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Optimal periodic scheduling of sensor networks: A branch and bound approach

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ABSTRACT

A periodic scheduling problem for sensor networks with communication constraints is considered for state estimation. The solvability of the problem is first discussed and a necessary and sufficient condition is presented based on the notion of periodic detectability. Since the calculation of the average prediction error variance requires the computation of the symmetric periodic positive-semidefinite stabilizing (SPPS) solutions to the periodic Riccati equations, a moving approximate cost function is proposed, which gradually converges to the exact cost function. Also, it is shown that the upper bound of the approximation error is independent of the SPPS solutions and converges to zero exponentially. Based on these results, a branch and bound based algorithm is proposed to compute the optimal periodic schedule, and the idea is to iteratively trim the set of schedules that are potentially robust optimal with respect to the approximation error. If the optimal schedule is unique, the algorithm solves the periodic scheduling problem by exploring a finite number of nodes. Moreover, given an arbitrary nonzero suboptimality specification, the algorithm results in a suboptimal schedule set containing all the optimal schedules at a manageable computation effort. A numerical example is presented to illustrate the proposed results.

1. Introduction

With the advent of wireless technology, applications of wireless sensor networks have emerged in a wide range of areas [1–4]. The adoption of these sensor networks provides more informative access to system information, and has brought on new opportunities to improve the performance of dynamic systems.

In this context, sensor scheduling problems for state estimation have received considerable attention in recent years. The main objective is to minimize cost functions related to the state estimation error, while the difficulty is mainly caused by the mixed integer nature of the problem and the constraints introduced by the sensor network.

Finite-horizon scheduling problems have been extensively studied in the literature. Optimal scheduling problems with uncorrelated and correlated sensor measurements were respectively considered in [5,6], and suboptimal solutions were obtained by solving relaxed convex optimization problems. In [7], a sensor scheduling problem with state-dependent measurement noise was formulated into a model predictive control problem, and a fast and optimal sensor scheduling algorithm was proposed. A two-step scheduling algorithm for networked sensor systems with heterogeneous sensors was proposed by [8], in which the sensor types and sensors of the same type were scheduled separately. In [9], an optimal partial broadcasting problem was considered, and a good-sensor–late-broadcast rule was proposed. In addition, it was shown that finite-horizon problems can be formulated as tree search problems [10], and different optimal/suboptimal pruning methods were proposed to reduce the computational complexity [11–13].

On the other hand, infinite-horizon scheduling problems are generally difficult to consider. One interesting exception was explored in [14], where a stochastic sensor selection algorithm was proposed to compute the optimal sensor selection distribution by minimizing the upper bound of the expected steady-state performance.

An intermediate class of problems between finite-horizon problems infinite-horizon problems is periodic scheduling problems. The optimal periodic schedules not only guarantee the steady-state estimation performance, but also provide insights of the optimal infinite-horizon aperiodic problems as well, since results indicated that some periodic phenomenon appears in optimal aperiodic schedules [15]. Moreover, the optimal schedules with finite periods can be computed offline and implemented online for an infinite time horizon. In [16], a periodic scheduling problem for heterogeneous sensor networks was formulated and a method to compute the optimal scheduling period was proposed. An optimal periodic scheduling problem for two sensors subject to communication and duty cycle constraints was considered by [17], where the sensors were assumed to send several most recent measurement data to the estimator, and the optimal periodic schedule was

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explicitly characterized for the first time. However, due to the combinatorial nature of these optimization problems, explicit optimal periodic schedules are normally not possible to characterize, especially for networks composed of multiple sensors. In addition, since the "initial value" of the steady-state error covariance is unknown and schedule dependent, the traditional tree search approaches and convex relaxation approaches utilized in finite-horizon problems are no longer applicable, making it cumbersome to design numerical optimization algorithms.

In this paper, the optimal periodic multiple-sensor scheduling problem is considered. The objective is to minimize the average state prediction error variance for linear Gaussian systems, and the constraint takes into account of the communication resource limitation. The contributions of the obtained results are threefold.

(1) The solvability of the periodic scheduling problem is discussed, and a necessary and sufficient condition is provided, which is numerically verifiable.
(2) A moving approximate cost function is proposed. Unlike the exact cost function, this function does not rely on the symmetric periodic positive-semidefinite stabilizing (SPPS) solutions to the periodic Riccati equations (PREs), and converges to the exact cost function asymptotically. In addition, an upper bound of the approximation error is obtained, which does not depend on the SPPS solutions and exponentially converges to zero.
(3) Based on the moving approximate cost function and the upper bound of the approximation error, a branch and bound based algorithm is proposed to identify the optimal periodic schedules without solving the PREs. Different from the results in [13], the lower bound of the objective function is designed based on the monotonicity properties of the Riccati equation solutions, and does not rely on the simultaneous diagonalization of the "sensor information matrices". It is shown that, given the uniqueness of the optimal schedule, the optimal solution can be computed by searching a finite number of nodes. Also, provided an arbitrary nonzero suboptimality specification exists, a set of suboptimal schedules containing all optimal schedules can be identified with known computational complexity bound.

The rest of the paper is organized as follows. Section 2 presents the problem formulation, the solvability condition, and preliminaries. In Section 3, the moving approximate cost function is proposed and its properties are discussed. The branch and bound based algorithm is presented in Section 4, and some implementation issues are discussed in Section 5. Section 6 presents a numerical example, and some concluding remarks are given in Section 7.

2. Problem formulation and preliminaries

2.1. Problem formulation

Consider the system in Fig. 1. The process is linear time invariant and evolves in discrete time:

$$x(k+1) = Ax(k) + w(k), \tag{1}$$

where $x(\cdot)$ is the state, and $w(\cdot)$ is the noise input, which is zero-mean Gaussian with covariance $Q \geq 0$.[1] We assume that (A, Q) is stabilizable. The initial value $x(0)$ of the state is also zero-mean Gaussian, with covariance P_0. The state information is measured using a network of sensors via the output equations:

$$y_i(k) = C_i x(k) + v_i(k), \tag{2}$$

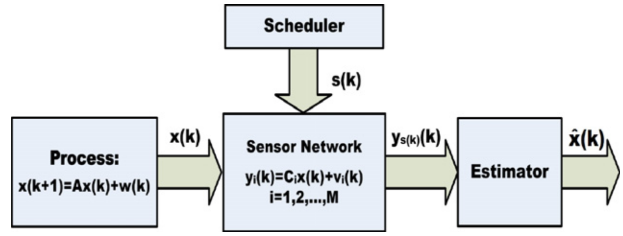

Fig. 1. Block diagram of the overall system.

where v_i is zero-mean Gaussian with covariance R_i for $i \in \{1, 2, \ldots, M\}$, M being the number of sensors, and v_i and v_j are uncorrelated if $i \neq j$. In addition, $x(0)$, w, and v_i are uncorrelated. Considering limited communication resources, assume that only one communication channel is available, and that only one sensor is allowed to access the channel at each time instant to transfer the measurement data. The sensors are chosen according to a periodic schedule s with period N, which is denoted as

$$s := [s(0), s(1), \ldots, s(N-1)],$$

where $s(k)$ takes values from $\{1, 2, \ldots, M\}$, indicating the index of the kth sensor scheduled within the N period. Assume that $N \geq M$. With a little abuse of notation, for $k > N - 1$, we still use $s(k)$ to denote the sensor used at time k, which is self-evident due to periodicity.

After updating the measurement, the optimal prediction $\bar{x}(k)$ of $x(k)$ is calculated based on the collected information $\{y_{s(0)}(0), y_{s(1)}(1), \ldots, y_{s(k-1)}(k-1)\}$, and the state estimation $\hat{x}(k)$ is computed according to [18]:

$$\hat{x}(k) = \bar{x}(k) + P(k)C_{s(k)}^T[C_{s(k)}P(k)C_{s(k)}^T + R_{s(k)}]^{-1} \times [y_{s(k)}(k) - C_{s(k)}\bar{x}(k)],$$

where

$$P(k) = \mathbf{E}[e(k)e'(k)], \tag{3}$$

and $e(k) = x(k) - \bar{x}(k)$. This estimate minimizes the prediction error covariance matrix $P(k)$.

Denote S_N as the set of periodic schedules with period N. We have the following definition.

Definition 1. A periodic schedule $s \in S_N$ is said to be well defined if there exists a linear periodic filter $K_{s(\cdot)}$ such that the closed-loop state matrix $A - K_{s(k)}C_{s(k)}$ is periodically stable [19].

Denote $\tilde{S}_N$ as the set of well-defined periodic schedules with period N. For a given schedule $s \in \tilde{S}_N$, define the average prediction error variance

$$J(s) := \lim_{T\to\infty} \frac{N}{T} \sum_{k=1}^{T} \mathbf{E}[e'(k)e(k)] = \lim_{T\to\infty} \frac{N}{T} \sum_{k=1}^{T} \mathbf{Tr}\{P(k)\}. \tag{4}$$

In this work, the following scheduling problem is considered:

$$\min_{s\in\tilde{S}_N} J(s). \tag{5}$$

Because the schedules are integer-valued functions, the optimization problem is nonconvex and of combinatorial nature. This type of problem is generally NP-complete, and analytical characterization of optimal schedules for general sensor networks is normally not possible. To exactly solve this problem, solutions to the PREs induced by all schedules in $\tilde{S}_N$ need to be considered, which requires the solution of M^N algebraic Riccati equations corresponding to the lifted periodic systems [20], and is computationally prohibitive. In this work, the objective is to design an iterative algorithm so that the optimal schedule can be numerically identified without computing the solutions to the PREs.

[1] In this paper, the notation $\Psi > (\geq)\Phi$ and $\Psi - \Phi > (\geq)0$ represents that matrix $\Psi - \Phi$ is positive definite (positive semidefinite), Ψ and Φ being two symmetric matrices.