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Global inverse optimal stabilization of stochastic nonholonomic systems

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ABSTRACT

Optimality has not been addressed in existing works on control of (stochastic) nonholonomic systems. This paper presents a design of optimal controllers with respect to a meaningful cost function to globally asymptotically stabilize (in probability) nonholonomic systems affine in stochastic disturbances. The design is based on the Lyapunov direct method, the backstepping technique, and the inverse optimal control design. A class of Lyapunov functions, which are not required to be as nonlinearly strong as quadratic or quartic, is proposed for the control design. Thus, these Lyapunov functions can be applied to design of controllers for underactuated (stochastic) mechanical systems, which are usually required Lyapunov functions of a nonlinearly weak form. The proposed control design is illustrated on a kinematic cart, of which wheel velocities are perturbed by stochastic noise.
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1. Introduction

This paper presents a design of optimal controllers with respect to a meaningful cost function for global asymptotic stabilization in probability of the following stochastic nonholonomic system

$$\begin{aligned} dx_0 &= u_0 dt + \boldsymbol{\varphi}_0^T(x_0) d\boldsymbol{w}, \\ dx_i &= x_{i+1} u_0 dt + \boldsymbol{\varphi}_i^T(x_0, u_0, \bar{\boldsymbol{x}}_i) d\boldsymbol{w}, \quad 1 \le i \le n-1 \\ dx_n &= u_1 dt + \boldsymbol{\varphi}_n^T(x_0, u_0, \boldsymbol{x}) d\boldsymbol{w}, \end{aligned} \tag{1}$$

where u_0 and u_1 are controls, x_0 and $\boldsymbol{x} = \text{col}(x_1, \ldots, x_n)$ are system states, $\bar{\boldsymbol{x}}_i = \text{col}(x_1, \ldots, x_i)$, $\boldsymbol{w}$ is an independent r-dimensional standard Wiener process, and $\boldsymbol{\varphi}_0(\bullet)$ and $\boldsymbol{\varphi}_i(\bullet)$ are r-vector valued smooth functions satisfying the following assumption:

Assumption 1.1. The vector valued smooth functions $\boldsymbol{\varphi}_0(x_0)$, $\boldsymbol{\varphi}_i(x_0, u_0, \bar{\boldsymbol{x}}_i)$, and $\boldsymbol{\varphi}_n(x_0, u_0, \boldsymbol{x})$ vanish at the origin.

The above assumption implies that the origin is the equilibrium point of the system (1) and is imposed to guarantee controllability of the $\boldsymbol{x}$-subsystem, i.e., the last two equations of (1), in the limit when $x_0 \to 0$ as $t \to \infty$. For clarity, the system (1) does not include nonlinear deterministic functions and unknown noise covariance. Including these terms does not add contributions but increases complexity of presentation because if there are deterministic functions (either containing unknown parameters or not) and/or unknown noise covariance, it is rather straightforward to combine the control design proposed in this paper together with techniques in [1,2] to deal with these functions containing both linear and nonlinear appearance of unknown parameters and noise covariance.

Let us consider the following kinematic cart that motivates the study of the stochastic nonholonomic system (1).

Example 1.1. The kinematic cart, see Fig. 1, is described by [3]:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \frac{s}{4} \begin{bmatrix} \cos(\phi) & \cos(\phi) \\ \sin(\phi) & \sin(\phi) \\ \dfrac{2}{b} & -\dfrac{2}{b} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix}, \tag{2}$$

where (x, y) and ϕ denote the position and orientation of the cart, s is the diameter of the actuated wheel, b is the width, P_0 is the middle point between the two actuated wheels, and ν_1 and ν_2 are the angular velocities of the actuated wheels. We now suppose that the angular velocities ν_1 and ν_2 are subject to some stochastic disturbances, and are assumed to be expressed as [4]:

$$\nu_i = \bar{\nu}_i(x, y, \phi) + \zeta_0(x, y, \phi)\dot{w}, \quad i = 1, 2, \tag{3}$$

where $\bar{\nu}_i(x, y, \phi)$, $i = 1, 2$ are viewed as controls, $\zeta_0(x, y, \phi)$ is a function of (x, y, ϕ) and vanishes at the origin, and w is a standard

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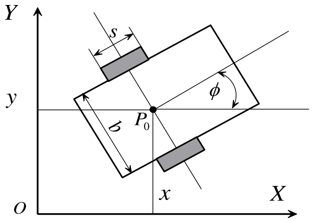

Fig. 1. Cart parameters and coordinates.

Wiener process. The following coordinate changes

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ \sin(\phi) & -\cos(\phi) & 0 \\ \cos(\phi) & \sin(\phi) & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ \phi \end{bmatrix}, \quad u_0 = \frac{s}{2b}(\bar{\upsilon}_1 - \bar{\upsilon}_2), \qquad u_1 = \frac{s}{4}(\bar{\upsilon}_1 + \bar{\upsilon}_2) + x_1 u_0 \tag{4}$$

transform the kinematic cart (2) together with (3) to

$$\begin{aligned} dx_0 &= u_0 dt, \\ dx_1 &= u_0 x_2 dt, \\ dx_2 &= u_1 dt + \frac{s}{2}\zeta_0(x_0, x_1, x_2) dw, \end{aligned} \tag{5}$$

which is a special form of the stochastic nonholonomic system (1). We will continue this example in Sections 3.3, 4.2 and 5.3.

When $d\boldsymbol{w}/dt$ is an either known or unknown constant vector, the system (1) becomes deterministic. By Brockett's condition [5], deterministic nonholonomic systems cannot be stabilized at the origin by any static continuous state feedback though they are open loop controllable. To overcome this obstacle, researchers have developed novel approaches to design asymptotic/exponential stabilizers, see for example [6–10,1,11] on the discontinuous time-invariant approach, and [12–15] on the time-varying approach.

Systems frequently contaminated by stochastic noise in practice plus development in control and stability analysis of stochastic nonlinear systems [16–18] motivate us to consider the problem of controlling stochastic nonholonomic systems. In comparison with deterministic systems, stochastic nonholonomic systems have received much less attention. This is mainly due to appearance of Hessian terms in the infinitesimal generator of a Lyapunov function if the powerful Lyapunov direct method is used for control design. The Hessian terms cause difficulties in design of control inputs to ensure that the infinitesimal generator negative definite. Moreover, nonholonomic constraints, especially the x_0-subsystem, i.e., the first equation of (1), create an obstacle in control design. By assuming that the x_0-subsystem is deterministic, there are several works on design of asymptotic stabilizers in probability for stochastic nonholonomic systems, see [19] where unknown noise covariance is considered, and [20] where nonlinear appearance of unknown parameters is treated. These works are based on the input-to-state scaling proposed in [1], and the control design techniques for high order nonholonomic systems in power chained form in [21] and nonlinear systems with nonlinear appearance of unknown parameters in [2]. When the x_0-subsystem is also stochastic, there are some results available in [22] where the results are incorrect, and in [23] where the x_0-subsystem is linear.

The controllers in all of the above works on both deterministic and stochastic nonholonomic systems are not optimal in the sense that no meaningful cost function is resulted from their control designs. The aforementioned issues motivate contributions of this paper on design of optimal control inputs u_0 and u_1 with respect to a meaningful cost function to globally asymptotically stabilize the system (1) at the origin in probability. In particular, this paper addresses the following control objective:

Control Objective 1.1. Design the control inputs $u_0 = \varpi_0(x_0)$ and $u_1 = \varpi_1(x_0, \boldsymbol{x})$ such that they guarantee global asymptotic stability in probability of the equilibrium $x_0 = 0$ and $\boldsymbol{x} = 0$ and minimize a meaningful cost functional of the form

$$J(\bar{\boldsymbol{u}}) = \mathbb{E}\left\{\int_{t_0}^{\infty}\Big(l(\bar{\boldsymbol{x}}(t)) + \sigma(\|\boldsymbol{R}^{1/2}(\bar{\boldsymbol{x}}(t))\bar{\boldsymbol{u}}(t)\|)\Big)dt\right\}, \tag{6}$$

where $\bar{\boldsymbol{u}} = \mathrm{col}(u_0, u_1)$, $\bar{\boldsymbol{x}} = \mathrm{col}(x_0, \boldsymbol{x})$, $l(\bullet)$ is a positive definite radially unbounded function, $\sigma(\bullet)$ is a class $\mathcal{K}_\infty$ function such that its derivative with respect to $\bullet$ is also a class $\mathcal{K}_\infty$ function, and $\boldsymbol{R}(\bullet)$ is a matrix-valued function satisfying $\boldsymbol{R}(\bullet) = \boldsymbol{R}^T(\bullet) > 0$.

2. Preliminaries

2.1. Legendre–Fenchel transform

Lemma 2.1 (*Krstic and Li [24]*). *Let $\ell\sigma(\chi)$ denote the Legendre–Fenchel transform defined by*

$$\ell\sigma(\chi) = \chi(\sigma^\star)^{-1}(\chi) - \sigma((\sigma^\star)^{-1}(\chi)), \tag{7}$$

where $\sigma : \mathbb{R} \to \mathbb{R}$ is a class $\mathcal{K}_\infty$ function whose derivative $\sigma^\star(\chi) = \frac{d\sigma(\chi)}{d\chi}$ is also a class $\mathcal{K}_\infty$ function, and $(\sigma^\star)^{-1}(\chi)$ denotes the inverse function of $\sigma^\star(\chi)$. The Legendre–Fenchel transform $\ell\sigma(\chi)$ has the following properties

(1) $\ell\sigma(\chi) = \int_0^\chi (\sigma^\star)^{-1}(s)ds$;
(2) $\ell\ell\sigma(\chi) = \sigma(\chi)$;
(3) $\ell\sigma(\chi)$ *is a class* $\mathcal{K}_\infty$ *function;*
(4) $\ell\sigma(\sigma^\star(\chi)) = \chi\sigma^\star(\chi) - \sigma(\chi)$.

2.2. Young's inequality

For $(x, y) \in \mathbb{R}^2$, the following Young inequality holds [25]:

$$xy \le \frac{\epsilon^p}{p}|x|^p + \frac{1}{q\epsilon^q}|y|^q, \tag{8}$$

where ϵ is a positive constant, and the constants $p > 1$ and $q > 1$ satisfy $(p-1)(q-1) = 1$.

2.3. Solution of a linear time-varying stochastic system

Lemma 2.2. *Consider the scalar linear time-varying stochastic system*

$$dx = (a(t)x + b(t))dt + x\sum_{i=1}^{r} c_i(t)dw_i, \tag{9}$$

where $a(t)$, $b(t)$ and $c_i(t)$ are real-valued Borel measurable bounded functions for $t \ge t_0$ and $w_i(t)$ is a standard Wiener process. Assume that the system (9) has a unique solution. Then this solution is given by

$$x(t) = \phi(t)\left(x(t_0) + \int_{t_0}^{t}\frac{1}{\phi(s)}b(s)ds\right) \tag{10}$$

where

$$\phi(t) = \exp\left[\int_{t_0}^{t}\left(\left(a(s) - \frac{1}{2}\sum_{i=1}^{r}c_i^2(s)\right)ds + \sum_{i=1}^{r}c_i(s)dw_i(s)\right)\right]. \tag{11}$$

Proof. See Appendix A.

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