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Distributed extremum seeking and formation control for nonholonomic mobile network☆

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ABSTRACT

In this paper, an integrated control and optimization problem is studied in the context of formation and coverage of a cluster of nonholonomic mobile robots. In particular, each communication channel is modeled by its outage probability, and hence, connectivity is maintained if the outage probability is less than a certain threshold. The objective of the communication network is to not only maintain resilient communication quality but also extend the network coverage. An information theory based performance index is defined to quantify this control objective. Unlike most of the existing results, the proposed cooperative control design does not assume the knowledge of any gradient (of the performance index). Rather, a distributed extremum seeking algorithm is designed to optimize the connectivity and coverage of the mobile network. The proposed approach retains all the advantages of cooperative control, and it can not only perform extremum seeking individually, but also ensures a consensus of estimates between any pair of connected systems. Simulation results demonstrate effectiveness of the proposed methodology.

1. Introduction

Formation control of multi-agent systems, a distributive strategy dedicated to driving a group of networked systems to the desired position or distances [1], has received significant amount of interests from researchers worldwide in the past decade, leading to breakthroughs in various fields of applications. However, it should be pointed out that existing results on this topic do not address or fully exploit the implications of communication quality to formation control strategies. In this paper, we propose a distributive strategy that integrates formation control with a communication performance metric (which captures the trade off between network coverage and communication quality). As is well known, such integration leads to an inherent dilemma. Specifically, communication quality and network connectivity favor agents/vehicles that are close and are moving closer, while network coverage demands them to stay separated and even further apart. In other words, each agent needs to make its decision, preferably distributively, to balance communication quality, connectivity, and network coverage such that overall performance can be optimized.

In general, formation control can be categorized further into position based formation and distance based formation [2], the former is devoted to maintaining a specific relative geometric configuration [1,3], while the later considers only the relative distance and bearing information [4] and is studied in the context of graph rigidity [5]. In this regard, coverage control [6] or flocking [7,8] can also be treated as special cases of distance-based formation control, and the majority of existing results use either a potential field function or its variations to achieve a desired formation as well as to avoid collision [9]. However, singular configuration (i.e., local minimum) is inevitable in any application of these approaches [8], and common solutions to this problem employ some mild assumptions on either velocity [10] or explicit knowledge of the leader [7]. In addition, formation control of nonholonomic systems has also received considerable attentions. Specifically, I/O linearization for formation control of nonholonomic robots under a digraph is introduced in [4], and a synchronization approach is proposed in [3] to study time varying formation control of mobile robots. Other notable contributions include [11–13], where a hierarchical (three layer) control conjecture is presented in [11] for coordination of mobile robots, and formation control and trajectory following of

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unicycles using saturated control scheme is studied in [12], while a distributed virtual structure approach is proposed in [13], all of which show global stability.

Mobile platforms are often equipped with wireless communication capabilities to provide and maintain connectivity of the communication network. To analyze this, modeling of communication channel quality is expected. For instance, metrics of communication quality such as SNR (i.e., signal-to-noise ratio) or Shannon capacity [14] are used for online measurements so that the current formation configuration (or relative distances) can be evaluated. As such, formation control can be accomplished by using communication quality as feedback instead of position information. Moreover, as shown in [15], the quality of a wireless communication link in a vehicular ad hoc network can be estimated by examining received data packets. In [16], motion control of networked robotic routers is investigated to maintain connectivity of a single user to a base station, which could be either stationary or adversarial. Recent advances on this topic include motion planning and gradient-based control of a robotic sensing network [17] to improve communication quality, optimization of SISO (i.e., single-input–single-output) communication chain under the assumption that gradient of SNR field is known [18], an online planing method is introduced in [19] to find a navigation path to meet network connectivity and bandwidth requirement, and an opportunistic communication strategy with energy constraint can be found in [20]. One closely related result can be found in [21], where formation control of single-integrator system is investigated with the help of classical extremum seeking scheme. However, there are the following key shortcomings in most of the existing studies: absence of an analytical investigation of integrating communication and control issues in nonholonomic mobile network, requirement of online extremum seeking algorithm with the knowledge of gradients, and possible inconsistency of an extremum seeking scheme in multi-agent scenario.

In this paper, a distance based formation control scheme is introduced to separate each pair of connected vehicles with a specified/optimized distance, which represents the aforementioned tradeoff between communication quality and network coverage. The optimal distance can be estimated using a model-free and distributed extremum seeking scheme. Note that the classical extremum seeking scheme needs to be enhanced when applied to networked control systems, where estimates between any pair of connected systems are expected to be consistent. If consistency is not ensured, no stable formation can be achieved. The contribution of this paper is twofold: (i) a new distance based formation strategy is proposed for nonholonomic robots that admits a communication performance metric; (ii) a new distributive extremum seeking scheme is designed to not only estimate the desired separation with acceptable accuracy but also ensure a consensus among estimates.

2. Preliminaries

2.1. Graph theory

In this paper, we consider an undirected graph $\mathscr{G} = (V, E)$, where $V = \{1, 2, \ldots, n\}$ and E denote the sets of vertices/nodes and edges, respectively. Unless otherwise specified, vertex j is said to be adjacent/connected to vertex i if $(j, i) \in E$, or equivalently $(i, j) \in E$. Analogously, neighborhood set $\mathscr{N}_i \subseteq V$ of vertex i is $\{k \in V \mid (k, i) \in E\}$, the set of all vertices that are adjacent to vertex i. If $j \in \mathscr{N}_i$, then $i \in \mathscr{N}_j$ holds as well.

Without loss of any generality, adjacency matrix $A(\mathscr{G})$ considered in this paper is weighted and normalized as

$$[A(\mathscr{G})]_{ik} = \begin{cases} a_{ik} > 0 & \text{if } (k, i) \in E \\ 1 - \sum_{k \neq i} a_{ik} > 0 & \text{if } k = i \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

That is, $A(\mathscr{G})$ is designed to be nonnegative, row-stochastic, and positive semi-definite. Furthermore, all its nonzero and hence positive weighting factors are both uniformly lower and upper bounded, i.e., $a_{ij} \in [\underline{a}\ 1]$, where $0 < \underline{a} < 1$, $\forall j \in \mathscr{N}_i$. Moreover, $\mathscr{G}$ is connected if its corresponding $A(\mathscr{G})$ is irreducible [1]. In addition, matrix $S(\mathscr{G})$ is defined as the connectivity matrix for graph $\mathscr{G}$, that is

$$[S(\mathscr{G})]_{ij} = \begin{cases} 1 & \text{if } (i, j) \in E \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

In order to accommodate the switching topologies, we define the time sequences $\{t_k : k \in \aleph\}$ for $\aleph = \{0, 1, \ldots, \infty\}$, and $\mathscr{G}$ is assumed to be time invariant during each of the intervals in the form of $[t_k,\ t_{k+1})$, and its corresponding adjacency matrix $\mathscr{A}$ is piecewise continuous as a result. That is, $\mathscr{G}(t_k) = \mathscr{G}(t_{k+1}^-)$. Before proceeding further, the following assumption is stated regarding the connectivity and switching of the underlying network.

Assumption 1. $\mathscr{G}$ is initially connected at time t_0 in the sense that there is an undirected link between nodes i and j for all $i \neq j$ in $\mathscr{G}$.

2.2. Outage probability of SISO communication

In a mobile ad hoc network, communication quality of service (measured at the physical layer by packet error rate, or outage rate between a transmitter and a receiver) depends on many unknown parameters beyond their relative position such as multipath fading, shadowing, noise, and interfering. In particular, the Shannon–Hartley law provides the relationship between distance and communication quality when combined with the empirical radio propagation model [14]. In what follows, outage probability is introduced to quantitatively characterize the implications of distance r_{ij} to data rate δ, whose value is commonly used as the criterion to determine whether a viable communication channel is plausible between any given pair of mobile nodes. Specifically, for a SISO communication link [22],

$$P\left[C_{SISO} < \delta\right] = 1 - \exp\left(-(2^{\delta} - 1)\frac{\sigma^2}{P_0}\left(\frac{r_{ij}}{r_0}\right)^{\nu}\right), \tag{3}$$

where C_{SISO} is the Shannon capacity, P_0 is the transmitting power, σ is the noise variance, P_0/σ^2 denotes SNR (i.e., signal–noise-ratio) at a reference distance, ν is the path loss exponent, r_0 is the reference distance from the transmitter to the receiver, and r_{ij} is the effective distance.

The outage probability in (3) has the intuitive behavior that, as r_{ij} grows, $P\left[C_{SISO} < \delta\right] \to 1$ and, as $r_{ij} \to 0$, $P[C_{SISO} < \delta] \to 0$, which is shown in Fig. 1. As such, in order to establish a viable communication channel, outage probability should always be less than a certain threshold ζ. That is,

$$1 - \exp\left(-(2^{\delta} - 1)\frac{\sigma^2}{P_0}\left(\frac{r_{ij}}{r_0}\right)^{\nu}\right) \leq \frac{\zeta}{100}. \tag{4}$$

Solving (4) for r_{ij} yields the maximum distance between transmitter and receiver that will give the worst outage probability of ζ%, which is referred to as r_x. In other words, any communication attempt beyond distance r_x will be too corrupted to be considered effective, due to the signal drop rate and data outage. Taking Fig. 1 for instance, $r_x = 68$, $r_x = 105$, and $r_x = 150$ correspond to outage probability being 9% (point A), 30% (point B), and 60% (point C), respectively.

3. Problem formulation

Consider a group of n nonholonomic mobile robots performing formation mission in a plane. For each robot $i \in \{1, 2, \ldots, n\}$, we