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Stable controllers for robust stabilization of systems with infinitely many unstable poles[☆]

Masashi Wakaiki [a,*], Yutaka Yamamoto [a], Hitay Özbay [b]

[a] Department of Applied Analysis and Complex Dynamical Systems, Graduate School of Informatics, Kyoto University, Kyoto 606-8501, Japan

[b] Department of Electrical and Electronics Engineering, Bilkent University, Bilkent, Ankara TR-06800, Turkey

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ABSTRACT

This paper studies the problem of robust stabilization by a stable controller for a linear time-invariant single-input single-output infinite dimensional system. We consider a class of plants having finitely many simple unstable zeros but possibly infinitely many unstable poles. First we show that the problem can be reduced to an interpolation–minimization by a unit element. Next, by the modified Nevanlinna–Pick interpolation, we obtain both lower and upper bounds on the multiplicative perturbation under which the plant can be stabilized by a stable controller. In addition, we find stable controllers to provide robust stability. We also present a numerical example to illustrate the results and apply the proposed method to a repetitive control system.

1. Introduction

In this paper, we study *robust stabilization by a stable controller* for a single-input single-output infinite dimensional system. The advantage of stable controllers is well appreciated in that such controllers are robust against a sensor or actuator failure [1] and the saturation of the control input [2]. Typical examples are flexible structures [3] and traffic networks [2]. Additionally, stable controllers are preferred for control of electromechanical positioning devices [4]. We also recall that two plants are simultaneously stabilizable if and only if an associated plant derived from these two plants is stabilizable by a stable controller [5].

For finite dimensional systems, several design methods of stable $\mathcal{H}^\infty$ controllers have been developed: linear matrix inequalities or algebraic Riccati equations [6,7] and non-smooth, non-convex optimization [8]. On the other hand, for infinite dimensional systems, while sensitivity reduction by a stable controller has been studied in [9–11], robust stabilization by a stable controller still remains to be an open problem.

Let us briefly summarize the difference between these two problems. Sensitivity reduction by a stable controller can be transformed to the modified Nevanlinna–Pick interpolation [9,12–14], and the associated $\mathcal{H}^\infty$-norm condition is $\|F\|_\infty < \rho$, where F is a solution of the unit interpolation problem. On the other hand, in robust stabilization by a stable controller, the counterpart is $\|W - mF\|_\infty < \rho$, where $W,\ 1/W \in \mathcal{H}^\infty$ and $m \in \mathcal{H}^\infty$ is inner. Since F needs to be a unit element, we cannot change this norm condition to a simpler one, although we can in the usual robust stabilization problem. We overcome this difficulty by extending the technique of [14]. We will discuss this technique in Section 3.

This paper studies a class of plants having *finitely many simple unstable zeros* but possibly *infinitely many unstable poles*. An example of such plants is a system with delayed feedback such as repetitive control systems [15,16]. The objective of the present paper is to obtain lower and upper bounds on the multiplicative perturbation under which the plant can be stabilized by a stable controller. We also develop a design method of stable controllers achieving robust stability by the method of [9,10].

The paper is organized as follows: Section 2 gives the statement of the robust stabilization problem with stable controllers. In Section 3, we obtain a sufficient condition for the problem and find stable controllers for robust stabilization. A necessary condition follows along similar lines. We present a numerical example and apply the proposed method to a repetitive control system in Section 4.

Notation and definitions

Let $\mathbb{C}_+$ denote the open right half-plane $\{s \in \mathbb{C} \mid \mathrm{Re}\, s > 0\}$. For $s \in \mathbb{C} \setminus \{0\}$, the principal value $\mathrm{Log}\, s$ is the complex logarithm whose imaginary part lies in the interval $(-\pi,\ \pi]$.

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[*] Corresponding author. Tel.: +81 75 753 5904; fax: +81 75 753 5517.
E-mail addresses: wakaiki@acs.i.kyoto-u.ac.jp (M. Wakaiki), yy@i.kyoto-u.ac.jp (Y. Yamamoto), hitay@bilkent.edu.tr (H. Özbay).

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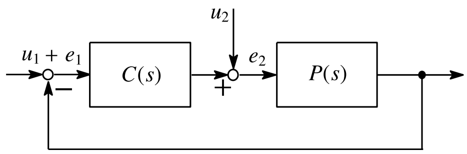

Fig. 1. Closed-loop system.

The space $\mathcal{H}^\infty$ denotes the Hardy space of functions that are bounded and analytic in $\mathbb{C}_+$, and $\mathcal{RH}^\infty$ denotes the subset of $\mathcal{H}^\infty$ consisting of real-rational functions. $U \in \mathcal{H}^\infty$ is called a *unit element* in $\mathcal{H}^\infty$ if $U,\ 1/U \in \mathcal{H}^\infty$. For $G \in \mathcal{H}^\infty$, the $\mathcal{H}^\infty$ norm is defined as $\|G\|_\infty := \sup_{s\in\mathbb{C}_+} |G(s)|$. The field of fractions of $\mathcal{H}^\infty$ is denoted by $\mathcal{F}^\infty$.

Two functions $N, D \in \mathcal{H}^\infty$ are *strongly coprime* in the sense of [17] if $NX + DY = 1$ for some $X, Y \in \mathcal{H}^\infty$. By the corona theorem [5], N and D are strongly coprime if and only if there exists $\delta > 0$ such that $|N(s)| + |D(s)| \geq \delta$ for all $s \in \mathbb{C}_+$.

To denote the interpolation data $G(s_i) = \alpha_i$ $(i = 1, \ldots, n)$ for $G \in \mathcal{H}^\infty$, we use the notation $(s_i;\ \alpha_i)_{i=1}^n$.

2. Problem statement

Consider the linear, continuous-time, time-invariant, single-input single-output closed-loop system given in Fig. 1. Let the plant P and the controller C belong to $\mathcal{F}^\infty$. P is said to be *stabilizable* if there exists C such that $S := 1/(1 + PC)$, CS, and PS belong to $\mathcal{H}^\infty$. For a given P, the set of all C leading to $S,\ CS,\ PS \in \mathcal{H}^\infty$ is denoted by $\mathscr{C}(P)$. P is *strongly stabilizable* if $\mathcal{H}^\infty \cap \mathscr{C}(P) \neq \emptyset$. We say that C *stabilizes* P if $C \in \mathscr{C}(P)$, and that C *strongly stabilizes* P if $C \in \mathcal{H}^\infty \cap \mathscr{C}(P)$.

Let P be a real-rational proper function. Then P is stabilizable by $C \in \mathcal{RH}^\infty$ if and only if P has the parity interlacing property [18]. On the other hand, if we do not require $C \in \mathcal{RH}^\infty$ but $C \in \mathcal{H}^\infty$ allowing complex coefficients, every stabilizable $P \in \mathcal{F}^\infty$ is strongly stabilizable [19], via a complex-valued controller in general.

We make the following assumption on the plant throughout this paper:

Assumption 2.1. $P \in \mathcal{F}^\infty$ can be factorized into the following form:

$$P = \frac{M_n}{M_d} N_o, \tag{1}$$

where $M_d \in \mathcal{H}^\infty$, $M_n \in \mathcal{RH}^\infty$ are inner functions and N_o, $1/N_o \in \mathcal{H}^\infty$. We assume that M_n possesses simple zeros $z_1, \ldots, z_n$ only and that M_d, M_n are strongly coprime.

Under Assumption 2.1, P has only finitely many unstable zeros arising from M_n, but P is allowed to possess infinitely many unstable poles arising from M_d. In [20], it is shown how to factorize retarded or neutral time delay systems into the form (1) under some mild conditions.

Let P be the *nominal* model of the plant. In this paper, we assume that the transfer function of the *actual* plant belongs to the following model set with multiplicative perturbations:

$$\mathscr{P}_\rho := \left\{ P_\Delta = (1 + W\Delta)P \ :\ \Delta \in \mathcal{H}^\infty,\ \|\Delta\|_\infty < 1/\rho \right\}$$

for some $\rho > 0$.

Recall that the controller C stabilizes all $P_\Delta \in \mathscr{P}_\rho$ if and only if C stabilizes the nominal model P and satisfies

$$\|WT\|_\infty \leq \rho, \quad \text{where } T := \frac{PC}{1 + PC}. \tag{2}$$

See, e.g., [1,5,21] for details.

We impose the following assumption on the weighting function:

Assumption 2.2. Both W and $1/W$ belong to $\mathcal{H}^\infty$.

Then robust stabilization by a stable controller can be formulated as follows:

Problem 2.3. Let Assumptions 2.1 and 2.2 hold. Suppose $\rho > 0$. Determine whether there exists a controller $C \in \mathcal{H}^\infty \cap \mathscr{C}(P)$ satisfying (2). Also, if one exists, find such a controller C.

We call Problem 2.3 *strong and robust stabilization*. Our aim is to provide both a sufficient and a necessary condition for strong and robust stabilization. These conditions give lower and upper bounds on the multiplicative perturbation.

3. Strong and robust stabilization

In this section, we first transform Problem 2.3 to the problem of an interpolation–minimization by a unit element in $\mathcal{H}^\infty$. Next we obtain a sufficient condition as well as a necessary condition for the interpolation–minimization problem using the modified Nevanlinna–Pick interpolation [22].

Lemma 3.1 below is a scalar version of Lemma III.1 of [11]. This result provides a necessary and sufficient condition that a controller strongly stabilizes the plant. The next statement is different from that of Lemma III.1 in [11], but the modification is easy. So we omit the proof.

Lemma 3.1 ([11]). *Suppose $P = N/D$, where $N,\ D \in \mathcal{H}^\infty$ are strongly coprime. Then C strongly stabilizes P if and only if C, $1/(D + NC) \in \mathcal{H}^\infty$.*

The following result shows that Problem 2.3 can be reduced to an interpolation–minimization by a unit element.

Theorem 3.2. *Consider Problem 2.3 under Assumptions 2.1 and 2.2. Problem 2.3 is solvable if and only if there exists a function F such that*

$$F,\ 1/F \in \mathcal{H}^\infty, \tag{3}$$

$$\|W - M_d F\|_\infty \leq \rho, \tag{4}$$

$$F(z_i) = \frac{W(z_i)}{M_d(z_i)}, \quad i = 1, \ldots, n. \tag{5}$$

Furthermore, once such a function F is constructed, the solution of Problem 2.3 is given by

$$C = \frac{W - M_d F}{M_n N_o F}. \tag{6}$$

Proof (*Necessity*). Let C be a solution of Problem 2.3. Define $F := W/(M_d + M_n N_o C)$. Then F satisfies (3) by Lemma 3.1. Since

$$WT = W\left(1 - \frac{M_d F}{W}\right) = W - M_d F, \tag{7}$$

F also achieves the norm constraint (4). In addition,

$$F(z_i) = \frac{W(z_i)}{M_d(z_i) + M_n(z_i) N_o(z_i) C(z_i)} = \frac{W(z_i)}{M_d(z_i)}, \quad i = 1, \ldots, n.$$

Thus F satisfies (3)–(5).

(*Sufficiency*). Suppose F satisfies (3)–(5), and define C by (6).

We show $C \in \mathcal{H}^\infty$ as follows. Since $1/N_o,\ 1/F \in \mathcal{H}^\infty$, it follows from (6) that

$$M_n C = \frac{W - M_d F}{N_o F} \in \mathcal{H}^\infty. \tag{8}$$

Suppose $C \notin \mathcal{H}^\infty$. Then the unstable poles of C must be the zeros of M_n by (8). Let z_i be such a pole. Since the zeros of M_n are simple,