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Memoryless reconstruction of an extended state of a control delayed system

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ABSTRACT

The general feedback controller for a class of control variable delayed systems has recently been shown to be the general linear functional of an extended state of the system. Using only plant input/output data, this work formulates a memoryless observer for the reconstruction of this extended state. It is also shown that the observer and the feedback controller can be independently designed in conformity with the separation principle.

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1. Introduction

Consider a control variable delayed system,

$$\Sigma_d : \dot{x}(t) = A_0x(t) + B_0u(t) + B_1u(t-h) \tag{1.1}$$

$$y(t) = C_0x(t), \tag{1.2}$$

with initial condition

$$x(0) = \phi^0 \in \mathbb{R}^n, \qquad (u_0, u(0)) \in \mathcal{C}_{ad}([-h, 0); \mathbb{R}^m) \times \mathbb{R}^m. \tag{1.3}$$

Here, t is time, the dot on x represents derivative with respect to $t, h > 0$ is the time lag, $u(t) \in \mathbb{R}^m$ and $x(t) \in \mathbb{R}^n$. Also, $A_0 \in \mathbb{R}^{n\times n}$, $B_0, B_1 \in \mathbb{R}^{n\times m}$ where $A_0 \neq 0$ and $B_1 \neq 0$. For simplicity, we consider $u \in \mathcal{C}_{ad}([-h, \infty); \mathbb{R}^m)$, the class of $\mathbb{R}^m$-valued piecewise continuous functions with the property that at a point of discontinuity, an element of $\mathcal{C}_{ad}([-h, \infty); \mathbb{R}^m)$ has left limit and right continuity. For $t \geq 0$, $u_t \in \mathcal{C}_{ad}([-h, 0); \mathbb{R}^m)$ is defined by $u_t(\theta) = u(t + \theta)$ where $\theta \in [-h, 0)$. Given the initial condition (1.3), the solution of (1.1) is an absolutely continuous function that satisfies (1.1) for $t > 0$.

Define $\mathbf{T}_{A_0} : \mathcal{C}_{ad}([-h, 0); \mathbb{R}^m) \to \mathbb{R}^n$ by $\mathbf{T}_{A_0}[g_t] = \int_{-h}^{0} e^{-(r+\theta)A_0} g_t(\theta)d\theta$. It is well known that under $z(t) = x(t) + \mathbf{T}_{A_0}[B_1u_t]$, namely,

$$z(t) = x(t) + \int_{-h}^{0} e^{-(r+\theta)A_0} B_1u_t(\theta)d\theta, \tag{1.4}$$

(1.1) is reduced to the ordinary system, $\dot{z}(t) = A_0z(t) + Bu(t)$ where $B = B_0 + e^{-hA_0}B_1$. Assume that (A_0, B) is controllable and choose $K \in \mathbb{R}^{m\times n}$ such that $A_0 - BK$ is a stability matrix. Then according to [1],

$$u(t) = -Kz(t) \tag{1.5}$$

feedback stabilizes (1.1). That this result is in error was brought to light by the numerical computations in [2] where the integral in (1.4) is approximated by numerical quadrature. There, it is shown that the closed loop system under (1.5) becomes unstable when the norm of the partition of the interval $[-h, 0]$ goes to zero. Following this result, we have shown in [3] that the closed loop instability observed in [2] is the result of inputs originating from ker $\mathbf{T}_{A_0}$. This closed loop instability, labeled as the *hidden input problem* in [3], was resolved in [4] under a new transformation theory called the *stable transformation theory*. According to the stable transformation theory, the general feedback stabilizing controller for (1.1) is a linear functional of the extended state $(x(t), z_{\mathcal{F}}(t)) \in \mathbb{R}^n \times \mathbb{R}^n$ where $z_{\mathcal{F}}(t)$ is a new transformation variable to be defined in (2.2). That is,

$$u(t) = -K_1x(t) - K_2z_{\mathcal{F}}(t) \tag{1.6}$$

is the general controller for (1.1) where $K_1, K_2 \in \mathbb{R}^{m\times n}$. The aim of this work is to reconstruct the extended state $(x(t), z_{\mathcal{F}}(t))$ from the system input/output data $(u(t), y(t))$ where $t > 0$ in order to realize (1.6) as an observer based feedback controller.

The change of structure of the control law from (1.5) to (1.6) renders obsolete the existing literature on the observation of $z(t)$ as a prelude to implementing the feedback control law, (1.5). To illustrate, [5] reconstructs $z(t)$ in order to generate the controller

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(1.5); however, the theory therein does not appear capable of easily generating $x(t)$. What is more, the need to implement delayed control terms in the construction of the observer [5] is undesirable. A second illustration is provided by [6] which also reconstructs $z(t)$ in order to implement (1.5); the novelty therein appears to be the adoption of a frequency domain approach based on a Diophantine type equation. Again, their approach does not appear able to easily yield $x(t)$. Thus, our contention that (1.6), rather than (1.5), is the general feedback controller places this paper in a ground breaking position since the need to recreate the (new) transformation variable, $z_{\mathcal{F}}(t)$ as well as $x(t)$ has hitherto not arisen.

Section 2 gives a summary of the relevant background results from [3,4]. Section 3 contains the principal result, namely the memoryless observer. This is illustrated with a numerical example. Section 4 contains the observer based general feedback controller and the validity of the *separation property* [7, p. 271]. Concluding remarks are made in Section 5.

2. Background results

Let $\nu_0 \geq 0$ be specified and define $\mathbb{C}^-_{-\nu_0} = \{\lambda \in \mathbb{C} : \operatorname{Re}\lambda < -\nu_0\}$ as the open left half of the complex plane bounded by $\operatorname{Re}\lambda = -\nu_0$. Define the class of $n \times n$ real valued stable matrices as $\mathcal{H} = \{\mathcal{M} \in \mathbb{R}^{n\times n} : |\lambda I_n - \mathcal{M}| = 0 \Rightarrow \lambda \in \mathbb{C}^-_0\}$. Relevant background results from [4] are summarized below where it is assumed that $\mathcal{F} \in \mathcal{H}$.

Lemma 2.1. *Let $\mathcal{F} \in \mathbb{R}^{n\times n}$ where $\mathcal{F} \neq A_0$. Define*

$$G_{\mathcal{F}}(\theta) = \mathrm{e}^{-(h+\theta)\mathcal{F}}, \quad -h \leq \theta \leq 0. \tag{2.1}$$

If $x(\cdot)$ denotes the solution of Σ_d, then the transformation

$$z_{\mathcal{F}}(t) = x(t) + \mathbf{T}_{\mathcal{F}}[B_1 u_t] = x(t) + \int_{-h}^{0} G_{\mathcal{F}}(\theta) B_1 u_t(\theta)\mathrm{d}\theta \tag{2.2}$$

reduces the initial value problem (1.1), (1.3) to the delay-free initial value problem:

$$\Sigma_0 : \dot{z}_{\mathcal{F}}(t) = \mathcal{F} z_{\mathcal{F}}(t) - (\mathcal{F} - A_0)x(t) + B_{\mathrm{eq}} u(t) \tag{2.3}$$

$$z_{\mathcal{F}}(0) = \phi^0 + \int_{-h}^{0} \mathrm{e}^{-(h+\tau)\mathcal{F}} B_1 u_0(\tau)\mathrm{d}\tau \tag{2.4}$$

where

$$B_{\mathrm{eq}} = B_0 + \mathrm{e}^{-h\mathcal{F}} B_1. \tag{2.5}$$

Lemma 2.2. *Let $u \in \mathcal{C}_{ad}([-h, \infty); \mathbb{R}^m)$ be given and $x(\cdot) \in C((0, \infty); \mathbb{R}^n)$. If $z_{\mathcal{F}}(\cdot)$ is an $\mathbb{R}^n$-valued absolutely continuous function on $(0, \infty)$ satisfying (2.2) and (2.3), then $x(\cdot)$ satisfies (1.1).*

Theorem 2.1. *The delayed control function, $B_1 u_t$, admits the decomposition*

$$B_1 u_t(\theta) = G'_{\mathcal{F}}(\theta) W_{\mathcal{F}}^{-1}[z_{\mathcal{F}}(t) - x(t)] + w_t^c(\theta) \tag{2.6}$$

where $w_t^c \in \ker \mathbf{T}_{\mathcal{F}} \cap \mathcal{C}_{ad}([-h, 0); \mathbb{R}^n)$,

$$W_{\mathcal{F}} := \int_{-h}^{0} G_{\mathcal{F}}(\theta) G'_{\mathcal{F}}(\theta)\ \mathrm{d}\theta > 0 \tag{2.7}$$

and $'$ denotes matrix transposition. In particular,

$$B_1 u(t-h) = W_{\mathcal{F}}^{-1}[z_{\mathcal{F}}(t) - x(t)] + w^c(t-h). \tag{2.8}$$

We have also shown (see [4]) that $w_t^c \in \ker \mathbf{T}_{\mathcal{F}}$ admits the representation

$$w^c(t-h) = \begin{cases} w_0^c(t-h) & \text{if } 0 \leq t < h \\ \mathrm{e}^{h\lfloor t/h\rfloor\mathcal{F}} w_0^c(\theta_0) & \text{if } t \geq h \end{cases} \tag{2.9}$$

where w_0^c is the initial segment w_t^c on $[-h, 0)$, $\lfloor\cdot\rfloor$ is the greatest integer function and $\theta_0 \in [-h, 0)$. The next lemma provides an estimate of $w^c(t-h)$, $t \geq 0$.

Lemma 2.3. *Let $\mathcal{F} \in \mathcal{H}$. If $w_t^c \in \ker \mathbf{T}_{\mathcal{F}}$, then there exists a constant $k_w > 0$ such that*

$$\|w^c(t-h)\| < k_w \mathrm{e}^{-\nu_{\mathcal{F}} t}, \quad t \geq 0 \tag{2.10}$$

for some $\nu_{\mathcal{F}} > 0$.

Proof. Let $\sigma(\mathcal{F})$ denote the spectrum of $\mathcal{F}$. Recall that $\mathcal{F} \in \mathcal{H} \Rightarrow \sigma(\mathcal{F}) \subset \mathbb{C}^-_{-\nu_{\mathcal{F}}}$ for some $\nu_{\mathcal{F}} > 0$. Furthermore, there exists a constant $c_{\mathcal{F}} > 0$ such that

$$\left\|\mathrm{e}^{t\mathcal{F}}\right\| \leq c_{\mathcal{F}} \mathrm{e}^{-\nu_{\mathcal{F}} t}, \quad t \geq 0. \tag{2.11}$$

Starting from the standard inequality $t/h - 1 < \lfloor t/h\rfloor \leq t/h$, multiplication by $\nu_{\mathcal{F}} h$ gives $\nu_{\mathcal{F}}(t-h) < \nu_{\mathcal{F}} h\lfloor t/h\rfloor \leq \nu_{\mathcal{F}} t$. Then exponentiation gives $\mathrm{e}^{\nu_{\mathcal{F}}(t-h)} < \mathrm{e}^{\nu_{\mathcal{F}} h\lfloor t/h\rfloor}$ or $\mathrm{e}^{-\nu_{\mathcal{F}} h\lfloor t/h\rfloor} < \mathrm{e}^{-\nu_{\mathcal{F}}(t-h)}$. With the help of this inequality and (2.11), we deduce that

$$\|\mathrm{e}^{\mathcal{F}\lfloor t/h\rfloor h}\| \leq c_{\mathcal{F}} \mathrm{e}^{-\nu_{\mathcal{F}}\lfloor t/h\rfloor h} \leq c_{\mathcal{F}} \mathrm{e}^{-\nu_{\mathcal{F}}(t-h)}, \quad t \geq 0. \tag{2.12}$$

Now, consider $w_t^c \in \ker \mathbf{T}_{\mathcal{F}}$. For $t \geq h$, application of the second case of (2.9) and (2.12) gives the estimate

$$\|w^c(t-h)\| \leq \|\mathrm{e}^{h\mathcal{F}\lfloor t/h\rfloor}\|\,\|w_0^c(\theta_0)\| \leq k_c \mathrm{e}^{-\nu_{\mathcal{F}} t} \tag{2.13}$$

where $k_c = c_{\mathcal{F}}\mathrm{e}^{\nu_{\mathcal{F}} h}\|w_0^c(\theta_0)\|$. Let $\mu_c > \max\left(1, \frac{\mathrm{e}^{\nu_{\mathcal{F}} h}\|w_0^c\|_{[-h,0]}}{k_c}\right)$ where $\|w_0^c\|_{[-h,0]} = \sup_{-h\leq\theta\leq 0}\|w_0^c(\theta)\|$. Then $\mu_c > 1$ and $\|w_0^c\|_{[-h,0]}\mathrm{e}^{\nu_{\mathcal{F}} h} < \mu_c k_c$. Since $\mu_c > 1$, (2.13) can be weakened to the form

$$\|w^c(t-h)\| < \mu_c k_c \mathrm{e}^{-\nu_{\mathcal{F}} t}, \quad t \geq h. \tag{2.14}$$

For $0 \leq t < h$, observe that $\mathrm{e}^{-\nu_{\mathcal{F}}(t-h)} > 1$. Then the first part of (2.9) yields

$$\begin{aligned} \|w^c(t-h)\| &\leq \sup_{0\leq t\leq h}\|w_0^c(t-h)\| = \|w_0^c\|_{[-h,0]} \\ &< \|w_0^c\|_{[-h,0]}\mathrm{e}^{-\nu_{\mathcal{F}}(t-h)} = \|w_0^c\|_{[-h,0]}\mathrm{e}^{\nu_{\mathcal{F}} h}\mathrm{e}^{-\nu_{\mathcal{F}} t} \\ &< \mu_c k_c \mathrm{e}^{-\nu_{\mathcal{F}} t}. \end{aligned} \tag{2.15}$$

Putting (2.14) and (2.15) together proves (2.10) on defining $k_w := \mu_c k_c$. □

Using (2.8), one can embed the given delayed system in an extended state space as follows:

Theorem 2.2. *Let $\bar{x}(t) = \begin{pmatrix} x(t) \\ z_{\mathcal{F}}(t) \end{pmatrix}$ and $\mathcal{R}_c(t-h) = \begin{bmatrix} w^c(t-h) \\ O_{n\times 1} \end{bmatrix}$ where $O_{n\times 1}$ is the $n \times 1$ matrix of zeros. Then Σ_d can be embedded in the extended system*

$$\frac{\mathrm{d}}{\mathrm{d}t}\bar{x}(t) = \mathcal{A}_{\mathcal{F}}\bar{x}(t) + \mathcal{B}_{\mathcal{F}} u(t) + \mathcal{R}_c(t-h) \tag{2.16}$$

where

$$\mathcal{A}_{\mathcal{F}} = \begin{bmatrix} A_0 - W_{\mathcal{F}}^{-1} & W_{\mathcal{F}}^{-1} \\ -(\mathcal{F} - A_0) & \mathcal{F} \end{bmatrix}, \qquad \mathcal{B}_{\mathcal{F}} = \begin{pmatrix} B_0 \\ B_{\mathrm{eq}} \end{pmatrix}. \tag{2.17}$$

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