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Output feedback networked control with persistent disturbance attenuation[☆]

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HIGHLIGHTS

- Present a method for stabilization and rejection of persistent disturbances.
- Consider output feedback using the state observer to estimate states and disturbances.
- Apply in NCS to reduce the number of transmitted messages.
- Improved performance is shown with respect to recent and similar work.

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ABSTRACT

This paper presents a model-based control approach for output feedback stabilization and disturbance attenuation of continuous time systems that transmit measurements over a limited bandwidth communication network. Necessary and sufficient conditions for asymptotic stability of the networked system in the presence of persistent external disturbances are given. The results in this paper provide a significant improvement in the performance of the system and provide a considerably reduction of the necessary network bandwidth with respect to similar approaches in the literature.

1. Introduction

In Networked Control Systems (NCS) a digital communication network is used to transfer information among the components of a control system including actuators, controllers, and sensors. This type of implementation differs significantly from classical control systems where all system components are attached directly to the control plant exchanging information using dedicated wiring [1]. The use of a shared communication channel for control systems makes feedback measurements inaccessible to the controller for long intervals of time. An approach followed by different authors is to keep the latest received feedback measurement constant until new information arrives from the controller, i.e. using a Zero-Order-Hold (ZOH), [2–4].

The use of a model of the system in the controller node provides better performance in general since an estimate of the state of the system can be used for control during the time intervals that feedback measurements are not available.

Model-based frameworks have been used by different authors for control of networked systems [5–14] or for control of systems with limited feedback communication [15]. The work in [5–7,13,14] considers model uncertainties and no external disturbances while the authors of [8,9] focus on attenuation of constant input disturbances by considering no plant-model mismatch. Similar model-based frameworks have been studied by different authors, for instance, the work in [11] considers discrete time systems subject to packet dropouts and the model dynamics are used to generate the control inputs when no feedback measurements are successfully received. The authors of [10,12] used the model-based framework in [6] for, respectively, stabilization of singularly perturbed systems and stabilization of coupled networked uncertain systems.

In the present paper we address the case of disturbance attenuation and consider the situation of output feedback using state observers. The main contributions of this paper are as follows: first,

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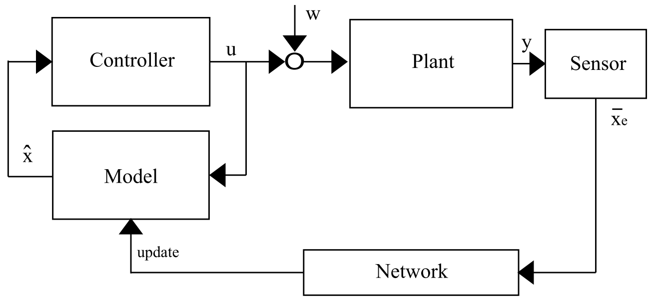

Fig. 1. Model-based networked system with external disturbance.

we introduce an augmented state observer that provides estimates not only of the states of the disturbed system but also estimates of the unknown external disturbance. We provide observability conditions for the augmented system in terms of the original plant parameters. Second, the augmented state observer is implemented in the networked system and we provide stability conditions as a function of the update interval parameter which dictates how often the sensor transmits the observed variables to the controller node. Constant update intervals are used throughout this paper. It is shown that this approach considerable improves the performance of the networked system compared to similar work in [8,9].

The paper is organized as follows. Section 2 introduces the model-based architecture for control over networks. Section 3 describes the augmented state observer and provides observability conditions on the augmented system. Section 4 provides stability results for continuous time systems that transmit measurements using a limited bandwidth communication network. Illustrative examples are given in Section 5, and Section 6 concludes the paper.

2. Model-based networked architecture

The model-based networked setup illustrated in Fig. 1 makes use of an explicit model of the plant which is added to the actuator/controller node to compute the control input based on the state of the model rather than on the plant state. The state of the model is updated when the controller receives the measured state of the plant that is sent from the sensor node every h seconds.

Consider a Model-Based Networked Control System (MB-NCS) as shown in Fig. 1. We concern ourselves with continuous time systems that are disturbed by an unknown external disturbance as shown in that figure. Assume that the disturbance is a persistent disturbance; in particular, we assume that it is a constant signal. This approach can be used in practice for piece-wise constant disturbances. In this case the output of the system is not asymptotically stable since there could be an infinite number of discrete changes on the disturbance as time goes to infinity. However, bounded stability can be obtained as it is shown in this paper.

The dynamics of the system are given by

$$\begin{aligned}\dot{x}(t) &= Ax(t) + B(u(t) + w(t)) \\ y(t) &= Cx(t)\end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, and $y(t) \in \mathbb{R}^p$ represent the state, control input, and measurable output of the system, respectively. $w(t) \in \mathbb{R}^m$ represents the unknown constant external disturbance. It is assumed that the disturbance is bounded as follows $\|w(t)\| \leq W$. The model dynamics are given by the nominal system dynamics:

$$\begin{aligned}\dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) \\ \hat{y}(t) &= C\hat{x}(t).\end{aligned} \tag{2}$$

We do not assume that the entire state vector is available for measurement but only the output $y(t)$. We implement a state observer in the controller node in order to estimate the state of the system and use the observer state to update the state of the model. Since the input disturbance $w(t)$ cannot be measured we augment the system in order to estimate both the states and the disturbance. This approach will be used in Section 4 in order to attenuate the undesired effects of the disturbance when the system is controlled using a networked architecture as shown in Fig. 1.

3. Augmented state observer

Consider a linear system given by (1). In this section we consider an augmented state observer under the assumption of continuous access to the input and output signals, $u(t)$ and $y(t)$ as in a traditional closed loop control system. This state observer will be used later in the networked implementation represented in Fig. 1. The purpose of this augmented observer is to provide estimates not only of the state of the system $x(t)$ but also to obtain estimates of the unknown disturbance $w(t)$. Define the augmented state vector:

$$x_e(t) = \begin{bmatrix} x(t)^T & w(t)^T \end{bmatrix}^T. \tag{3}$$

The augmented system is given by

$$\begin{aligned}\dot{x}_e(t) &= \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix} x_e(t) + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) = A_e x_e(t) + B_e u(t) \\ y(t) &= \begin{bmatrix} C & 0 \end{bmatrix} x_e(t) = C_e x_e(t).\end{aligned} \tag{4}$$

A state observer is designed for the augmented system (4) and it is given by

$$\dot{\bar{x}}_e(t) = (A_e - LC_e)\bar{x}_e(t) + B_e u(t) + Ly(t). \tag{5}$$

Theorem 1. *Assume that the continuous time pair (A, C) is observable, then (A_e, C_e) is observable if $\mathcal{N}(M) = \{0\}$, the nullspace of M contains only the zero vector, where*

$$M = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}. \tag{6}$$

Proof. Let λ_i represent the eigenvalues of A for $i = 1, \ldots, n$, with associated eigenvectors v_i, and let λ_i^e represent the eigenvalues of A_e for $i = 1, \ldots, n+m$, with associated eigenvectors v_i^e. Note that

$$\begin{aligned}\lambda_i^e &= \lambda_i \quad \text{for } i = 1, \ldots, n. \\ \lambda_i^e &= 0 \quad \text{for } i = n+1, \ldots, n+m.\end{aligned} \tag{7}$$

For any $\lambda_i^e \neq 0$ we have that

$$(\lambda_i^e I - A_e)v_i^e = \begin{bmatrix} \lambda_i I - A & -B \\ 0 & \lambda_i I \end{bmatrix} \begin{bmatrix} v_n \\ v_m \end{bmatrix} = 0 \tag{8}$$

since $\lambda_i \neq 0$ then $v_m = 0$ and

$$\begin{aligned}&(\lambda_i I - A)v_n = 0 \Rightarrow v_n = v_i \\ &\Rightarrow v_i^e = \begin{bmatrix} v_i \\ 0 \end{bmatrix}.\end{aligned} \tag{9}$$

Additionally, assume that $\lambda_i^e \neq 0$ is an unobservable eigenvalue of (A_e, C_e), then by the PBH unobservable eigenvalue condition [16] we have that

$$\begin{bmatrix} \lambda_i^e I - A_e \\ C_e \end{bmatrix} \hat{v} = 0 \tag{10}$$

for some $\hat{v} \neq 0$. Eq. (10) can be expressed as

$$\begin{bmatrix} \lambda_i I - A & -B \\ 0 & \lambda_i I \\ C & 0 \end{bmatrix} \begin{bmatrix} \hat{v}_n \\ \hat{v}_m \end{bmatrix} = 0 \tag{11}$$