Systems & Control Letters 75 (2015) 1–7

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Design of stabilizing controllers of upper triangular nonlinear time-delay systems[☆]

Zhishan Liang [a], Qingrong Liu [a,b,*]

[a] College of Geophysics and Information Engineering, China University of Petroleum-Beijing, Beijing, 102249, China

[b] School of Mathematic and Quantitative Economics, Shandong University of Finance and Economics, Jinan, 250014, China

ARTICLE INFO

Article history:
Received 25 May 2013
Received in revised form
23 April 2014
Accepted 29 October 2014
Available online 19 November 2014

Keywords:
Upper triangular systems
Time-delay systems
Lyapunov method
Dynamic gain controllers

ABSTRACT

In this paper, it is considered the state feedback controller design for a class of upper triangular nonlinear systems with simultaneous input and state delays. By using the state transformation of nonlinear systems, the problem of designing controller can be converted into that of designing a dynamic parameter, which is dynamically regulated by a dynamic equation. Then, by appraising the nonlinear terms of the given systems, a dynamic equation can be delicately constructed. At last, with the help of Lyapunov stability theorem, it is provided the stability analysis for the closed-loop system consisting of the designed controller and the given systems. Both discrete delays and continuous delays with integral form are considered here. Different from many existing control designs for upper triangular nonlinear systems, neither forwarding recursive nor saturation computation is utilized here, and thus our design procedure is simpler. A simulation example is given to demonstrate the effectiveness of the proposed design procedure.

1. Introduction

In real control systems, input delays are often encountered because of transmission of the measurement information. The existence of these delays may be the source of instability or serious deterioration in the performance of the closed-loop system (see [1,2], and references therein). Hence, the control design problem of input-delayed systems has attracted considerable attention in the recent decades (see, for example, [3–5]).

Combined the saturation design with the recursive design method, the authors in [3] solved the problem of the global stabilization by bounded feedback of a chain of integrators with a constant delay in the input. The authors in [4] considered a stabilization problem of a chain of integrators which allows a time-varying delay in the input. The authors in [5] considered a stabilization problem of a chain of integrators with an unknown delay in the input.

Over the last decade, there has been constant progress on the problem of global stabilization of the triangular nonlinear systems with a chain of integrators as their special case. One class of triangular nonlinear systems are upper triangular nonlinear systems, which are also called feedforward nonlinear systems. Many applications of upper triangular nonlinear systems have been also reported (see, e.g., [6,7]). Generally speaking, it is more difficult to study upper triangular nonlinear systems. Based on the integrator forwarding method, which is a powerful tool to design the stabilizing controllers for the upper triangular systems, the problem of the asymptotic stabilization by state feedback of upper triangular systems in the absence of delay has been studied by many researchers (see, [8,9], and references therein).

For the upper triangular time-delay systems, some interesting results have been proposed recently.

For upper triangular linear systems with discrete delays simultaneous in input and state, the stabilizing controller was provided in [10] when the delays were known, and in [11] when the delays were unknown, respectively.

For upper triangular nonlinear systems, most of existing results dealt with systems with a delay in the input (see, for example, [12–16], and references therein). Specifically, under the condition that the nonlinear terms satisfying quadratic growth condition, in [12], it was solved the problem of stabilizing a family of upper triangular systems with a delay in the input; in [13], it was considered the stabilization problem of an input-delayed chain of integrators with polynomial nonlinearity; in [14], it was proposed a design

☆ This work was supported by the National Natural Science Foundation of China under Grant 61174208.

* Corresponding author at: College of Geophysics and Information Engineering, China University of Petroleum-Beijing, Beijing, 102249, China.
E-mail address: liuqingrong18@126.com (Q. Liu).

http://dx.doi.org/10.1016/j.sysconle.2014.10.011
0167-6911/

scheme of a feedback controller for a class of input-delayed nonlinear systems that are dominated by an upper triangular system satisfying the linear growth condition. In the results of [12–14], the involved nonlinearities must satisfy some growth conditions with constant gains. Recently, the predictor-based feedback law (see [15]) and the sampled-data feedback law (see [16]) were proposed for controlling upper triangular nonlinear systems with a delay in the input.

For a class of upper triangular nonlinear system with discrete delays in the state, the nested saturation feedback law was proposed in [17]. For a class of high order upper triangular nonlinear systems with discrete delays in the state, the problem of stabilization by state feedback was investigated in [18].

In this paper, the globally asymptotically stabilizing controller will be designed for a class of upper triangular nonlinear time-delay systems. The contributions of the work can be characterized by the following novel features: (i) the involved nonlinear terms can admit a function incremental rate depending on the delayed input; (ii) the delays can appear simultaneously in input and state of the given systems; (iii) the delays can be the form of discrete delays or continuous delays with integral term.

Motivated by the works of [14,19], we will construct the controllers for the given systems by the dynamic gain design method, whose design procedure can be outlined as follows. First, by choosing the appropriate state transformation of time-delay systems, the problem of controller design can be converted into that of finding a parameter $r(t)$, which is the state of a dynamic equation to be specified later. Second, the dynamic equation, with $r(t)$ as its state, can be constructed by appraising the nonlinear terms of the systems. At last, the stability analysis of the closed-loop system is conducted by using Lyapunov–Razumikhin theorem. Our controller for the time-delay systems has a feedback of the current state and the past input history.

Throughout this paper, we adopt the following notations (see [16,20]):

- The notation will be simplified whenever no confusion can arise from the context.
- R denotes the set of real numbers, R^+ the set of nonnegative real numbers, and R^n the n-dimensional real number space.
- For any matrix or the vector X, X^T denotes the transpose. I is used to represent an identity matrix of appropriate dimension.
- For a symmetric matrix $P \in R^{n\times n}$, $\bar{\lambda}(P)$ denotes the largest and $\underline{\lambda}(P)$ the smallest eigenvalue of P, respectively.
- $C^0([-d,0],R^n)$ means the set of continuous functions from $[-d,0]$ to R^n for $d>0$, and $C^0([-d,0],R^n)=R^n$ for $d=0$.
- Let $z \in C^0([a-\tau,b];R^n)$, $v \in C^0([a-\tau,b];R)$, where $b>a\geq 0$ and $\tau\geq 0$. Then, $T_\tau(t)z$ denotes the "history" of z from $t-\tau$ to t, i.e., $(T_\tau(t)z)(\theta) := z(t+\theta)$, $\theta\in[-\tau,0]$, for $t\in[a,b)$; $\|z\|$ denotes the Euclidean norm of the real vector function $z(t)$; and $\|z\|_\tau = \sup_{-\tau\leq\theta\leq 0}\|z(t+\theta)\|$. Similarly, $(T_\tau(t)v)(\theta) := v(t+\theta)$, $\theta\in[-\tau,0]$, for $t\in[a,b)$; $|v|$ denotes the absolute value of the real scalar function $v(t)$; and $|v|_\tau = \sup_{-\tau\leq\theta\leq 0}|v(t+\theta)|$.

2. System modeling and control objective

2.1. System modeling

Consider the nonlinear system described by the equations:

$$\begin{cases} \dot{x}_i = x_{i+1}\left(t-d_{i,1}\right) + f_i\left(t, T_{d_{i,2}}(t)x, T_{d_{i,3}}(t)u\right), \\ \quad i=1,2,\ldots,n-1 \\ \dot{x}_n = u\left(t-d_{n,1}\right), \end{cases} \tag{1}$$

where $x=(x_1,x_2,\ldots,x_n)^{\mathrm{T}} \in R^n$ is the state, $u\in R$ is the input, $d_{i,j}$, $i=1,2,\ldots,n-1$, $j=1,2,3$, are unknown time-varying delays and satisfy $0\leq d_{i,j}\leq d$, with d being a known constant, non-negative real number $d_{n,1}$ is a known constant delay, $\bar{d}=\max\{d,d_{n,1}\}$, uncertain nonlinear functions $f_i(\cdot): R^+\times C^0([-d_{i,2},0];R^n)\times C^0([-d_{i,3},0];R)\to R$, $i=1,2,\ldots,n-1$, are unknown continuous functions with respect to their arguments.

2.2. Control objective

For system (1), the objective of this paper is to find a state feedback controller of the form:

$$\begin{cases} u = F_2\left(r, x, \displaystyle\int_{t-d_{n,1}}^{t} u(s)ds\right), \\ \dot{r} = F_1\left(r, x, \displaystyle\int_{t-d_{n,1}}^{t} u(s)ds\right), \quad r(0)=r_0\geq 1 \end{cases} \tag{2}$$

such that

(i) All solutions of the closed-loop system consisting of (1) and (2) are bounded on $[0,+\infty)$.
(ii) For any initial condition $x(t)=x_0(t)\in C^0([-\bar{d},0],\ R^n)$, $u(t)=u_0(t)\in C^0([-\bar{d},0],R)$, $r(0)=r_0\geq 1$, the state x of the closed-loop system consisting of (1) and (2) converges to zero.

Next, we introduce a lemma that will serve as the basis for the development of our novel state feedback controller for system (1).

Lemma 1 (*See [19]*). *Let $D\in R^{n\times n}$, $G_0\in R^{n\times n}$ and $F\in R^{n\times 1}$ be the matrices as:*

$$D=\mathrm{diag}[n,n-1,\ldots,1],$$

$$G_0=\begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}, \qquad F=\begin{pmatrix} 0\\0\\ \vdots \\ 0\\1\end{pmatrix}.$$

Then, there exist a positive constant α, a positive definite matrix P and a row vector $K_a=(a_1,a_2,\ldots,a_n)$, satisfying:

$$PA+A^{\mathrm{T}}P\leq -I, \qquad PD+DP\geq \alpha I \tag{3}$$

where $A=G_0-FK_a$.

3. State feedback controller design

We give an assumption first, and then based on the assumption, a stabilizing controller of system (1) is proposed.

Assumption 1. For any $(t,x,u)\in R^+\times C^0([0,+\infty);R^n)\times C^0([0,+\infty);R)$, the following inequalities hold:

$$\begin{aligned} &\left|f_i\left(t, T_{d_{i,2}}(t)x, T_{d_{i,3}}(t)u\right)\right| \\ &\quad\leq \rho_i(|u|_{d_{i,3}})\left(\left(\sum_{j=i+2}^{n+1}|x_j|_{d_{i,2}}\right)+|u|_{d_{i,3}}\right), \\ &\quad i=1,2,\ldots,n-1 \end{aligned} \tag{4}$$

where $|x_{n+1}|_{d_{i,2}}=0$, $\rho_i(\cdot)$, $i=1,2,\ldots,n-1$ are known non-decreasing continuous functions.

Remark 1. From the form of the right hand of the sign of inequality in (4), one can see that system (1) satisfying Assumption 1 is indeed a upper triangular nonlinear system (see [8,12]). It should be pointed that system (1) satisfying Assumption 1 can include a wide variety of nonlinear systems.