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Stability analysis of a class of switched linear systems on non-uniform time domains

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ABSTRACT

This paper deals with the stability analysis of a class of switched linear systems on non-uniform time domains. The considered class consists of a set of linear continuous-time and linear discrete-time subsystems. First, some conditions are derived to guarantee the exponential stability of this class of systems on time scales with bounded graininess function when the subsystems are exponentially stable. These results are extended when considering an unstable discrete time subsystem or an unstable continuous-time subsystem. Some examples illustrate these results.

1. Introduction

The theory of system dynamics on an arbitrary time scale $\mathbb{T}$ was found promising because it demonstrates the interplay between the theories of continuous-time and discrete-time systems [1–3]. It enables to analyze the stability of dynamical systems on non-uniform time domains which are subsets of $\mathbb{R}$ [4]. As expected, when $\mathbb{T} = \mathbb{R}$, time scale dynamic equations reduce to standard continuous differential equations. When $\mathbb{T} = h\mathbb{Z}$ (h is a real number), they reduce to standard difference equations. Besides these two cases, there are many interesting time scales with non-uniform step sizes (for instance, $\mathbb{T} = \{t_n\}_{n\in\mathbb{N}}$ of so-called harmonic numbers with $t_n = \sum_{k=1}^{n} \frac{1}{k}$, the Cantor set). Exponential stability has been derived for linear systems using the time scale exponential function [5–7]. Some extensions to time-varying dynamic equations [8], dynamic equations with general structured perturbations [9] and nonlinear finite-dimensional control systems [10] on time scales have also been investigated. However, this analysis cannot be easily extended to the class of switched systems.

This paper deals with the stability analysis of a specific class of switched linear systems. Switched systems are systems involving both continuous and discrete dynamics. They consist of a finite number of subsystems and a discrete rule that dictates switching between these subsystems [11]. They have been widely studied during these two last decades (see for instance [12–14]) because they can describe a wide range of physical and engineering systems. Most of the existing methods to analyze the stability of switched linear systems can only be applied to systems evolving on either continuous [12–14] or discrete uniform time domains [15].

Motivated by this observation, in this paper, the stability is analyzed for a special class of switched linear systems where the dynamical system commutes between a continuous-time linear subsystem and a discrete-time linear subsystem during a certain period of time. There are many applications involving such switched systems. A cascaded system composed of a continuous-time plant, a set of discrete-time controllers and switchings among the controllers is one example [16]. Impulsive systems (which are a relevant class of switched systems, in which the state jumps occur only at some time instances [17]) with non-instantaneous state jumps are another examples. Indeed, their temporal nature cannot be represented by the continuous line (i.e. $\mathbb{R}$) or the discrete line (i.e. $\mathbb{Z}$). The distributed control over network has also attracted a great deal of attention in the last few years due to its broad range of applications in many areas [18,19]. This example is of great interest because the network dictates not only the switching modes but also the timing of the system [20]. In this case, the time domain is neither continuous nor uniformly discrete due to possible intermittent information transmissions for instance.

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In [16], some stability conditions are derived for switched normal linear systems which are given by two subsystems evolving on continuous time domain and discrete uniform time domains with fixed sampling periods. The stability analysis is based on a common quadratic Lyapunov function. However, the extension to a larger class of systems evolving on a non-uniform time domain is not trivial. To solve this issue, the theory of system dynamic on an arbitrary time scale $\mathbb{T}$ seems to be appropriate. The analysis of switched systems on arbitrary time scales with additional constraints imposed upon the graininess of the time scales is performed in [20,21] using common quadratic Lyapunov function. In the same way, the stability of a class of switched linear systems which consist of a set of stable linear continuous-time and stable linear discrete-time subsystems with fixed graininess function is studied in [22]. However, finding a common Lyapunov function is not an easy task for switched systems. Furthermore, the approaches given in [20–22] do not work if one individual subsystem is not asymptotically stable.

This paper aims to extend the existing results for a non uniform time domain $\mathbb{T} = \mathbb{P}_{\{a_k,b_k\}}$ formed by a union of disjoint intervals with variable length a_k and variable gap b_k. The studied system switches between a continuous-time dynamic subsystem and a discrete-time subsystem with bounded graininess function. Either continuous-time or discrete-time subsystem may be unstable. Using the time scale exponential function properties, some conditions are derived to guarantee the exponential stability of this class of systems under bounded graininess condition when the subsystems are exponentially stable. These results are extended when considering an unstable discrete time subsystem or an unstable continuous-time subsystem using the spectrum of the system matrices.

The outline of this paper is as follows. Section 2 states the problem and recalls some useful concepts on time scale theory. Section 3 derives sufficient conditions to guarantee the exponential stability of a particular class of switched systems on non-uniform time domains. Some examples illustrate these results.

2. Preliminaries and problem statement

In this section, we recall the basic notations, main definitions and properties regarding time scale calculus [1–3]. Then, the studied class of switched systems is described.

2.1. Basic concepts on time scales

Definition 2.1. A *time scale* $\mathbb{T}$ is an arbitrary nonempty closed subset of $\mathbb{R}$.

For $t \in \mathbb{T}$, the *forward jump operator* $\sigma(t) : \mathbb{T} \to \mathbb{T}$ is defined by

$$\sigma(t) := \inf\{s \in \mathbb{T} : s > t\}. \quad (1)$$

The mapping $\mu : \mathbb{T} \to \mathbb{R}^+$, called the *graininess function*, is defined by

$$\mu(t) = \sigma(t) - t. \quad (2)$$

Definition 2.2. A point $t \in \mathbb{T}$ is called *right-scattered* if $\sigma(t) > t$ and *right-dense* if $\sigma(t) = t$.

The set $\mathbb{T}^\kappa$ is defined as follows: if $\mathbb{T}$ has a left-scattered maximum m, then $\mathbb{T}^\kappa = \mathbb{T} - \{m\}$; otherwise $\mathbb{T}^\kappa = \mathbb{T}$.

These necessary definitions are required to define the differential operator for functions with time scale domains.

Definition 2.3. Let $f : \mathbb{T} \to \mathbb{R}$ be Δ-differentiable on $\mathbb{T}^\kappa$. The *Δ-derivative* of f at $t \in \mathbb{T}^\kappa$ is defined as

$$f^\Delta(t) = \lim_{s\to t} \frac{f(\sigma(t)) - f(s)}{\sigma(t) - s}. \quad (3)$$

One can notice that if $\mathbb{T} = \mathbb{R}$, then $f^\Delta(t) = \dot{f}(t)$, which is the euclidian derivative of f; and if $\mathbb{T} = h\mathbb{Z}$, then $f^\Delta(t) = \frac{f(t+h)-f(t)}{h}$. Hence, using the time scale theory, the theory of both differential and difference equations is unified.

Considering the theory of dynamic equations, let us first introduce the following class of functions.

Definition 2.4. A function $f : \mathbb{T} \to \mathbb{R}$ is said to be *right-dense continuous* or *rd-continuous*, if it is continuous (in the usual sense) over any right-dense interval within $\mathbb{T}$.

We say that a matrix A is rd-continuous on $\mathbb{T}$, if each entry of A is rd-continuous on $\mathbb{T}$.

Definition 2.5. A function $p : \mathbb{T} \to \mathbb{R}$ is *regressive* if $(1 + \mu(t)p(t)) \neq 0$, $\forall t \in \mathbb{T}^\kappa$. We denote the set of all regressive and rd-continuous functions by $\mathscr{R}$ and by $\mathscr{R}^+$ if they satisfy $1+\mu(t)p(t) > 0, \forall t \in \mathbb{T}^\kappa$ (i.e positively regressive functions).

Similarly, a function matrix $A : \mathbb{T} \to M_n(\mathbb{R})$ is called *regressive*, if $\forall t \in \mathbb{T}^\kappa, I + \mu(t)A(t)$ is invertible, where I is the identity matrix. Equivalently, a function matrix $A(t)$ is regressive if and only if all its eigenvalues are regressive (i.e $1 + \mu(t)\lambda_i(t) \neq 0, \ \forall 1 \leq i \leq n, \ \forall t \in \mathbb{T}^\kappa$, where $\lambda_i(t)$ are the eigenvalues of $A(t)$). The class of all regressive and rd-continuous functions A from $\mathbb{T}$ to $M_n(\mathbb{R})$ is denoted by $\mathscr{R}(\mathbb{T}, M_n(\mathbb{R}))$.

The *generalized exponential function* of $p \in \mathscr{R}$ on time scale $\mathbb{T}$ is expressed by

$$e_p(t,s) = \exp\left(\int_s^t \xi_{\mu(\tau)}(p(\tau))\Delta\tau\right)$$
$$\text{with } \xi_{\mu(t)}(z) = \begin{cases} \dfrac{\log(1+\mu(t)z)}{\mu(t)} & \text{if } \mu(t) \neq 0 \\ z & \text{if } \mu(t) = 0 \end{cases} \quad (4)$$

where $s, t \in \mathbb{T}$, log is the principal logarithm function and the delta integral is used [23]. For $\mathbb{T} = \mathbb{R}$, $e_p(t, t_0) = e^{p(t-t_0)}$ and for $\mathbb{T} = h\mathbb{Z}$, $e_p(t,s) = \prod_{\tau=s}^{t}(1 + hp(\tau))$.

Theorem 2.6 ([1]). *Let $p(t) \in \mathscr{R}$ and $t_0 \in \mathbb{T}$, the generalized exponential function $e_p(t, t_0)$ is the unique solution of the initial value problem*

$$x^\Delta(t) = p(t)\,x(t), \qquad x(t_0) = 1. \quad (5)$$

The unique solution of

$$x^\Delta(t) = A(t)x(t) \quad (6)$$

with $x(t_0) = I$, $t \in \mathbb{T}$, $A \in \mathscr{R}(\mathbb{T}, M_n(\mathbb{R}))$, is called the transition matrix and is denoted by $\Phi_A(t, t_0)$. If A is a constant matrix, the generalized exponential function $\Phi_A(t, t_0) = e_A(t, t_0)$ is the unique solution of (6).

Theorem 2.7 ([1]). *Suppose that matrix A is regressive, and $C : \mathbb{T} \to M_n(\mathbb{R}^n)$ is Δ-differentiable. If $C(t)$ is a solution of the matrix dynamic equation $C^\Delta(t) = A(t)C(t) - C(\sigma(t))A(t)$, then $C(t)e_A(t,s) = e_A(t,s)C(s)$.*

Corollary 2.8. *Suppose A is regressive and C is a constant matrix. If C commutes with A, then C commutes with e_A. In particular, if A is a constant matrix, then A commutes with e_A.*