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Adaptive coordinated tracking of multi-agent systems with quantized information

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ABSTRACT

In this paper, we study the adaptive coordinated tracking problem for continuous-time first-order integrator systems with quantized information under switching undirected and fixed directed communication graphs, respectively. The combined effect of quantized relative information error and quantized absolute information error on the tracking result is investigated. Both the logarithmic quantizers and uniform quantizers are considered. It is shown that when logarithmic quantizers are used, exact coordinated tracking can still be achieved by properly choosing the design parameters in the controller while when uniform quantizers are used, practical coordinated tracking can be achieved with tracking error bounds proportional to the quantizer parameter. Simulation examples are provided to illustrate the results.

1. Introduction

In recent years, the coordination problems of multi-agent systems have attracted intensive attentions from various disciplines due to their wide range of potential applications in areas such as sensor networks, unmanned vehicle formation and satellite altitude alignment [1,2]. Among the different coordination tasks of multi-agent systems such as consensus, synchronization, flocking and swarming control, one of the important control topics is coordinated tracking control. In this problem, there exist some agents playing the role of leaders while the others as the followers and the objective is to design distributed controllers for the followers using only local information to track the trajectory of the leaders. Many useful results have been reported on this topic regarding different agent dynamics and communication topologies, [3–12] to name just a few.

A simple observation is that most of the results on the coordination problems of multi-agent systems rely on exact information communication among the neighboring agents [1–11]. However, this may be an unrealistic assumption in practical. Since the agents usually acquire information about their neighbors through information transmission via digital channels or relative state measurement through digital sensors, various restrictions exist which may severely degrade the performance of the distributed controllers. One of the important restriction induced by the finite bandwidth of the digital channels or the finite precision of the digital sensors is the quantization effect. The information available is quantized information instead of the precise information. Early results on the quantized control of multi-agent systems mainly focused on the average consensus problem for discrete-time first-order integrator systems under undirected communication graphs [13–18]. In [19–24], dynamic coding/decoding digital channels with dynamic uniform quantizers were employed and exact consensus with quantized information under general directed communication graphs was achieved.

Since most of the practical multi-agent systems are more naturally modeled by continuous-time systems and the quantized communication usually takes place asynchronously, quantization effect on continuous-time multi-agent systems attracts more and more attention recently. A few works have appeared recently focusing on this class of problems. In [25], the authors considered the problems of state agreement and distance-based formation control of first-order integrators with quantized relative measurement when the communication graph is an undirected tree. In [26], asymmetrically and symmetrically quantized consensus protocols for first-order integrator systems were proposed which guaranteed that the closed-loop system was Lyapunov stable and the agents converged to an appropriately defined set in finite time. In [27], the authors studied the consensus problem for first-order integrator systems with uniformly quantized absolute information and a hysteretic quantizer was introduced to cope with the undesired

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chattering phenomena. In [28], quantization effect of different types of quantizers on the synchronization problem of second-order mobile agents was investigated. More recently, Ref. [29] studied the consensus problem of a network of first-order integrators with quantized measurement and time-varying directed topology. A passivity approach to consensus and synchronization problems in the presence of quantized measurements was investigated in [30].

Note that most of the existing results on the quantized control of continuous-time multi-agent systems have focused on the consensus problem under undirected communication graphs [25–28,30]. This graph limitation mainly comes from the fact that the iteration based method used in [20–24] for discrete-time systems is no longer valid in the continuous-time setting. In this situation, it is much harder to fully exploit the symmetrical quantizer properties when the Laplacian matrices of the directed communication graphs are asymmetrical. Another observation is that only one kind of quantization process, either absolute information quantization or relative information quantization is considered in most of the existing results. As pointed out earlier, coordinated tracking problem plays an important role in the application of multi-agent systems. The communication graphs are usually directed due to link failure or energy constraint. Moreover, it is typical to have both quantized relative information and quantized absolute information in the control of multi-agent systems to achieve better performance. Thus in this paper, we study the adaptive coordinated tracking problem for continuous-time first-order integrator systems with two types of quantized information under both switching undirected and fixed directed communication graphs. The dynamic coding/decoding scheme proposed for discrete-time systems is generalized to the continuous-time setting to achieve exact tracking with only quantized information. Both the effect of logarithmic quantizers and uniform quantizers are considered.

The rest of the paper is organized as follows. In Section 2, some preliminaries and the problem setup are given. In Section 3, adaptive coordinated tracking with logarithmically and uniformly quantized information under switching undirected and fixed directed graphs are studied respectively. In Section 4, two simulation examples are provided to illustrate the results. Finally, some concluding remarks are given in Section 5.

Notation. I_N is the $N \times N$ identity matrix. $\mathbf{1}_N = [1, \ldots, 1]^T \in \mathbb{R}^N$. $\mathbf{0}$ is a vector or matrix with all the elements equal to zero. $\mathrm{col}(x_i)$ is the stack column vector of x_i with i in some index set S. For a vector x, $\|x\|_1$, $\|x\|_2$ and $\|x\|_\infty$ are the 1-norm, 2-norm and infinity-norm of x, respectively. For a matrix $A \in \mathbb{R}^{N\times N}$, $\|A\|_2$ denotes the induced 2-norm and $\{\lambda_i(A),\ i = 1, \ldots, N\}$ is the eigenvalue set of A. When all $\lambda_i(A),\ i = 1, \ldots, N$ are real, $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ are the maximum and minimum eigenvalues of A, respectively.

2. Preliminaries and problem setup

2.1. Graph theory

The communication relation among the agents in the leader–follower system can be represented by graphs. A directed graph $\mathcal{G} = (\mathcal{V}(\mathcal{G}), \mathcal{E}(\mathcal{G}))$ consists of a finite set of vertices $\mathcal{V}(\mathcal{G}) = \{v_0, v_1, \ldots, v_N\}$ and a finite set of edges $\mathcal{E}(\mathcal{G}) \subset \mathcal{V}(\mathcal{G}) \times \mathcal{V}(\mathcal{G})$. Each agent is represented by a vertex in $\mathcal{V}(\mathcal{G})$ and an edge is an ordered pair (v_i, v_j) which represents the information flow from agent j to agent i. Graph $\mathcal{G}$ is said to be undirected if for any edge $(v_i, v_j) \in \mathcal{E}(\mathcal{G})$, $(v_j, v_i) \in \mathcal{E}(\mathcal{G})$. A path $\mathcal{P}$ in $\mathcal{G}$ is a sequence $\{v_{i_0}, \ldots, v_{i_k}\}$ where $(v_{i_{j-1}}, v_{i_j}) \in \mathcal{E}(\mathcal{G})$ for $j = 1, \ldots, k$ and the vertices are distinct. If there exists a path from vertex v_i to v_j, we say that v_j is reachable from v_i. An induced subgraph $\mathcal{G}_s$ of $\mathcal{G}$ is a graph such that $\mathcal{V}(\mathcal{G}_s) \subset \mathcal{V}(\mathcal{G})$ and for any $v_i, v_j \in \mathcal{V}(\mathcal{G}_s)$, $(v_i, v_j) \in \mathcal{E}(\mathcal{G}_s)$ if and only if $(v_i, v_j) \in \mathcal{E}(\mathcal{G})$. In this paper, we use the vertex set $\mathcal{V}(\mathcal{G}_s) = \{v_1, \ldots, v_N\}$ of subgraph $\mathcal{G}_s$ to represent the follower agents. For a vertex v_i of $\mathcal{G}_s$, the set of in neighbors is denoted by $N_i^+ = \{j : (v_i, v_j) \in \mathcal{E}(\mathcal{G}_s)\}$ while the out neighbors by $N_i^- = \{j : (v_j, v_i) \in \mathcal{E}(\mathcal{G}_s)\}$. The adjacency matrix $A = [a_{ij}]$ associated with $\mathcal{G}_s$ is defined as $a_{ii} = 0$ and $a_{ij} > 0$ if $(v_i, v_j) \in \mathcal{E}(\mathcal{G}_s)$ where $i \neq j$. The Laplacian matrix of $\mathcal{G}_s$ is defined as $L = [l_{ij}]$ where $l_{ii} = \sum_{j\neq i} a_{ij}$ and $l_{ij} = -a_{ij}$ where $i \neq j$. The communication relation between the leader and the followers can be represented by a diagonal matrix $B = \mathrm{diag}\{b_1, \ldots, b_N\}$ where $b_i > 0$ if there exists an edge from follower i to the leader and $b_i = 0$ otherwise. The in degree of follower i is defined as $\deg(v_i) = \sum_{j\neq i} a_{ij} + b_i$. To facilitate the following analysis, a matrix $H = L + B$ is defined and we have the following two lemmas about the properties of H.

Lemma 1 ([10]). *If the subgraph $\mathcal{G}_s$ is undirected and the leader is reachable from all the followers in the graph $\mathcal{G}$, then H is symmetric and positive definite.*

Lemma 2 ([31]). *If the subgraph $\mathcal{G}_s$ is directed and the leader is reachable from all the followers in $\mathcal{G}$, then H is of full rank. Furthermore, define*

$$
\begin{aligned}
q &= [q_1, \ldots, q_N]^T = H^{-1}\mathbf{1}_N \\
P &= \mathrm{diag}\{p_1, \ldots, p_N\} = \mathrm{diag}\left\{\frac{1}{q_1}, \ldots, \frac{1}{q_N}\right\} \qquad (1)\\
Q &= PH + H^T P,
\end{aligned}
$$

then the diagonal matrix P and symmetric matrix Q are both positive definite.

In the following, we use the notations $p^+ = \max_{i=1,\ldots,N}\{p_i\}$, $p^- = \min_{i=1,\ldots,N}\{p_i\}$, $\lambda_q^+ = \lambda_{\max}(Q)$ and $\lambda_q^- = \lambda_{\min}(Q)$.

2.2. Quantizers

A uniform quantizer [15,27] is a map $q_u : \mathbb{R} \to \mathbb{R}$ such that

$$
q_u(x) = \Delta \left\lfloor \frac{x}{\Delta} + \frac{1}{2} \right\rfloor
$$

where $\Delta > 0$ is the quantizer parameter and $\lfloor a \rfloor$ denotes the greatest integer that is less than or equal to a. For a uniform quantizer, the quantization error is always bounded by $\frac{\Delta}{2}$, i.e., $|q_u(x) - x| \leq \frac{\Delta}{2}$ for all $x \in \mathbb{R}$.

A logarithmic quantizer [19,28] is an odd map $q_l : \mathbb{R} \to \mathbb{R}$ defined by

$$
q_l(x) = \begin{cases} \exp\left(\Delta \left\lfloor \dfrac{\ln(x)}{\Delta} \right\rfloor\right), & x > 0 \\ 0, & x = 0 \\ -q_l(-x), & x < 0. \end{cases}
$$

The quantization error for the logarithmic quantizer satisfies

$$
|q_l(x) - x| \leq \delta_l |x|, \qquad (2)
$$

where the quantizer parameter $\delta_l = 1 - e^{-\Delta}$. It holds that $x q_l(x) \geq (1 - \delta_l)\,|x|^2$.

We can easily generalize the above scalar-valued definitions of quantizers to vector-valued cases. For example, for any $x = [x_1, \ldots, x_n]^T \in \mathbb{R}^n$, the vector-valued logarithmic quantizer $q_l(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$ is defined to be $q_l(x) \triangleq [q_l(x_1), \ldots, q_l(x_n)]^T$. It is easy to verify that $x^T q_l(x) \geq (1 - \delta_l) x^T x$ for any $x \in \mathbb{R}^n$.

Remark 1. The considered uniform and logarithmic quantizers are the two types of quantizers which are most commonly used