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# Stability of nonlinear discrete repetitive processes with Markovian switching☆

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ABSTRACT

Repetitive processes are a class of 2D systems that operate over a subset of the upper-right quadrant of the 2D plane. Applications include iterative learning control where experimental verification has been reported based on a linear time-invariant model approximation of the dynamics. This paper considers discrete nonlinear repetitive processes with Markovian switching and applies, as one application, the resulting stability theory to iterative learning control for a class of networked systems where time-varying dynamics arise.



## 1. Introduction

The systems considered in this paper repeat the same finite duration operation over and over again. Each repetition is termed a pass and the duration the pass length. One industrial application is long-wall coal mining where the coal is cut by a machine that passes along the coal face and the objective is to maximize the volume of coal cut without penetrating the coal/stone interface. During each pass the machine rests on the pass profile cut during the previous pass, i.e., the height of the stone/coal interface above some datum line. Once a pass is complete, the machine is returned to the starting location and then pushed across to rest on the newly cut floor profile ready for the start of the next pass.

The geometry of the long-wall coal mining process means that the previous pass profile acts as a forcing function on the next pass profile and hence contributes to its dynamics. This interaction between successive pass profiles can result in oscillations that increase in amplitude from pass-to-pass. These oscillations are the distinguishing feature of repetitive processes [1] and they cannot be removed by standard control action. Instead these processes must be treated as systems operating over a subset of the upper-right quadrant of the 2D plane.

This paper uses the notation $y_k(t)$, $0 \leq t \leq T$, where $y$ is the scalar or vector valued variable, $k \geq 0$, is the pass number and $T < \infty$ is the pass length. Given that these process operate over the domain defined by $(k, t) \in [0, \infty] \times [0, T]$, boundary conditions need to be specified for $k = 0$ and $t = 0$, i.e, the starting, or initial, condition on each pass and the initial pass profile respectively. The detailed modeling of long-wall coal mining as a repetitive process is given in [1], which also references the original work and details the modeling of other repetitive processes, such as forms of metal rolling.

In physical examples, such as long-wall coal mining, the interaction between success pass profiles is part of the evolution of the dynamics. Of direct relevance to the focus of this paper are applications where the repetitive process structure arises from the control action applied. Consider the commonly encountered industrial task where a gantry robot is undertaking a pick and place operation over and over again and the sequence of operations is: (i) collect an object from a specified location, (ii) transfer it over a

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finite duration, (iii) deposit it at a fixed location or onto a moving conveyor, (iv) return to the starting location, and (v) repeat (i)–(iv) as many times as required or until a stop for maintenance is required. The transfer of each object can be viewed as completing a pass over the finite pass length and once complete, all information generated is available to update the control law to be applied on the next pass.

Iterative Learning Control (ILC), see the survey papers [2,3], has been developed for applications such as the gantry robot operation outlined briefly above. The control law is usually computed in the resetting time between successive passes as a function of the control used on the previous pass and a corrective term computed using information from the previous pass (also termed a trial in some literature) input and error or a finite number thereof. Suppose that a reference signal, $y_{ref}(t)$ that can be realized by the gantry robot is given and on pass $k$ let $y_k(t), 0 \leq t \leq T$, be the pass profile and $u_k(t)$ the control input. Then $e_k(t) = y_{ref}(t) - y_k(t)$ is the error on this pass and the ILC design problem to force tracking of the reference can be formulated as control law design to achieve convergence, as measured by the norm on the underlying function space, to zero of the error sequence $\{e_k\}_k$ and convergence of the input sequence $\{u_k\}_k$ to $u_\infty$, which is termed the learned control. Another example where the repetitive process structure arises as a result of control action or the iterative solution method is nonlinear dynamic optimal control problems based on the maximum principle (described in [1] with references to the original work).

Given the possibility of oscillations that increase in amplitude from pass-to-pass, stability of a repetitive process is defined as requiring that a bounded initial pass profile produces a bounded sequence of pass profiles, defined in terms of the norm on the underlying function space, either over the finite pass length or for all possible values of this parameter. If the dynamics are linear then an abstract model in a Banach space setting can be used [1] where the conditions are expressed in terms of the bounded linear operator describing the contribution of the previous pass profile to the dynamics of the next. This theory has been applied to ILC design, assuming that the dynamics are time-invariant, with experimental verification [4] on a gantry robot that replicates the pick and place operation discussed briefly above.

A significant proportion of the literature on the control of 2D systems is based on a linear model of the dynamics. Comparatively little work has been reported on the stability of nonlinear or linear time-varying examples, see, for example, [5]. In many possible applications for repetitive processes/2D systems, the dynamics are nonlinear and the new results in this paper address this issue with an application to ILC where it is shown that control law design by Linear Matrix Inequalities (LMIs) is possible for cases where linearization of the nonlinear dynamics about an operating point is possible. One more recent addition to applications for repetitive process control theory where the use of a nonlinear model will be required is laser metal deposition processes [6].

In the application of control systems, failures in operation can arise and the representation used in this paper for this problem is based on random switching. In particular, a process with failures is modeled by a state-space model with jumps in the parameter values and/or structure governed by a Markov chain with a finite set of states, often termed Markovian jump systems or systems with random structure, see, for example, [7]. Results on the development of control theory for Markovian jump systems, which address issues such as stability, optimal and robust control problems, in the standard, or 1D, case can be found in, for example, [8–13]. These results cannot be applied to 2D systems. Progress on the development of a systems theory for 2D linear systems with Markovian jumps is reported in [14,15] and references therein.

This paper considers nonlinear and time-varying discrete repetitive processes where, with applications such as ILC over a network in mind, the dynamics also have Markovian switching. The property of exponential stability in the mean square is defined and characterized, leading to results on stabilization and $\mathcal{H}_\infty$ control with an application to ILC. Moreover, the results are developed for time-invariant dynamics but have an immediate extension to the time-varying case. As in other control systems areas, it is to be expected that progress towards control law design will for physical examples with nonlinear dynamics make use of particular features in the corresponding models. Exactly this feature is present in the ILC design analysis that forms the second major part of this paper.

## 2. Process description and stability theory

If the dynamics of a repetitive process are linear then stability analysis can proceed from the abstract model and the task for a given example is to obtain conditions that can be tested. Let the pass profile $y_k \in E_T$ where $E_T$ is a Banach space. Then the pass-to-pass dynamics of a linear repetitive process with constant pass length $T < \infty$ are described by $y_{k+1} = L_T y_k, k \geq 0$, where $L_T$ is a bounded linear operator mapping $E_T$ into itself. In this case $L_T$ is a convolution operator over the finite interval $t \in [0, T]$ and contributions that enter on the current pass can be represented by adding a term that lies in a linear subspace of $E_T$.

The stability problem for repetitive processes is that the pass profile sequence $\{y_k\}_{k\geq 1}$ for a given initial profile $y_0$ can contain oscillations that increase in amplitude with $k$, as discussed in the previous section for the coal cutting example. Bounded-Input Bounded-Output (BIBO) stability for these processes is therefore defined, in terms of the norm on the underlying function space, as the requirement that a bounded initial pass profile produces a bounded sequence $\{y_k\}_{k\geq 1}$, either over the finite pass length or else independent of this parameter. The latter property is the most general case and for dynamics described by the abstract model, i.e., by $L_T$, requires the existence of real numbers $M_\infty > 0$ and $\lambda_\infty \in (0, 1)$, which are independent of $T$, such that $\|L_T^k\| \leq M_\infty \lambda_\infty^k$ where $\|\cdot\|$ denotes both the norm on $E_T$ and the induced operator norm.

The vast majority of the systems theory currently available for repetitive processes is for linear deterministic examples or those for which such a description is an adequate basis for initial analysis. In this paper, the process state-space model considered is

$$\begin{aligned} x_{k+1}(t+1) &= F_1(x_{k+1}(t), y_k(t), u_{k+1}(t), w_k(t), r_k(t)), \\ y_{k+1}(t) &= F_2(x_{k+1}(t), y_k(t), u_{k+1}(t), w_k(t), r_k(t)), \\ &\quad 0 \leq t \leq T, k = 0, 1, 2, \ldots \end{aligned} \tag{1}$$

where the integer $T$ denotes the pass length and on pass $k$, $x_k(t) \in \mathbb{R}^{n_x}$ is the current pass state vector, $y_k(t) \in \mathbb{R}^{n_y}$ is the pass profile vector, $u_k(t) \in \mathbb{R}^{n_u}$ is the control input vector, $w_k(t) \in \mathbb{R}^{n_w}$ is the disturbance vector, $F_1$ and $F_2$ are nonlinear functions, $r(t)$ is the homogeneous Markov chain whose state-space is the set of integers $\mathbb{N} = \{1, 2, \ldots, \nu\}$ and the transition probabilities are given by

$$\mathrm{P}[r_k(t+1) = j | r_k(t) = i] = \pi_{ij},$$
$$\mathrm{P}[r_{k+1}(t) = j | r_k(t) = i] = \omega_{ij}.$$

The boundary conditions are the pass state initial vector sequence and the initial pass profile and in this paper have the form

$$\begin{aligned} x_{k+1}(0) &= d_{k+1}, \quad k \geq 0, \\ y_0(t) &= f(t), \quad 0 \leq t \leq T-1, \end{aligned} \tag{2}$$

where the entries in the $n \times 1$ vector $d_{k+1}$ are known constants and $f(t)$ is an $m \times 1$ vector whose entries are known functions of $t$. Also the presence of the time shift on the left-hand side of the first equation in (1) means that the state vector $x_k(t)$ is defined over