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Instability of stochastic switched systems ☆

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ABSTRACT

The instability problem of stochastic switched systems is investigated in this paper. Definitions of instability are given in the forms of instability in probability, *m*th instability, moment exponential instability and almost sure exponential instability. By the aid of Dynkin's formula, Itô's formula and strong law of large numbers, the criteria on instability of stochastic switched systems under arbitrary switching are established based on Lyapunov-like techniques. Simulation examples are presented to illustrate the validity of the results.

1. Introduction

Recent years have witnessed a considerable growth of interest in dynamical systems [1] described by a family of continuous-time subsystems with discrete switching events. Such systems are referred to as switched systems or hybrid systems in the scientific literature [2] and are commonly found in many practical applications, such as in power systems [3], computer-controlled systems [4], transportation systems and control [5], communication networks [6] and many other fields. Theoretical analysis for switched systems is usually based on Lyapunov's second method, such as the existence of a common Lyapunov function for individual systems guarantees stability of the switched system for arbitrary switching sequences [7,8], the multiple-Lyapunov-function method is a useful tool for stability analysis of switched systems with some constrained switchings [9]. In general, the piecewise Lyapunov function approach [10] or the switched Lyapunov function approach [11] is employed to study the stability problem of switched systems.

The research of switched systems has attracted a great deal of attention. The most important consideration in the analysis of switched systems is their stability. Many researchers focused on stochastic switched systems with time-dependent switching which was a right-continuous and piecewise constant function. By combining the multiple Lyapunov function method with a comparison principle, stability of nonlinear switched systems was presented in [12]. The problems of stabilization and H_∞ state-feedback control of systems with state-multiplicative noise and dwell time constraint were studied in Chapter 11 of Ref. [13]. Moment stability for stochastic switched systems with stable and/or unstable subsystems was studied in [14–17]. Some authors investigated stochastic switched systems with state-dependent switching, which was trigged by the active regions of subsystems, and the switching law can be viewed as a closed-loop control strategy. Stability analysis of stochastic switched systems with state-dependent switching recently was investigated in [18,19]. Particularly, by means of designing active regions of unstable subsystems, stochastic switched systems were exponentially stable under the state-dependent switching signal in examples of [19]. It is well known that switching between unstable subsystems might stabilize the switched system, and switching between stable subsystems may result in an unstable switched system. As a complex system, the switched system is operating as a result of interaction between subsystems and the switching signal. Therefore, it is critical to find out the essential design of switching signals and/or subsystems configuration with good performance to assure stability of switched systems or to keep switched systems stable with the great external disturbance, and the switched system may become unstable without proper subsystems configuration and suitable switching strategy.

Instability is one of the basic problem of stability theory of stochastic systems. For stochastic control problems, the instability study of stochastic systems has a vital significance to judge stability of stochastic systems and explore stable regions of stochastic systems or the instability conditions for the first approximation of

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some stochastic nonlinear systems [20–22]. Roughly speaking, the sample paths of stochastic systems may leave the instability set because of purely random forces, for example, the unstable deterministic system is impaired by the addition of a small diffusion, which causes that the stochastic system obtained is asymptotically stable in the large. Consequently, the problem of instability conditions is even more complicated. The analogs of the instability theorems of Lyapunov and Chetaev do not hold for stochastic systems. The basic theory of instability of stochastic differential systems was only established earlier in [23,24]. However, little work has been done on instability of stochastic switched systems so far.

The purpose of this paper is to extend some appropriate forms of instability to stochastic switched systems. Definitions of instability in probability, mth instability, moment exponential instability and almost sure exponential instability are first presented for stochastic switched systems as preliminaries in a new background different from [23,24]. By the aid of Dynkin's formula, Itô's formula and strong law of large numbers, the criteria on instability of stochastic switched systems with arbitrary switching are established based on Lyapunov-like techniques. To establish the criterion on the instability in probability, we should firstly prove that any solution of a stochastic switched system beginning in a bounded region, almost surely reaches the boundary of this domain in a finite time. Itô's integral of a switched function should be verified to be a higher-order infinitesimal about time using strong law of large numbers, then, the almost sure exponential instability is proved in this paper.

The paper is organized as follows. Stochastic switched systems and definitions of instability are presented in Section 2; instability criteria of stochastic switched systems for arbitrary switching sequences are researched in Section 3; simulation examples are given in Section 4; the paper is concluded in Section 5.

Notations: The following notations are used throughout the paper: For a vector x, $|x|$ denotes its usual Euclidean norm and x^T denotes its transpose; the Frobenius norm of a matrix X is denoted by $|X|_F = (\mathrm{Tr}\{XX^T\})^{1/2}$, where $\mathrm{Tr}(\cdot)$ denotes the square matrix trace, i.e., the sum of all elements on the main diagonal line; $\mathbb{R}_+$ denotes the set of all nonnegative real numbers; $\mathbb{R}^n$ denotes the real n-dimensional space; $\mathbb{R}^{n\times r}$ denotes the real $n \times r$ matrix space; $\mathcal{C}^{2,1}(\mathbb{R}^n \times [t_0,\infty); \mathbb{R}_+)$ denotes the family of all nonnegative functions $V(x(t), t)$ on $\mathbb{R}^n \times [t_0,\infty)$ which are $\mathcal{C}^2$ in x and $\mathcal{C}^1$ in t; $\mathcal{C}_0^{2,1}(\mathbb{R}^n \times [t_0,\infty); \mathbb{R}_+)$ denotes the family of all nonnegative functions $V(x(t), t) \in \mathcal{C}^{2,1}(\mathbb{R}^n \times [t_0,\infty); \mathbb{R}_+)$ except possibly at the point $x = 0$.

2. Stochastic switched systems and definitions of instability

In this section, we illustrate stochastic switched systems and define several forms of stochastic instability.

Consider a family of stochastic nonlinear systems described by

$$\begin{aligned} \Sigma_p : dx(t) &= f_p(x(t), t)dt + g_p(x(t), t)dW(t), \\ x(t_0) &= x_0,\ p \in \mathcal{P}, \end{aligned} \tag{1}$$

where $\mathcal{P} = \{1, 2, \ldots, m\}$, $x(t) \in \mathbb{R}^n$ is the state of system, $W(t)$ is an r-dimensional independent standard Wiener process (or Brownian motion), and the underlying complete probability space is taken to be the quartet $(\Omega, \mathcal{F}, \mathcal{F}_t, P)$ with a filtration $\mathcal{F}_t$ satisfying the usual conditions (i.e., it is increasing and right continuous while $\mathcal{F}_0$ contains all P-null sets), functions $f_p : \mathbb{R}^n \times [t_0,\infty) \to \mathbb{R}^n$, $g_p : \mathbb{R}^n \times [t_0,\infty) \to \mathbb{R}^{n\times r}$ are locally Lipschitz in $x \in \mathbb{R}^n$ and piecewise continuous in t for all $t \geq t_0$ and $p \in \mathcal{P}$, and $f_p(0, t) = 0,\ g_p(0, t) = 0,\ \forall p \in \mathcal{P}, t \in [t_0,\infty)$.

For system family (1), a time-dependent switching signal is given based on [18] by

$$\sigma(t) = p_j, \quad t \in [\tau_{j-1}, \tau_j),\ j \in N, \tag{2}$$

where $\tau_0 = t_0, \tau_\infty = \infty$, $p_j \in \mathcal{P}$ with $p_j \neq p_{j+1}$, and N is the set of nonnegative integers. The switching signal (2) is a right-continuous and piecewise constant function. It should be noted that the switching instants $\tau_j, j = 1, 2, \ldots, n, \ldots$ with $\tau_0 = t_0$ can be stopping times or deterministic times in this paper. The sequence $\{\tau_j, j \in N\}$ is strictly increasing. When $t \in [\tau_{j-1}, \tau_j)$, the p_jth subsystem is active, and the corresponding active subsystem has a unique solution in the interval $[\tau_{j-1}, \tau_j)$.

A stochastic switched system generated by stochastic system family (1) and a switching signal (2) can be described by the equation

$$dx(t) = f_\sigma(x(t), t)dt + g_\sigma(x(t), t)dW(t), \quad x(t_0) = x_0. \tag{3}$$

It is assumed that there is no jump in the state x at the switching instants, and there is a finite number of switches on every bounded interval of time.

The trajectory of the switched system (3) is the trajectory of the p_jth subsystem at time interval $[\tau_{j-1}, \tau_j)$. By the recursive procedure [18,19], the solution of the stochastic switched system (3) is presented by

$$\begin{aligned} x(t) = x(t_0) &+ \sum_{j=1}^{\infty} \int_{t\wedge\tau_{j-1}}^{t\wedge\tau_j} f_\sigma(x(s), s)ds \\ &+ \sum_{j=1}^{\infty} \int_{t\wedge\tau_{j-1}}^{t\wedge\tau_j} g_\sigma(x(s), s)dW(s), \end{aligned} \tag{4}$$

which holds in the interval $[t_0,\infty)$. According to the switching signal (2), the solution (4) of the stochastic switched system (3) is unique and continuous in the interval $[t_0,\infty)$ and is adapted to $\mathcal{F}_t$.

Because of switching behavior between subsystems, we must give new definitions of stochastic instability of stochastic switched systems in the following.

Definition 1. The trivial solution $x(t) \equiv 0$ of stochastic switched system (3) is said to be unstable in probability or stochastically unstable, if for any $\epsilon \in (0, 1)$, $\delta = \delta(\epsilon, r, t_0) > 0$, there exist $r > 0$, and $0 < |x_0| < \delta$, such that

$$P\{|x(t, x_0, t_0)| \geq r\} \geq 1 - \epsilon, \quad \forall t \geq t_0. \tag{5}$$

Definition 2. For $m > 0$, the trivial solution $x(t) \equiv 0$ of stochastic switched system (3) is said to be mth unstable, if there exists $\epsilon = \epsilon(x_0, t_0) > 0$, such that

$$E|x(t)|^m \geq \epsilon, \quad \forall t \geq t_0,\ x_0 \in \mathbb{R}^n \setminus \{0\}. \tag{6}$$

When $m = 2$, we also say that it is unstable in mean square.

Definition 3. For $m > 0$, the trivial solution $x(t) \equiv 0$ of stochastic switched system (3) is said to be mth moment exponentially unstable, if there exist positive constants c, λ, such that

$$E|x(t)|^m \geq c|x_0|^m e^{\lambda(t-t_0)}, \quad \forall t \geq t_0,\ x_0 \in \mathbb{R}^n \setminus \{0\}. \tag{7}$$

When $m = 2$, we also say that it is exponentially unstable in mean square.

Definition 4. The trivial solution $x(t) \equiv 0$ of stochastic switched system (3) is said to be almost surely exponentially unstable, if there exists a positive constant λ, such that

$$\liminf_{t\to\infty} \frac{1}{t} \ln|x(t)| \geq \lambda, \quad \text{a.s. } \forall x_0 \in \mathbb{R}^n \setminus \{0\}. \tag{8}$$

At last, the useful strong law of large numbers is recited for the further argument.