Systems & Control Letters 58 (2009) 547–552

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Locally distributed control for a model of fluid–structure interaction

M. Tucsnak [a,*], M. Vanninathan [b]

[a] *Institut Élie Cartan, Nancy Universités/CNRS/INRIA, BP239, 54506 Vandœuvre-lès-Nancy Cedex, France*

[b] *TIFR-CAM, Post Bag 6503, GKVK Post, Bangalore-560065, India*

ARTICLE INFO

Article history:
Received 27 October 2008
Received in revised form
20 February 2009
Accepted 10 March 2009
Available online 11 April 2009

Keywords:
Fluid structure interaction
Controllability
Stabilizability
Hautus test

ABSTRACT

We consider the equations modeling the coupled vibrations of a fluid–solid system. The control acts in a subset of a domain occupied by the fluid. Our main result asserts that we have exact controllability and exponential stabilizability provided that the support of the control contains a neighborhood of the solid and a neighborhood of the exterior boundary. This improves the existing exact controllability results, which require a control which is active in the whole fluid domain. The proof is based on a frequency domain approach, combined with the use of appropriate multipliers. Moreover, we show that the strong stabilizability property holds for any open control region.

1. Introduction

The aim of this work is to study the the controllability and the stabilizability of a system describing the coupled vibrations of a potential fluid and of a structure immersed in it. The mathematical model used in this work has been introduced in C. Conca, Planchard, Thomas, Vanninathan [1] and Conca, Planchard, Vanninathan [2]. The exact controllability of this system by means of a control acting in the entire fluid domain has been investigated in Raymond and Vanninathan [3]. In order to describe the governing equations we introduce the connected bounded domain $\mathcal{A} \subset \mathbb{R}^2$ and its open subset S, with $\overline{S} \subset \mathcal{A}$, representing the solid. The fluid fills the domain $\Omega = \mathcal{A} \setminus \overline{S}$ and the control is supported in the open set $\mathcal{O} \subset \Omega$. Moreover, we denote $\Gamma = \partial S$, $\Gamma_e = \partial\mathcal{A}$ and $Q = \Omega\times(0,\infty)$, $\Sigma_e = \Gamma_e\times(0,\infty)$, $\Sigma = \Gamma\times(0,\infty)$. With the above notation, the equations modeling the vibrations of the coupled system with locally distributed control u are

$$\begin{cases} \ddot{w} - \Delta w = u\chi_{\mathcal{O}} & \text{in } Q, \\ w = 0 & \text{on } \Sigma_e, \\ \dfrac{\partial w}{\partial n} = \dot{s}\cdot n & \text{on } \Sigma, \\ w(0) = f \quad \text{and} \quad \dot{w}(0) = g & \text{in } \Omega, \\ \ddot{s} + s = -\displaystyle\int_\Gamma \dot{w}\, n \,\mathrm{d}\sigma & \text{in } (0,T), \\ s(0) = l \quad \text{and} \quad \dot{s}(0) = k & \text{in } \mathbb{R}^2. \end{cases} \tag{1.1}$$

In the above system w stands for the velocity potential in the fluid, $s \in \mathbb{R}^2$ denotes the displacement of the solid, n is the unit normal on Γ exterior to Ω and $\mathrm{d}\sigma$ is the line element of Γ. Moreover, $\chi_{\mathcal{O}}$ is the characteristic function of $\mathcal{O}$ and the input function u is extended by zero outside $\mathcal{O}$.

The main result in [3] asserts for that if $\mathcal{O} = \Omega$ then the system (1.1) is exactly controllable in any time, i.e., that for every $\tau > 0$ there exists a $u \in L^2(Q)$ such that

$$w(\cdot,\tau) = \dot{w}(\cdot,\tau) = s(\tau) = \dot{s}(\tau) = 0.$$

This was done by establishing a dual exact observability inequality via the method of multipliers. It is not clear how to use this approach to treat the case when $\mathcal{O}$ is different from Ω.

The main result of the present work asserts that we have the exact controllability of (1.1) under the assumption.

(**H**) There exists $\varepsilon > 0$ such that $\mathcal{O} \supset N_\varepsilon \cap \Omega$, where $N_\varepsilon = \{x \in \mathbb{R}^2 \mid \mathrm{d}(x, \partial\Omega) < \varepsilon\}$.

This will be done by using an infinite dimensional Hautus test. For reader's convenience, a sketch showing different domains in our problem is given in Fig. 1.

We also consider the feedback stabilization of (1.1) by means of collocated actuators and sensors. More precisely, let

$$E(t) = \frac{1}{2}\int_\Omega \left(|\dot{w}(x,t)|^2 + |\nabla w(x,t)|^2\right)\mathrm{d}x + \frac{1}{2}\left(|\dot{s}(t)|^2 + |s(t)|^2\right), \tag{1.2}$$

be the total energy of the fluid–solid system. Taking the derivative of the above formula with respect to t, integrating by parts and

* Corresponding author.
E-mail address: tucsnak@loria.fr (M. Tucsnak).

0167-6911/$ – see front matter © 2009 Elsevier B.V. All rights reserved.
doi:10.1016/j.sysconle.2009.03.002

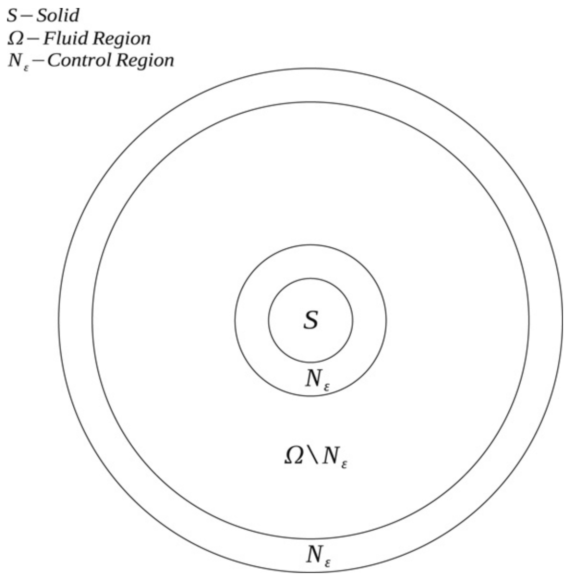

Fig. 1. Simple representation of the fluid–structure system.

using (1.1), we can easily check that

$$\dot{E}(t) = \int_{\mathcal{O}} u(x,t)\dot{w}(x,t)\mathrm{d}x \quad (t \geq 0).$$

Therefore, a simple feedback law for which the energy is decreasing is

$$u(x,t) = -\dot{w}(x,t) \quad (x \in \mathcal{O},\ t \geq 0).$$

We obtain in this way the following closed-loop version of (1.1):

$$\begin{array}{ll}
\ddot{w} - \Delta w = -\dot{w}(x,t)\chi_{\mathcal{O}} & \text{in } Q, \\
w = 0 & \text{on } \Sigma_e, \\
\dfrac{\partial w}{\partial n} = \dot{s}\cdot n & \text{on } \Sigma, \\
w(0) = f \quad \text{and} \quad \dot{w}(0) = g & \text{in } \Omega, \\
\ddot{s} + s = -\displaystyle\int_{\Gamma} \dot{w}\, n\, \mathrm{d}\sigma & \text{in } (0,T), \\
s(0) = l \quad \text{and} \quad \dot{s}(0) = k & \text{in } \mathbb{R}^2.
\end{array} \tag{1.3}$$

We show below that the system (1.3) is strongly stable for any nonempty open $\mathcal{O}$ and that it is exponentially stable if $\mathcal{O}$ satisfies the assumption (H) above.

As a motivation for our work, let us add the following comment: Our results corroborate the underlying hope of designers, namely that waves in a compact domain can be eventually damped out at walls. We are familiar with observing such designs in concert halls, for instance. The exact controllability result of [3] requires an actuation by a volume force present everywhere in fluid domain. Though this actuation results in immediate exact controllability, it is less practical.

2. Notation and preliminaries

In the first part of this section we gather, for easy reference, several essentially known results on exact controllability of general linear systems. We do not give proofs and we refer to the existing literature.

Let X and U be Hilbert spaces, let $A : \mathcal{D}(A) \to X$ be the generator of the C^0 semigroup $\mathbb{T}$ on X and let $B \in \mathcal{L}(U,X)$. We recall that the pair (A,B) is called *exactly controllable* if there exists $\tau > 0$ such that for every $\phi \in X$ there exists $u \in L^2([0,\tau];U)$ such that the solution z of

$$\dot{z}(t) = Az(t) + Bu(t), \qquad z(0) = 0,$$

satisfies $z(\tau) = \phi$. The dual concept of exact controllability is the exact observability. The duality between these two concepts is formalized in the result below, see Dolecki and Russell [4].

Proposition 2.1. *With the above assumptions on A and B, the pair (A,B) is exactly observable if and only if the pair (A^*,B^*) is exactly observable, i.e., if there exist $\tau,\ C_\tau > 0$ such that*

$$C_\tau^2 \int_0^\tau \|B^*\mathbb{T}_t^*\phi\|_U^2 \geq \|\phi\|_X^2 \quad (\phi \in \mathcal{D}(A^*)).$$

In the above proposition and in the remaining part of this work the adjoint of a linear operator L is denoted by L^*.

It has been remarked by several authors that in the case of a skew-adjoint generator (i.e., $A = -A^*$), the exact controllability of (A,B) (or the exact observability of (A^*,B^*)) can be characterized by using an infinite dimensional version of the Hautus test (Zhou and Yamamaoto [5], Liu [6], Miller [7]). We use in this work a version of these results which considers a decomposition of the state space X into "low frequency" and "high frequency" subspaces. More precisely, by combining Proposition 6.6.4 from Tucsnak and Weiss [8] and Proposition 2.1, we have:

Proposition 2.2. *Let $A : \mathcal{D}(A) \to X$ be skew-adjoint and with compact resolvents and let $B \in \mathcal{L}(U,X)$. Let $(\phi_k)_{k\in\mathbb{N}}$ be an orthonormal basis of eigenvectors of A and denote by $\mathrm{i}\mu_k$, with $\mu_k \in \mathbb{R}$ the eigenvalue corresponding to ϕ_k. For any $\lambda > 0$ we denote by E_λ the closure in X of* span $\{\phi_k \mid |\mu_k| > \lambda\}$. *Assume that*

1. *there exist $\delta, \alpha > 0$ such that for all $\omega \in \mathbb{R}$ with $|\omega| > \alpha$,*
$$\|(\mathrm{i}\omega I - A)\phi\|_X^2 + \|B^*\phi\|_U^2 \geq \delta^2\|\phi\|_X^2 \quad (\phi \in E_\alpha \cap \mathcal{D}(A)),$$
2. *$B^*\phi \neq 0$ for every eigenvector ϕ of A.*

Then (A,B) is exactly controllable.

In the second part of this section, we introduce some spaces and operators which are used below to write (1.1) and (1.3) as differential equations in an appropriate Hilbert space.

Let V be the space defined by

$$V = \{\varphi \in H^1(\Omega) \mid \varphi = 0 \text{ on } \Gamma_e\}, \tag{2.1}$$

endowed with the norm

$$\varphi \longmapsto \left(\int_\Omega |\nabla\varphi|^2\mathrm{d}x\right)^{\frac{1}{2}}.$$

The norm in V will be denoted by $\|\cdot\|_V$. The same kind of notation will be used for other Banach spaces. The norm in $\mathbb{C}^2$ will be simply denoted by $|\cdot|$. The inner product of $k,\ l \in \mathbb{C}^2$ is denoted by $k\cdot l$ and it is as usually defined by

$$k\cdot l = k_1\,\overline{l_1} + k_2\,\overline{l_2} \quad (k,l \in \mathbb{C}^2),$$

where $\overline{z}$ stands, as usual, for the complex conjugate of $z \in \mathbb{C}$. The natural state space for the systems (1.1) and (1.3) is

$$X = V \times L^2(\Omega) \times \mathbb{C}^2 \times \mathbb{C}^2. \tag{2.2}$$

The inner product in X will be simply denoted by $\langle\cdot,\cdot\rangle$ and it is defined by

$$\left\langle \begin{pmatrix} f_1 \\ g_1 \\ k_1 \\ l_1 \end{pmatrix}, \begin{pmatrix} f_2 \\ g_2 \\ k_2 \\ l_2 \end{pmatrix} \right\rangle = \langle\nabla f_1, \nabla f_2\rangle_{L^2(\Omega)} + \langle g_1, g_2\rangle_{L^2(\Omega)} + k_1\cdot k_2 + l_1\cdot l_2.$$