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Pairwise synchronization of multi-agent systems with nonuniform information exchange

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ABSTRACT

In this note, the pairwise output synchronization problem of heterogeneous linear multi-agent systems is proposed. Considering a pair of agents denoted by (i, j), the control objective of this problem is to achieve output synchronization between jth output of agent i and ith output of agent j, and for this reason it can be viewed as a generalization of existing works in the output synchronization literature. The communication network is undirected and time varying with relative output information available for intercommunication. When the agent dynamics are identical and the output signals are observable, this problem becomes a state synchronization problem with nonuniform information exchange, and our results show that state synchronization is achievable in jointly connected graph.

1. Introduction

The output synchronization of heterogeneous multi-agent systems is a hot topic of research in recent years with fruitful results [1–9]. In these works, the output signals of all agents synchronize to a common trajectory: consider a multi-agent system characterized by linear dynamics

$$\begin{aligned}\dot{x}_i(t) &= A_i x_i(t) + B_i u_i(t)\\ y_i(t) &= C_i x_i(t)\end{aligned} \quad ; i = 1, \dots, N \tag{1}$$

the distributed control law is designed to achieve $\lim_{t\to\infty} y_i(t) - y_j(t) = 0$ for every i, j. In this note, we generalize this issue by investigating the "pairwise output synchronization problem", which is described as follows: for each pair of agents in the system, there is a particular pair of output signals to be synchronized, specifically speaking, the jth output of agent i to be synchronized with the ith output of agent j is in the form of

$$y_{ij}(t) = C_{ij} x_i(t)$$

and $y_{ji}(t) = C_{ji} x_j(t)$ is defined similarly. Our control objective is to achieve $\lim_{t\to\infty} y_{ij}(t) - y_{ji}(t) = 0$. As can be readily seen, the synchronized trajectories of two distinct pairs of agents represented by (i_1, j_1) and (i_2, j_2) are not necessarily the same. When $C_{i1} = \cdots = C_{iN} = C_i$, our problem is reduced to (1).

When the agent dynamics are identical and the output signals are observable, the pairwise output synchronization problem can be viewed as a state synchronization problem with nonuniform information exchange. In this sense, we generalize the results in [10,11] where only uniform information exchange is considered.

The communication graph in our problem is allowed to be uniformly jointly connected (UJC). There are at least two well-known schemes for tackling synchronization (consensus) problems in UJC graphs, the first one is based on the consensus behavior of first-order integrators, developed by [12,13] and some other authors; the other scheme is based on the conservativeness of passive multi-agent system [14,1,2]. In the above-mentioned works related to output synchronization in UJC graphs, [6] is based on the first scheme, the drawback of which is the need for full dimensional internal state exchange, while our distributed controller only needs relative output information exchange; [1] is based on the second scheme, in which all agents are passive. This assumption is not needed in this note, but we do incorporate a distributed compensator that behaves passively, thus making output synchronization possible even in UJC topology. With the assumption that (1) is passive, a static feedback controller is sufficient to achieve asymptotic output synchronization as in [1,2]. When (1) is not passive, as is hypothesized in this note, a dynamic feedback controller is generally needed [3–5].

The rest of this note is organized as follows: in Section 2 the problem under consideration is formulated, after a brief introduction of graphs. The pairwise output synchronization problem is

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solved in Section 3 while Section 4 considers state synchronization with nonuniform information exchange. Numerical examples are provided in Section 5. Section 6 concludes this note.

Notation: $I_n \in \mathbb{R}^{n\times n}$ stands for the identity matrix, I for short. $O_{m\times n} \in \mathbb{R}^{m\times n}$ stands for the zero matrix, O for short.

2. Problem formulation

Some basic graphical notions are necessary for the problem formulation.

2.1. Graph

A graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consists of a finite set $\mathcal{V} = \{1, \ldots, N\}$ of vertices and an edge set $\mathcal{E} = \{(i,j) | i,j \in \mathcal{V}, i \neq j\} \subseteq \mathcal{V} \times \mathcal{V}$. We call j a neighbor of i if $(i,j) \in \mathcal{E}$. The edge (i,j) is called undirected if $(i,j) \in \mathcal{E}$ implies $(j,i) \in \mathcal{E}$. The graph is called undirected if every edge in $\mathcal{E}$ is undirected. An (undirected) path in an undirected graph from i_1 to i_l is a sequence of vertices $\{i_1, \ldots, i_l\}$ such that $(i_k, i_{k+1}) \in \mathcal{E}$, $k = 1, \ldots, l-1$. We call i_l is connected to i_1 if there is a path from i_1 to i_l. An undirected graph $\mathcal{G}$ is called connected if every two vertices are connected by an undirected path.

Let $\mathcal{S}$ represent the set that contains all possible undirected graphs with vertex set $\mathcal{V}$. For simplicity at a price of abuse of notations, denoting $\mathcal{G} : [0, +\infty) \to \mathcal{S}$ a time-varying graph, we can write $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$ where $\mathcal{E}(t)$ is the corresponding time-varying edge set. $\mathcal{G}(t)$ is piecewise constant by definition. The graph $\mathcal{G}([t_1, t_2)) \triangleq \left(\mathcal{V}, \bigcup_{t\in[t_1,t_2)} \mathcal{E}(t)\right)$ is called the union of $\mathcal{G}(t)$ in $[t_1, t_2)$.

Now we introduce the concept of uniformly jointly connectedness [15] as follows.

Definition 1. The undirected graph $\mathcal{G}(t)$ is called uniformly jointly connected (UJC) if there is $T > 0$ such that the union graph $\mathcal{G}([t, t+T))$ is connected for all $t \geq 0$.

Now we define the dwell time for the *edges* of $\mathcal{E}(t)$ [1].

Definition 2. Considering the undirected graph $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$, $0 \leq t_{e0}^{(i,j)} < t_{b0}^{(i,j)} \leq t_{e1}^{(i,j)} < t_{b1}^{(i,j)} \cdots$ are time instants that the edge (i,j) is established at $t_{eq}^{(i,j)}$ (at this time instant (i,j) exists) and broken down at $t_{bq}^{(i,j)}$ (at this time instant (i,j) does not exist). The dwell time $\tau_{ij} \geq 0$ for the undirected edge (i,j) is defined such that, for every $q = 0, 1\ldots$, the inequality $t_{bq}^{(i,j)} - t_{eq}^{(i,j)} \geq \tau_{ij}$ holds.

2.2. Problem statement

The communication mechanism of a multi-agent system can be conventionally described by a (time-varying) graph, viewing each agent as a vertex and the link from agent i to agent j, which indicates that agent i receives information from agent j, as a directed edge (i,j). Denote the agent set $\mathcal{V} = \{1, \ldots, N\}$. The underlying communication graph of the pairwise output synchronization problem (as well as the state synchronization problem) is denoted by $\mathcal{G}(t) = \{\mathcal{V}, \mathcal{E}(t)\}$.

In this note, the following assumptions are satisfied.

Assumption 1. $\mathcal{G}(t)$, $t \in [0, \infty)$ is pointwise undirected.

Assumption 2. The dwell time for every edge (i,j) in $\mathcal{G}(t)$ is nonvanishing, i.e. $\tau_{ij} > 0$.

Assumption 3. $a_{ij}(t) \in \{0\} \cup [\underline{b}, \bar{b}]$ is piecewise continuous in t, where $0 < \underline{b} \leq \bar{b}$ are constant numbers. $a_{ij}(t) \neq 0$ if and only if $(i,j) \in \mathcal{E}(t)$.

Consider a multi-agent system:

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i u_i(t) \\ z_i(t) &= x_i(t) \qquad\qquad ; i,j = 1, \ldots, N \\ y_{ij}(t) &= C_{ij} x_i(t) \end{aligned} \tag{2}$$

where $x_i \in \mathbb{R}^{n_i}$ is the state, $u_i \in \mathbb{R}^{p_i}$ is the control input, $z_i \in \mathbb{R}^{n_i}$ is the measurement output available for self-feedback, which is assumed to be the state variable. $y_{ij} \in \mathbb{R}^{m_{ij}}$ represents the jth output of agent i to be synchronized with the ith output y_{ji} of agent j. The control objective of the pairwise output synchronization problem is rendering $\lim_{t\to\infty} y_{ij}(t) - y_{ji}(t) = 0$. Then it is necessary to assume $m_{ij} = m_{ji}$. (A_i, B_i) is stabilizable.

Assumption 4. A_i is marginally stable for every i, i.e. there exists a positive definite matrix $P_i \in \mathbb{R}^{n_i\times n_i}$ such that $A_i^T P_i + P_i A_i \leq 0$.

Note that A_i is marginally stable if and only if all the unstable eigenvalues of A_i are semi-simple with zero real part [11].

Our distributed controller is designed as

$$\begin{aligned} \dot{w}_i(t) &= A_i w_i(t) - \sigma \sum_{j=1}^{N} a_{ij}(t) P_i^{-1} C_{ij}^T \left(y_{ij}(t) - y_{ji}(t)\right) \\ u_i(t) &= K_i \left(z_i(t) - w_i(t)\right) \end{aligned} \tag{3}$$

where $w_i \in \mathbb{R}^{n_i}$ is the internal state. $\sigma > 0$ is a scaler gain to be designed, which is identical to every agent to avoid introducing extra complexity. $A_i + B_i K_i$ is Hurwitz. Let $Q_i \in \mathbb{R}^{n_i\times n_i}$ be the positive definite matrix such that $(A_i + B_i K_i)^T Q_i + Q_i (A_i + B_i K_i) = -I_{n_i}$. Denote $e_i(t) = x_i(t) - w_i(t)$. The closed-loop system consisting of (2) and (3) reads

$$\begin{aligned} \dot{e}_i(t) \;=\; & (A_i + B_i K_i) e_i(t) + \sigma \sum_{j=1}^{N} a_{ij}(t) P_i^{-1} C_{ij}^T \left[C_{ij}(e_i(t) \right. \\ & \left. + w_i(t)) - C_{ji}\left(e_j(t) + w_j(t)\right)\right] \\ \dot{w}_i(t) \;=\; & A_i w_i(t) - \sigma \sum_{j=1}^{N} a_{ij}(t) P_i^{-1} C_{ij}^T \left[C_{ij}(e_i(t) + w_i(t)) \right. \\ & \left. - C_{ji}\left(e_j(t) + w_j(t)\right)\right]. \end{aligned} \tag{4}$$

Denote $e = [e_1^T, \ldots, e_N^T]^T$, $w = [w_1^T, \ldots, w_N^T]^T$, $A = \mathrm{diag}(A_1, \ldots, A_N)$, $B = \mathrm{diag}(B_1, \ldots, B_N)$, $K = \mathrm{diag}(K_1, \ldots, K_N)$, $P = \mathrm{diag}(P_1, \ldots, P_N)$. *For every $i < j$, denote $\mathcal{C}_{ij}$ is given in* Box I. Noting $a_{ij}(t) = a_{ji}(t)$, it is not hard to verify that system (4) is equivalent to

$$\begin{aligned} \dot{e}(t) \;=\; & (A + BK) e(t) \\ & + \sigma \left(\sum_{i<j} a_{ij}(t) P^{-1} \mathcal{C}_{ij}^T \mathcal{C}_{ij}\right)(w(t) + e(t)) \\ \dot{w}(t) =\; & A w(t) - \sigma \left(\sum_{i<j} a_{ij}(t) P^{-1} \mathcal{C}_{ij}^T \mathcal{C}_{ij}\right)(w(t) + e(t)) \end{aligned} \tag{5}$$

in the compact form.

3. Pairwise output synchronization

Assumption 5. Consider a pair of distinct agents (k, l). Without loss of generality, suppose $1 \leq k < l \leq N$. For any $s > 0$, there is $T > 0$ and an interval $\Gamma_s \subset [s, s+T)$ with the non-vanishing length such that

$$a_{kl}(t) \geq \underline{b} \quad \forall t \in \Gamma_s.$$