$$

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is discussed, where $\tau > 0$ is a constant delay, $A(t), B(t), C(t), D(t)$ are complex time-varying $n\times n$ matrices, $z \in \mathbb{C}^n$ is the state vector, $\bar{z}$ is the conjugate of z, and h is a complex-valued vector function with dimension n. Although system (1) with the case $A(t) = A \in \mathbb{C}$, $C(t) = C \in \mathbb{C}$ and $B(t) = D(t) = h = 0$ were discussed in [30,23,28], for the general case, it was only discussed by Kalas in [29,31,32]. However, Kalas just gave the stability conditions of system (1) with $n = 1$, and the stability criteria and derivation process were also very complicated. In this paper, the stability criteria of complex-valued delay differential system (1) are obtained by using the matrix measure, and the derived stability criteria not only improve some known results in literature, but also greatly reduce the complexity of analysis and computation. As an application, the exponential stability conditions of 2-dimensional real-valued time-varying delay systems are derived, the conditions are easier to verify in comparison with known results.

The remainder of the paper is organized as follows. In Section 2, the conceptions of matrix measure and the definition of stability of complex-valued nonlinear differential systems are presented. The exponential stability criteria of complex-valued nonlinear delay differential systems are established by virtue of the matrix measure theory in Section 3. The main points of the paper are illustrated by some numerical examples in Section 4. Finally, some conclusions are drawn in Section 5.

2. Preliminaries

First, the definition and essential properties of the matrix measure is given. Consider a complex square matrix $P = (p_{ij}(t))_{n\times n}$. Let $\|P\|_\theta$ denote a matrix norm which is the operator norm induced by the corresponding vector norm of $|\cdot|_\theta, \theta = 1, 2, \infty, \omega$.

Definition 2.1 (*[33,34]*)**.** The matrix measure induced from a given matrix norm $\|P\|_\theta$ is defined as

$$\mu_\theta(P) = \lim_{h\to 0^+} \frac{\|I + hP\|_\theta - 1}{h},$$

where I is the identity matrix.

Properties and calculations on the matrix measure can be found in [33,34], from which the matrix measure corresponding to the commonly-used matrix norms are collected. When the matrix norm

$$\|P\|_1 = \max_j \sum_i^n |p_{ij}|, \qquad \|P\|_2 = [\lambda_{\max}(P^*P)]^{1/2},$$

$$\|P\|_\infty = \max_i \sum_j^n |p_{ij}|, \qquad \|P\|_\omega = \max_j \sum_i^n \frac{\omega_j}{\omega_i}|p_{ij}|,$$

we can obtain the corresponding matrix measure

$$\mu_1(P) = \max_j \left\{ Re(p_{jj}) + \sum_{i\neq j, i=1}^n |p_{ij}| \right\},$$

$$\mu_2(P) = \frac{1}{2}\lambda_{\max}\left(P^* + P\right),$$

$$\mu_\infty(P) = \max_i \left\{ Re(p_{ii}) + \sum_{j\neq i, j=1}^n |p_{ij}| \right\},$$

$$\mu_\omega(P) = \max_j \left\{ Re(p_{jj}) + \sum_{i\neq j, i=1}^n \frac{\omega_j}{\omega_i}|p_{ij}| \right\},$$

where P^* denotes the conjugate transpose of P, $\omega = \mathrm{diag}\{\omega_1, \omega_2, \dots, \omega_n\} > 0$, the vector norm $|x|_\omega = |\omega^{-1}x|_1$, $Re(p)$ and $|p|$ are the real part and modules of complex number p, respectively.

Second, we give some definitions about complex-valued delay functional differential equations.

Suppose $\tau \geq 0$ is a given real number, $\mathbb{R} = (-\infty, +\infty)$, $\mathbb{C}^n$ is the n-dimensional complex linear vector space with norm $|\cdot|_\theta$. $C([a, b], \mathbb{C}^n)$ is the Banach space of continuous functions mapping the interval $[a, b]$ into $\mathbb{C}^n$ with the topology of uniform convergence. Denote $\Omega = C([-\tau, 0], \mathbb{C}^n)$ and designate the norm of an element $\varphi \in \Omega$ by $\|\varphi\|_\theta = \sup_{-\tau\leq s\leq 0}\{|\varphi(s)|_\theta\}$. If

$$t_0 \in \mathbb{R}, \qquad A \geq 0, \quad \text{and} \quad z \in C([t_0 - \tau, t_0 + A], \mathbb{C}^n),$$

then for any $t \in [t_0, t_0 + A]$, let $z_t \in \Omega$ be defined by $z_t(\gamma) = z(t + \gamma), \gamma \in [-\tau, 0]$. If D is a subset of $\mathbb{R}\times\Omega$, $f : D \to \mathbb{C}^n$ is a given function and "D^+" represents the right-hand derivative, we say that the equation

$$D^+z(t) = f(t, z_t) \tag{2}$$

is a complex-valued delay functional differential equation. Assume that for all $t \in \mathbb{R}$, $f(t, 0) \equiv 0$ and $z(t_0, \varphi)(t)$ is the unique solution of (2) with initial value $\varphi \in \Omega$ at $t_0 \in \mathbb{R}$.

Definition 2.2. The zero solution of (2) is said to be exponentially stable if there exist positive reals η, δ and k such that

$$|z(t)|_\theta \leq k\|z_{t_0}\|_\theta \exp\{-\eta(t - t_0)\}, \quad \forall \|z_{t_0}\|_\theta < \delta,$$

for all $t \geq t_0$.

In addition, the following lemma, which is slightly modified from [35], is needed in the proof of our main results.

Lemma 2.1. *Assume that the function $f(t)$ or $g(t)$ is bounded and $\inf_{t\geq t_0}\{f(t) - g(t)\} > 0$, for all $t \in [t_0, +\infty)$*

$$D^+v(t) \leq -f(t)v(t) + g(t)\|v_t\|_\theta,$$

where $v(t) \in C([t_0 - \tau, +\infty), R^+)$, $\tau > 0$, $f(t), g(t) \in C([t_0, +\infty), R^+)$, $\|v_t\|_\theta = \sup_{t-\tau\leq s\leq t}\{|v(s)|_\theta\}$, then there exists a constant $\eta > 0$ such that

$$v(t) \leq \|v_{t_0}\|_\theta \exp\{-\eta(t - t_0)\},$$

for $t \in [t_0 - \tau, +\infty)$.

3. Main results

In this section, some sufficient conditions of exponential stability of system (1) are given by the matrix measure theory.

Theorem 3.1. *If the following conditions hold:*

(i) *there exist $L_1(t), L_2(t) \in C([t_0, +\infty), R)$ such that, for any $z, w \in \mathbb{C}^n$*

$$|h(t, z, w)|_\theta \leq L_1(t)|z|_\theta + L_2(t)|w|_\theta; \tag{3}$$

(ii) *$f(t)$ or $g(t)$ is bounded, $\inf_{t\geq t_0}\{f(t) - g(t)\} > 0$ and $g(t) > 0$, where*

$$f(t) = -[\mu_\theta(A(t)) + \|B(t)\|_\theta + L_1(t)],$$
$$g(t) = \|C(t)\|_\theta + \|D(t)\|_\theta + L_2(t),$$

then the zero solution of system (1) is exponentially stable.

Proof. Assume that $z(t_0, \varphi)(t)$ is the unique solution of system (1) with initial value $\varphi \in \Omega$ at t_0, for convenience, we denote $z(t) := z(t_0, \varphi)(t)$ and

$$J = D^+|z(t)|_\theta - \mu_\theta(A(t))|z(t)|_\theta - \|C(t)\|_\theta|z(t - \tau)|_\theta - \|D(t)\|_\theta|\bar{z}(t - \tau)|_\theta - |h(t, z(t), z(t - \tau))|_\theta.$$

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