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Adaptive control design under structured model information limitation: A cost-biased maximum-likelihood approach[☆]

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ABSTRACT

Networked control strategies based on limited information about the plant model usually result in worse closed-loop performance than optimal centralized control with full plant model information. Recently, this fact has been established by utilizing the concept of competitive ratio, which is defined as the worst-case ratio of the cost of a control design with limited model information to the cost of the optimal control design with full model information. We show that an adaptive controller, inspired by a controller proposed by Campi and Kumar, with limited plant model information, asymptotically achieves the closed-loop performance of the optimal centralized controller with full model information for almost any plant. Therefore, there exists, at least, one adaptive control design strategy with limited plant model information that can achieve a competitive ratio equal to one. The plant model considered in the paper belongs to a compact set of stochastic linear time-invariant systems and the closed-loop performance measure is the ergodic mean of a quadratic function of the state and control input.

1. Introduction

Networked control systems are often complex large-scale engineered systems, such as power grids [1], smart infrastructures [2], intelligent transportation systems [3–5], or future aerospace systems [6,7]. These systems consist of several subsystems each one often having many unknown parameters. It is costly, or even unrealistic, to accurately identify all these plant model parameters offline. This fact motivates us to focus on optimal control design under structured parameter uncertainty and limited plant model information constraints.

There are some recent studies in optimal control design with limited plant model information [8–12]. The problem was initially addressed in [8] for designing static centralized controllers for a class of discrete-time linear time-invariant systems composed of scalar subsystem, where control strategies with various degrees of model information were compared using the competitive ratio, i.e., the worst-case ratio of the cost of a control design with limited model information scaled by the cost of the optimal control design with full model information. The result was generalized to the static decentralized controllers for a class of systems composed of fully-actuated subsystems of arbitrary order in [9]. More recently, the problem of designing optimal H_2 dynamic controllers using limited plant model information was considered in [10]. It was shown that, when relying on local model information, the smallest competitive ratio achievable for any control design strategy for distributed linear time-invariant controllers is strictly greater than one; specifically, equal to the square root of two when the B-matrix was assumed to be the identity matrix.

In this paper, we generalize the set of applicable controllers to include adaptive controllers. We use the ergodic mean of a quadratic function of the state and control as a performance measure of the closed-loop system. Choosing this closed-loop performance measure allows us to use certain adaptive algorithms available in the literature [13–16]. In particular, we consider an adaptive controller proposed by Campi and Kumar [13], which uses a cost-biased (i.e., regularized) maximum-likelihood estimator for learning the unknown parts of the model matrices. We prove that this adaptive control design achieves a competitive ratio equal to one and, hence, the smallest competitive ratio that a control design strategy using adaptive controllers can achieve is equal to one (since this ratio is always lower-bounded by one). This is contrary to control design strategies that construct linear time-invariant control laws [8–12]. This shows that, although the design of each subcontroller is only relying on local model information,

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the closed-loop performance can still be as good as the optimal control design strategy with full model information (in the limit).

The rest of the paper is organized as follows. In Section 2, we present the mathematical problem formulation. In Section 3, we introduce the Campi–Kumar adaptive controller using only local model information and show that it achieves a competitive ratio equal to one. We use this adaptive algorithm on a vehicle platooning problem in Section 4 and conclude the paper in Section 5.

1.1. Notation

The sets of natural and real numbers are denoted by $\mathbb{N}$ and $\mathbb{R}$, respectively. Let $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. Additionally, all other sets are denoted by calligraphic letters such as $\mathcal{P}$.

Matrices are denoted by capital Roman letters such as A. The entry in the ith row and the jth column of matrix A is a_{ij}. Moreover, A_{ij} denotes a submatrix of matrix A, the dimension and the position of which will be defined in the text. $A > (\geq)0$ means that symmetric matrix $A \in \mathbb{R}^{n\times n}$ is positive definite (positive semidefinite) and $A > (\geq)B$ means $A - B > (\geq)0$. Let $\mathcal{S}^n_{++}$ ($\mathcal{S}^n_{+}$) be the set of positive definite (positive semidefinite) matrices in $\mathbb{R}^{n\times n}$.

Let matrices $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}, Q \in \mathcal{S}^n_{+}$, and $R \in \mathcal{S}^m_{++}$ be given such that the pair (A, B) is stabilizable and the pair $(A, Q^{1/2})$ is detectable. We define $\mathbf{X}(A, B, Q, R)$ as the unique positive definite solution of

$$X = A^\top XA - A^\top XB\left(B^\top XB + R\right)^{-1} B^\top XA + Q.$$

In addition, we define

$$\mathbf{L}(A, B, Q, R) = -\left(B^\top \mathbf{X}(A, B, Q, R)B + R\right)^{-1} B^\top \mathbf{X}(A, B, Q, R)A.$$

When Q and R are not relevant or can be deduced from the text, we use $\mathbf{X}(A, B)$ and $\mathbf{L}(A, B)$ instead of $\mathbf{X}(A, B, Q, R)$ and $\mathbf{L}(A, B, Q, R)$, respectively.

All graphs G considered in this paper are directed with vertex set $\{1, \ldots, N\}$ for a given $N \in \mathbb{N}$. The adjacency matrix $S \in \{0, 1\}^{N\times N}$ of G is a matrix whose entry $s_{ij} = 1$ if $(j, i) \in E$ and $s_{ij} = 0$ otherwise for all $1 \leq i, j \leq N$.

A measurable function $f : \mathcal{Z} \to \mathbb{R}$ is said to be essentially bounded if there exists a constant $c \in \mathbb{R}$ such that $|f(z)| \leq c$ almost everywhere. The greatest lower bound of these constants is called the essential supremum of $f(z)$, which is denoted by $\text{ess sup}_{z\in\mathcal{Z}} f(z)$. Let mappings $f, g : \mathbb{Z} \to \mathbb{R}$ be given. Denote $f(k) = O(g(k))$ and $f(k) = o(g(k))$, respectively, if $\limsup_{k\to\infty} |f(k)/g(k)| < \infty$ and $\limsup_{k\to\infty} |f(k)/g(k)| = 0$. Finally, $\chi(\cdot)$ denotes the characteristic function, i.e., it gives a value equal to one if its statement is satisfied and a value equal to zero otherwise.

2. Problem formulation

2.1. Plant model

Consider a discrete-time linear time-invariant dynamical system composed of N subsystems, such that the state-space representation of subsystems i, $1 \leq i \leq N$, is

$$x_i(k+1) = \sum_{j=1}^{N} [A_{ij}x_j(k) + B_{ij}u_j(k)] + w_i(k); \quad x_i(0) = 0,$$

where $x_i(k) \in \mathbb{R}^{n_i}$, $u_i(k) \in \mathbb{R}^{m_i}$, and $w_i(k) \in \mathbb{R}^{n_i}$ are state, control input, and exogenous input vectors, respectively. We assume that $\{w_i(k)\}_{k=0}^{\infty}$ are independent and identically distributed Gaussian random variables with zero means $\mathbb{E}\{w_i(k)\} = 0$ and unit covariances $\mathbb{E}\{w_i(k)w_i(k)^\top\} = I$. The assumption of unit covariance is without loss of generality and is only introduced to simplify the presentation. To show this, assume that $\mathbb{E}\{w_i(k)w_i(k)^\top\} = H_i \in \mathcal{S}^{n_i}_{++}$ for all $1 \leq i \leq N$. Now, using the change of variables $\bar{x}_i(k) = H_i^{-1/2}x_i(k)$ and $\bar{w}_i(k) = H_i^{-1/2}w_i(k)$ for all $1 \leq i \leq N$, we get

$$\bar{x}_i(k+1) = \sum_{j=1}^{N} [\bar{A}_{ij}\bar{x}_j(k) + \bar{B}_{ij}u_j(k)] + \bar{w}_i(k),$$

in which $\bar{A}_{ij} = H_i^{-1/2}A_{ij}H_j^{1/2}$ and $\bar{B}_{ij} = H_i^{-1/2}B_{ij}$ for all $1 \leq i, j \leq N$. This gives $\mathbb{E}\{\bar{w}_i(k)\bar{w}_i(k)^\top\} = I$. In addition, let $w_i(k)$ and $w_j(k)$ be statistically independent for all $1 \leq i \neq j \leq N$. Note that this assumption is often justified by the fact that in many large-scale systems, such as smart grids, the subsystems are scattered geographically and, hence, the sources of their disturbances are independent. We introduce the augmented system as

$$x(k+1) = Ax(k) + Bu(k) + w(k); \quad x(0) = 0,$$

where the augmented state, control input, and exogenous input vectors are

$$x(k) = [x_1(k)^\top \cdots x_N(k)^\top]^\top \in \mathbb{R}^n,$$
$$u(k) = [u_1(k)^\top \cdots u_N(k)^\top]^\top \in \mathbb{R}^m,$$
$$w(k) = [w_1(k)^\top \cdots w_N(k)^\top]^\top \in \mathbb{R}^n,$$

with $n = \sum_{i=1}^{N} n_i$ and $m = \sum_{i=1}^{N} m_i$. In addition, the augmented model matrices are

$$A = \begin{bmatrix} A_{11} & \cdots & A_{1N} \\ \vdots & \ddots & \vdots \\ A_{N1} & \cdots & A_{NN} \end{bmatrix} \in \mathcal{A} \subset \mathbb{R}^{n\times n},$$

$$B = \begin{bmatrix} B_{11} & \cdots & B_{1N} \\ \vdots & \ddots & \vdots \\ B_{N1} & \cdots & B_{NN} \end{bmatrix} \in \mathcal{B} \subset \mathbb{R}^{n\times m}.$$

Let a directed plant graph $G_{\mathcal{P}}$ with its associated adjacency matrix $S^{\mathcal{P}}$ be given. The plant graph $G_{\mathcal{P}}$ captures the interconnection structure of the plants, that is, $A_{ij} \neq 0$ only if $s^{\mathcal{P}}_{ij} \neq 0$. Hence, the sets $\mathcal{A}$ and $\mathcal{B}$ are structured by the plant graph:

$$\mathcal{A} \subseteq \bar{\mathcal{A}} = \{A \in \mathbb{R}^{n\times n} \mid s^{\mathcal{P}}_{ij} = 0 \Rightarrow A_{ij} = 0 \in \mathbb{R}^{n_i\times n_j} \text{ for all } i, j \text{ such that } 1 \leq i, j \leq N\},$$

$$\mathcal{B} \subseteq \bar{\mathcal{B}} = \{B \in \mathbb{R}^{n\times m} \mid s^{\mathcal{P}}_{ij} = 0 \Rightarrow B_{ij} = 0 \in \mathbb{R}^{n_i\times m_j} \text{ for all } i, j \text{ such that } 1 \leq i, j \leq N\}.$$

From now on, we present a plant with its pair of corresponding model matrices as $P = (A, B)$ and define $\mathcal{P} = \mathcal{A} \times \mathcal{B}$ as the set of all possible plants. We make the following assumption on the set of all plants:

Assumption 1. The set $\mathcal{A} \times \mathcal{B}$ is a compact set (with nonzero Lebesgue measure in the space $\bar{\mathcal{A}} \times \bar{\mathcal{B}}$) and the pair (A, B) is controllable for almost all $(A, B) \in \mathcal{A} \times \mathcal{B}$.

The assumption that the pair (A, B) is controllable for almost all $(A, B) \in \mathcal{A} \times \mathcal{B}$ is guaranteed if and only if the family of systems is structurally controllable [17,18].

2.2. Adaptive controller

We consider (possibly) infinite-dimensional nonlinear controllers $\mathbf{K}_i = (\mathbf{K}_i^{(k)})_{k\in\mathbb{N}_0}$ for each subsystem i, $1 \leq i \leq N$, with control law

$$u_i(k) = \mathbf{K}_i^{(k)}(\{x(t)\}_{t=0}^{k} \cup \{u(t)\}_{t=0}^{k-1}), \quad \forall\, k \in \mathbb{N}_0,$$

where $\mathbf{K}_i^{(k)} : \prod_{i=1}^{k} \mathbb{R}^n \times \prod_{i=1}^{k-1} \mathbb{R}^m \to \mathbb{R}^{m_i}$ is the feedback control law employed at time $k \in \mathbb{N}_0$. Let $\mathcal{K}_i$ denote the set of all such control laws. We also define $\mathcal{K} = \prod_{i=1}^{N} \mathcal{K}_i$ as the set of all admissible controllers.